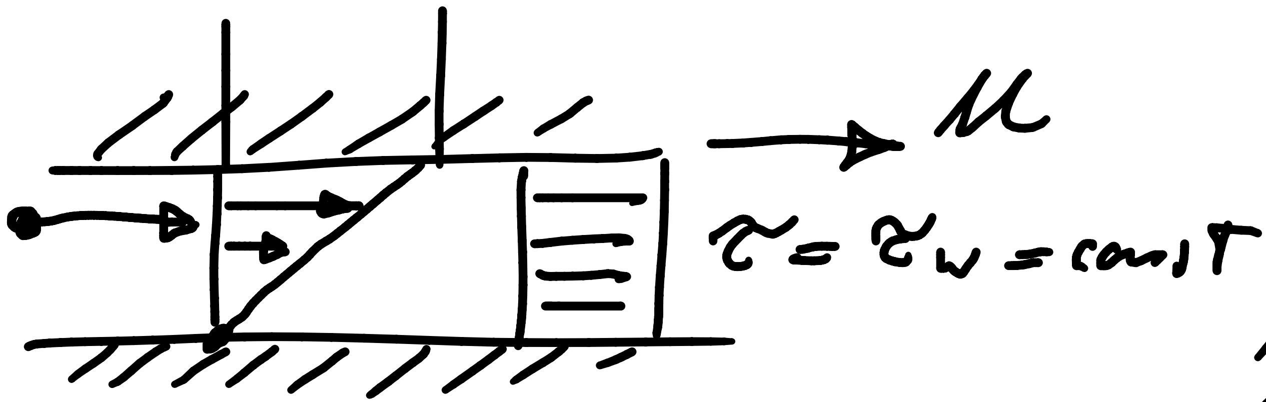
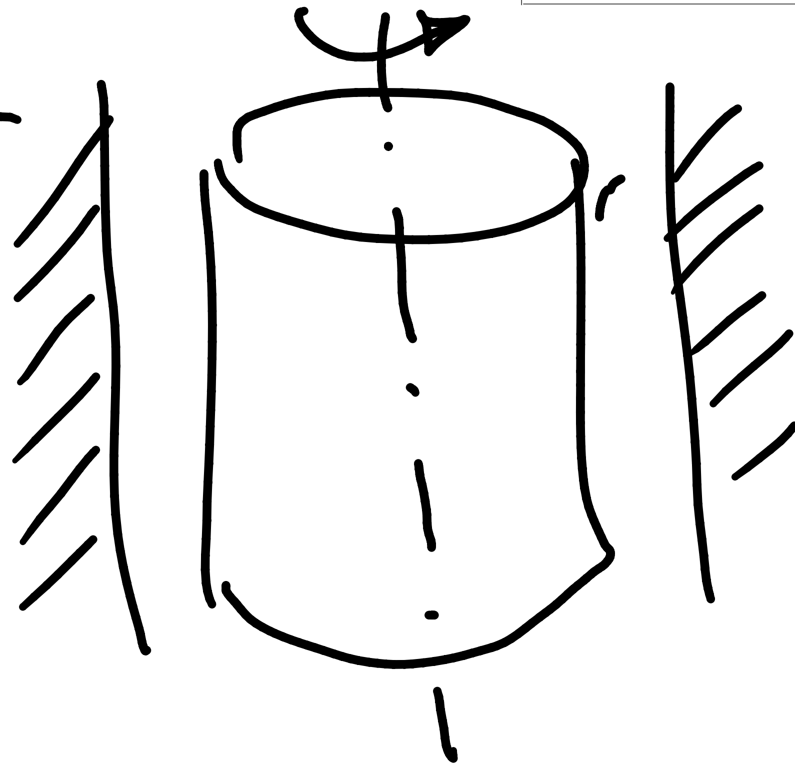
Turbulente Strömungen

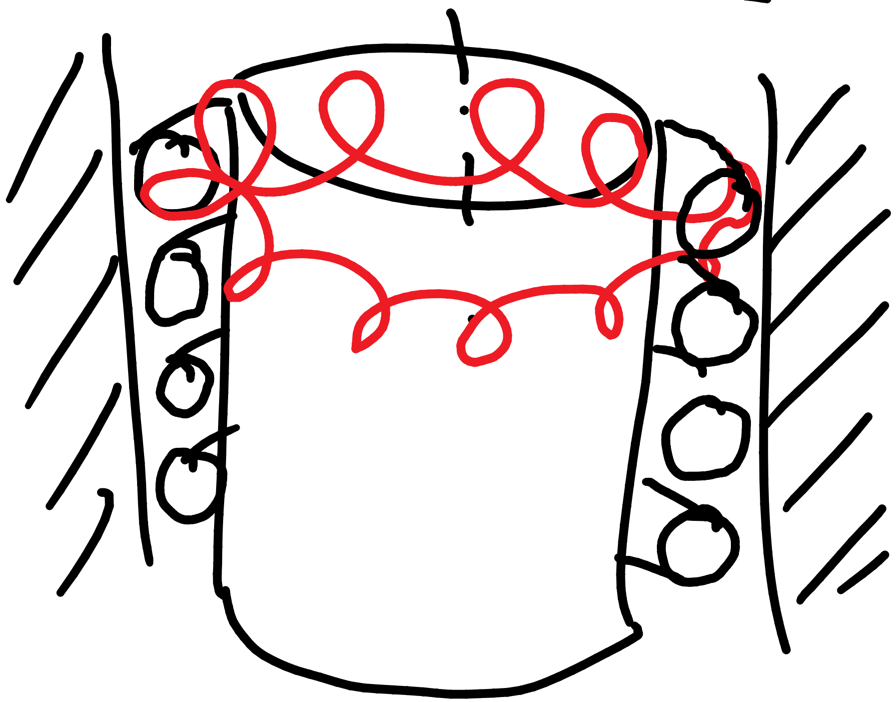


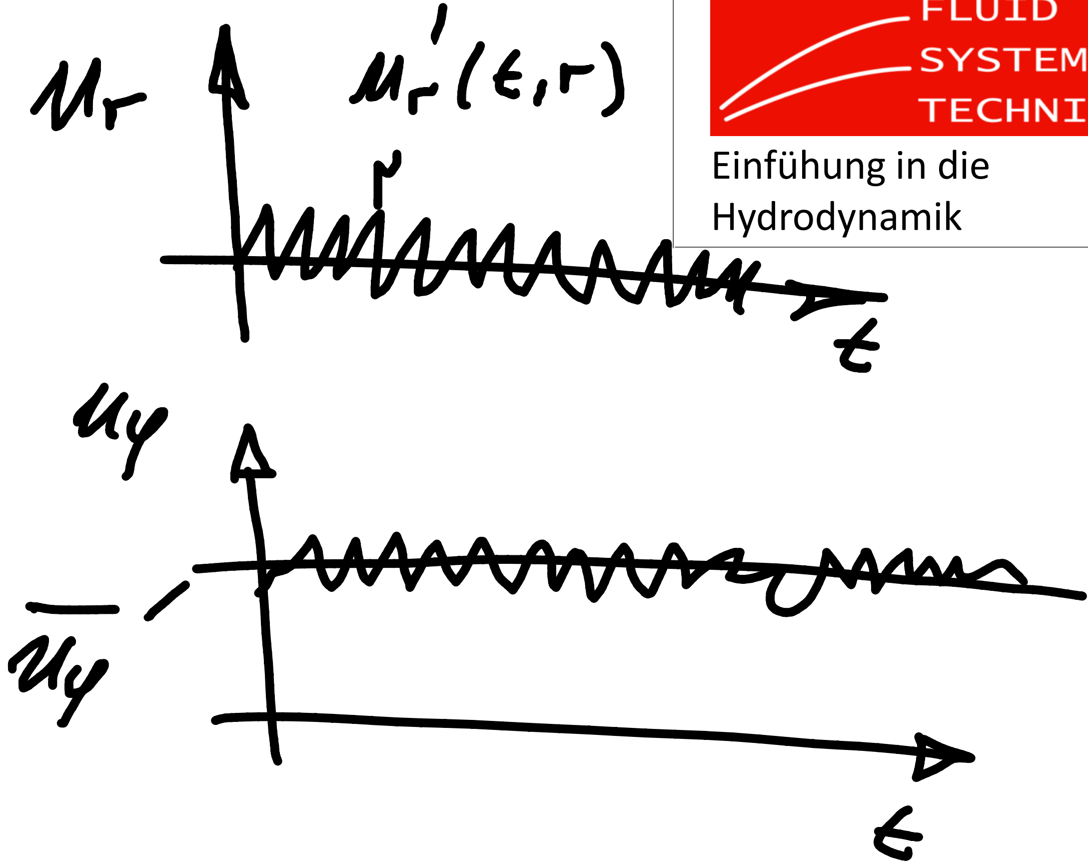
laminar
Geschwindigkeitsprofil
einer Couette-Strömung

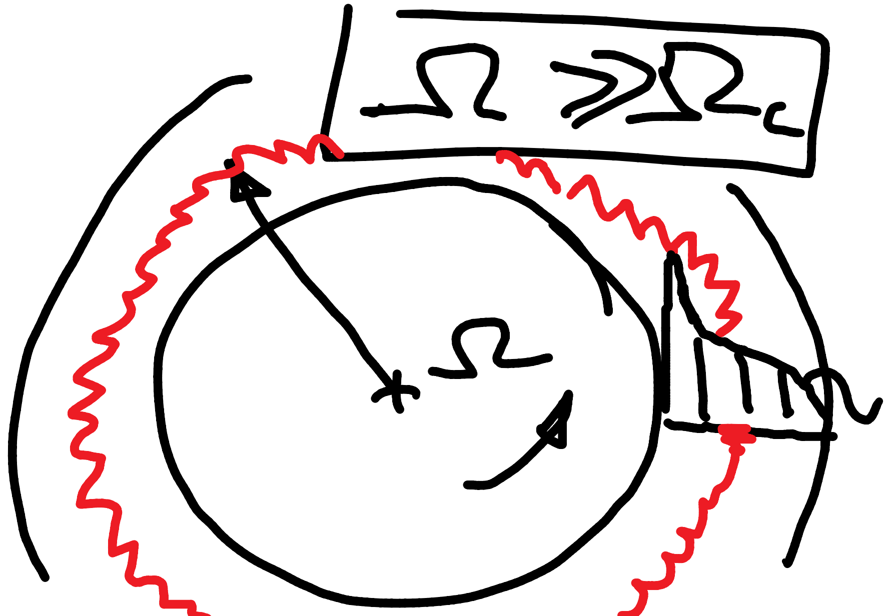
$\Omega < \Omega_c$



$\Omega > \Omega_c$







$$u_\varphi(r,t) = \overline{u_\varphi}(r) + u_\varphi'(r,t)$$

Der Impulsaustausch der Flüssigkeitsteilchen bewirkt eine scheinbare Erhöhung der Viskosität

↳ Wirbelviskosität μ_t

Eddy Viscosity.

In turbulenten Strömungen ist ~~total~~ die Diffusion von

- Rotation ⇝ $\frac{\partial \omega}{\partial t} = \nu \Delta \omega$ ν_T
- Temperatur $\frac{\partial T}{\partial t} = a \Delta T$ a_T
- Stoff $\frac{\partial c}{\partial t} = D \Delta c$ D_T

Bewegungsgleichung für ein Flüssigkeitsteilchen für Newtonsche-Strömungen.

$$\overline{\rho \frac{Du_i}{Dt}} = \overline{\rho k_i} - \overline{\frac{\partial p}{\partial x_i}} + \overline{\eta \frac{\partial u_i}{\partial x_j \partial x_j}} \qquad i = 1, 2, 3$$

$$\overline{\rho \frac{\partial u_i}{\partial t}} + \overline{\rho u_j \frac{\partial u_i}{\partial x_j}} = \dots\dots$$

1.

Ansatz

$$u_i(x_j, t) = \overline{u_i(x_j)} + u_i'(x_j, t)$$

$$\overline{\phi} = \lim_{T \to \infty} \int_t^{t+T} \phi \, dt \, \frac{1}{T}$$

2. zeitliche Mittelung.

$$\overline{\varrho \frac{\partial u_i}{\partial t}} = 0$$ für ein Ströng,

die im zeitlich Mittel stationär ist.

Kontigleichg für eine inkompressible Strög.

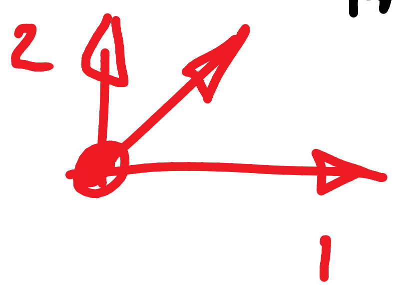
$$\underbrace{\frac{D\varrho}{Dt}}_{\equiv\, 0} + \varrho \frac{\partial u_i}{\partial x_i} = 0 \rightsquigarrow \overline{\frac{\partial u_i}{\partial x_i}} = 0$$

$$\rightarrow \frac{\partial \overline{u}_i}{\partial x_i} = 0$$

$$\frac{\partial u_i'}{\partial x_i} = 0.$$

$$\rho \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = \rho k_i - \frac{\partial \bar{p}}{\partial x_i} + \underbrace{\eta \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j}}_{\text{molekularer Wechselwirkung}} + $$

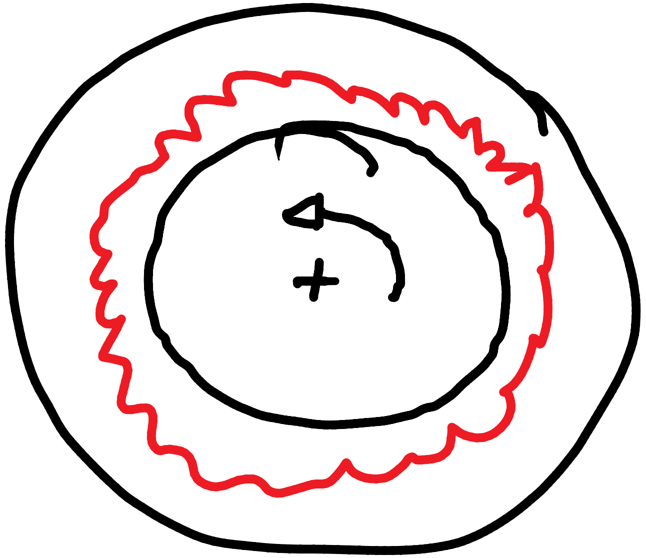
$$- \underbrace{\frac{\partial}{\partial x_j}\left(\overline{\rho u_i' u_j'}\right)}_{\text{Impulsaustausch infolge Turbulenz}}$$



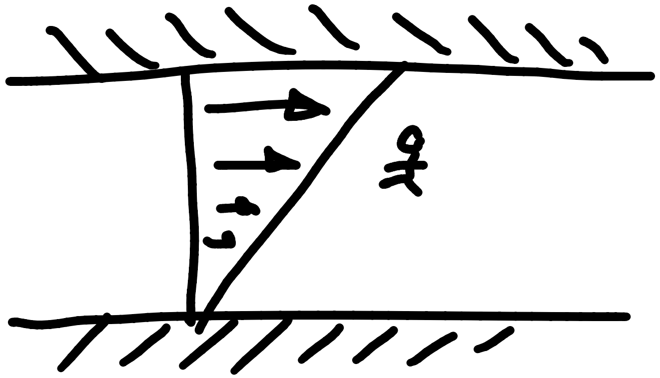
Reynoldsgemittelte Navier-Stokes-Gleichung

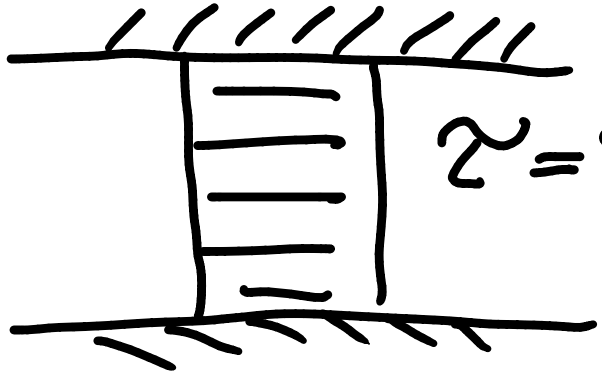
$$\rho \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = \rho k_i - \frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\Bigg[\underbrace{\eta \frac{\partial \bar{u}_i}{\partial x_j} - \overbrace{\rho\, \overline{u_i' u_j'}}^{\text{Modelle für turbulente Scheinspannung.}}}_{\text{Spannungstensor}} \Bigg]$$

Spannungstensor =
viskoser Anteil $\sim \eta$ +
turbulenter Anteil $\sim \rho$

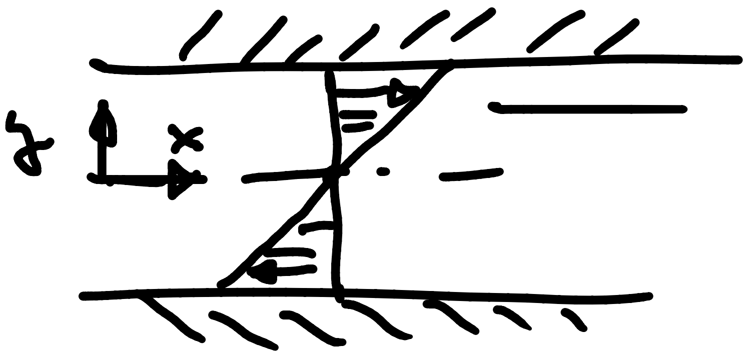


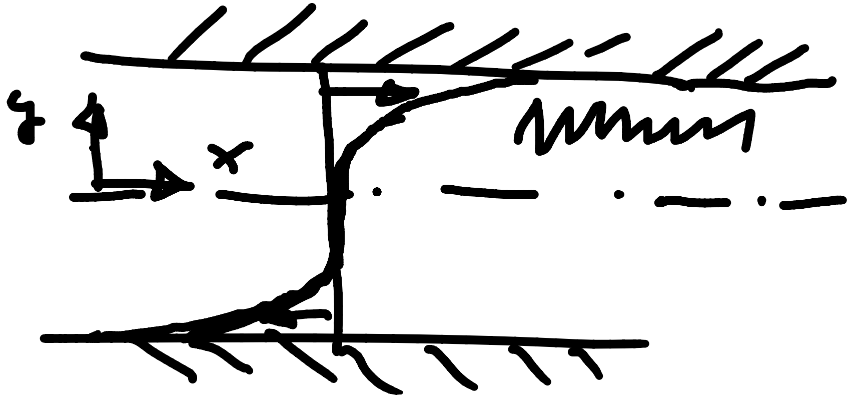
Zur turbulenten Couetteströmung





$\tau = \tau_w = \text{const}$





$u(y)$

laminares Profil

$\bar{u}(y)$

turbulentes Profil

Berühmte Turbulenzforscher:

Ludwig Prandtl ✓

Theodor von Karman ✓

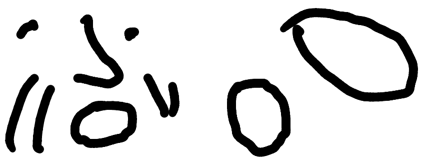
Kolmogorov

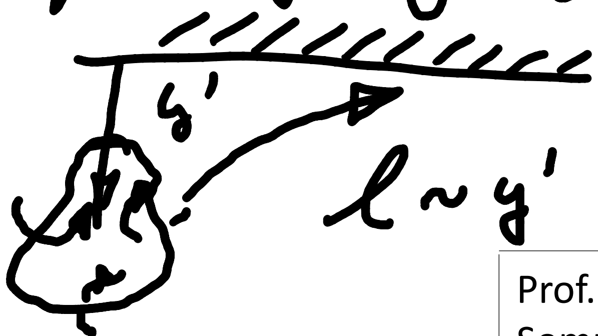
G.I. Taylor.

Prandtlsche Mischungsweg-Ansatz.

in der statistischen Mechanik → freie Länge l

Turbulenzballen = Ringwirbel





$l \sim y'$

turbulente Schubspannungen $\quad u_1' = u'$, $\quad u_2' = v'$

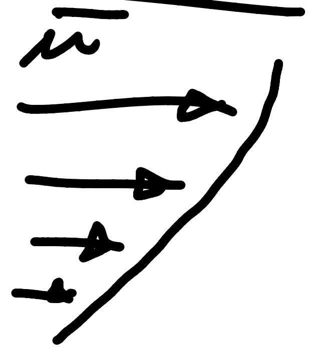
$$\tau_t := -\rho\,\overline{u'v'}$$

Ansatz Modell.

$$= \rho\, l^2 \left(\frac{d\bar{u}}{dy}\right)^2$$

Genauer mit Vorzeichen:

$$\tau_t = \rho\, l^2 \frac{d\bar{u}}{dy}\left|\frac{d\bar{u}}{dy}\right|$$

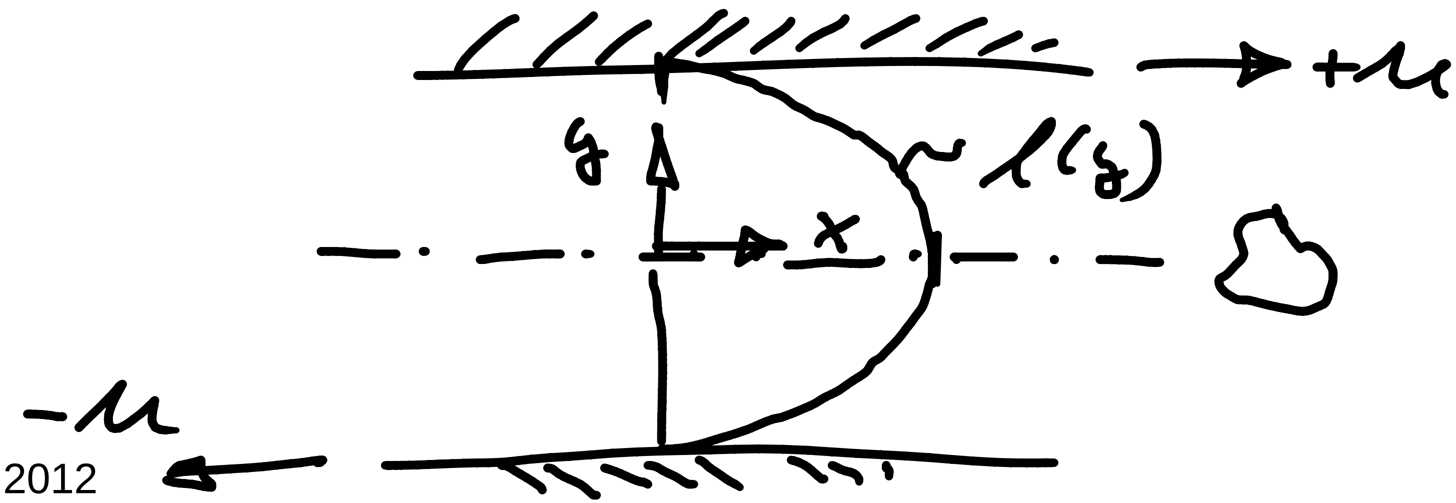


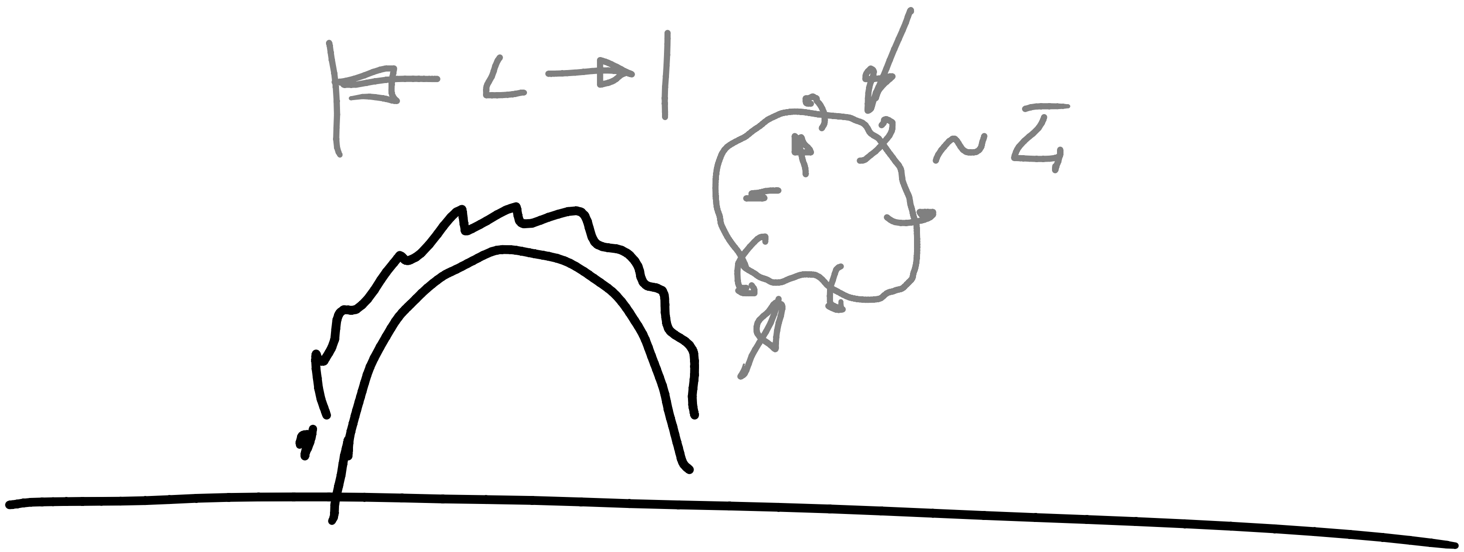
Frage: Wie wird der Mischungsweg gewählt???

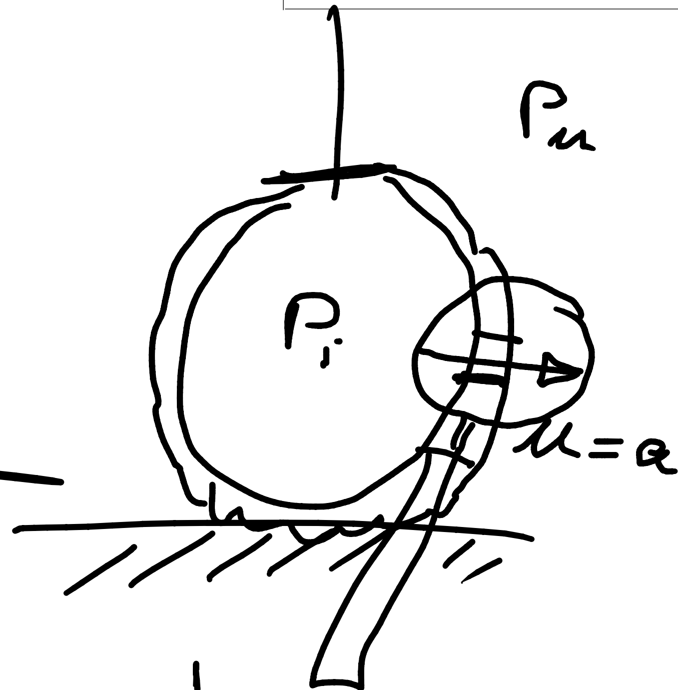
Wirbelviskosität $\frac{\mu_t}{\rho} = l^2 \left| \frac{d\bar{u}}{dy} \right|$

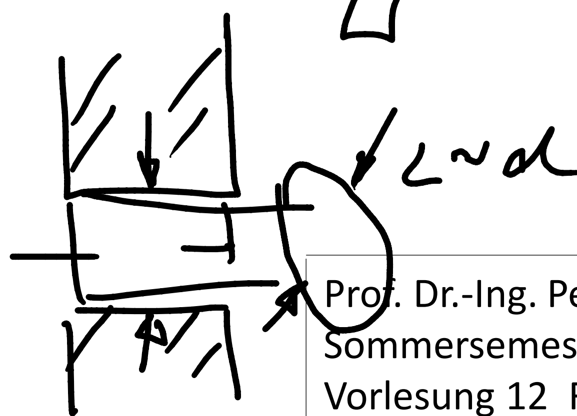
Anwendung am Beispiel der turbulenten Couetteströmung

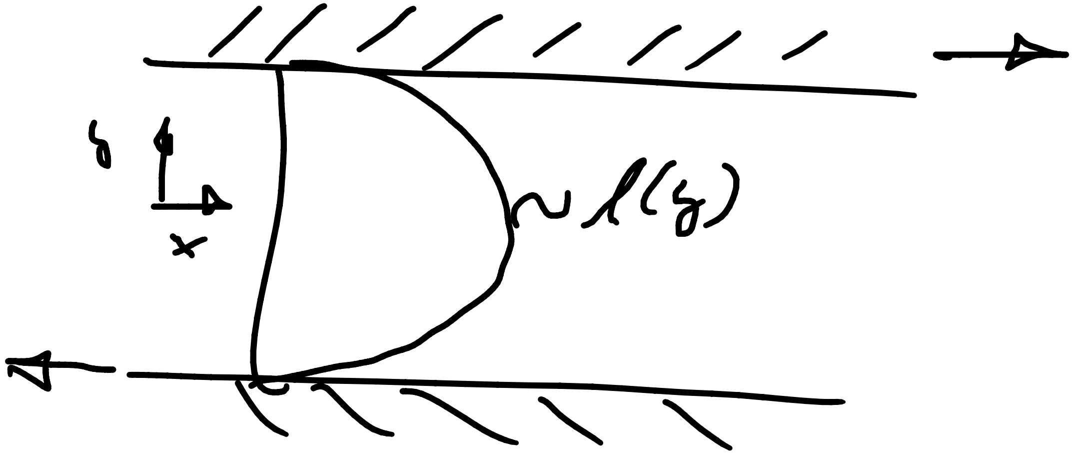
Ansatz für den Mischungsweg $l(y) = K(h^2 - y^2)$

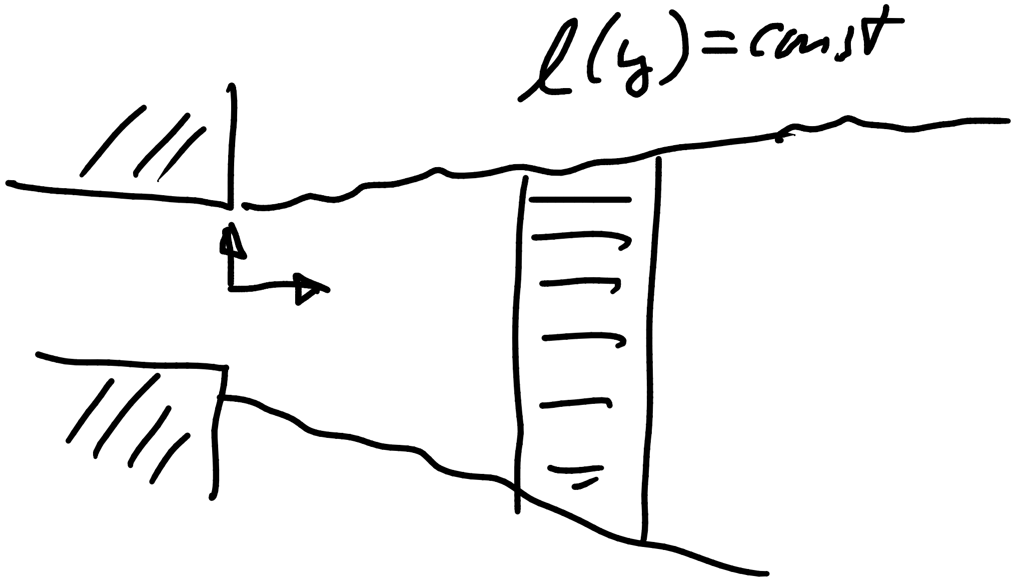


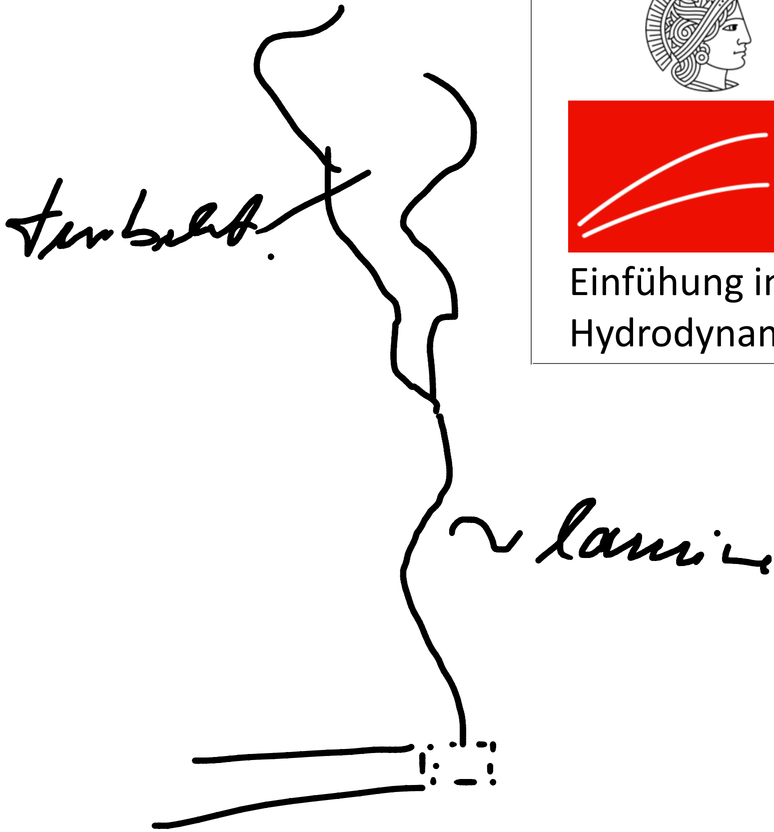





L ~ d

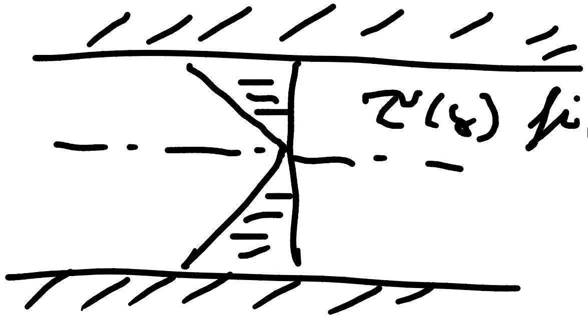

y
x
~ l(y)


l(y) = const


turbulent
~ laminar

Bewegungsgleichung in x-Richtung

$$0 = 0 + \underset{\frac{\partial \bar{p}}{\partial x}}{0} + \frac{d}{dy}\Big[\underbrace{\eta \frac{d\bar{u}}{dy} - \rho \overline{u'v'}}\Big]$$



$\tau(y)$ für $\frac{\partial \bar{p}}{\partial x} \neq 0$

$$\tau = \tau_v + \tau_t = \tau_w = \text{const}$$

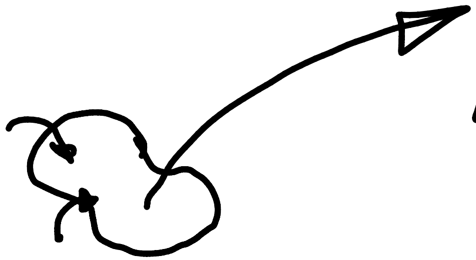
für die turbulente Couetteströmung.

$$\boxed{\eta \frac{d\bar{u}}{dy} - \rho \overline{u'v'} = \tau_w = \text{const}}$$

Definition: τ_w ✓ ρ ✓

Schubspannungsgeschwindigkeit

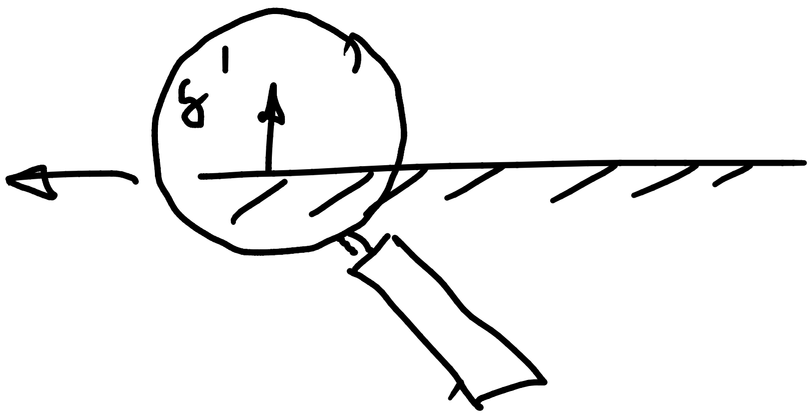
$$u_* := \sqrt{\frac{\tau_w}{\rho}}$$



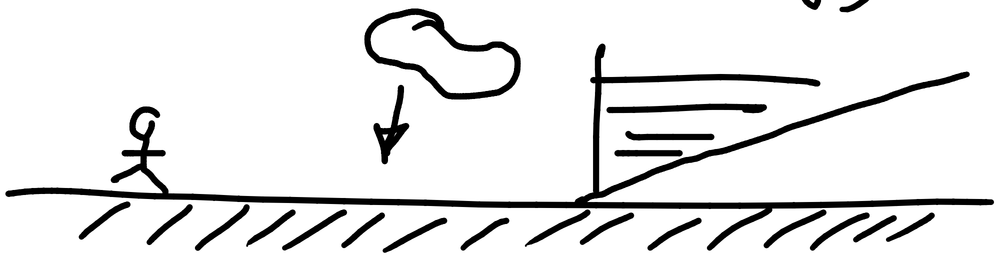
$\overline{|\vec{u}|} \sim u_*$

In der Nähe einer Wand muß die Strömung linear mit dem Wandabstand abfallen

$y' = 2h$



$\ell(y') = \kappa y'$



κ: Karmansche Konstante

$\kappa = 0.4$ universelle Konstante,
die bei allen turbulenten Strömungen
in Wandnähe zu beobachten ist.

↳ Bedingung an den Mindsprung:

$$-\left.\frac{d\ell}{dy}\right|_{y=\pm h} \overset{!}{=} \pm \varkappa$$

↳ $$K = \frac{\varkappa}{2h}.$$

$$\ell = \frac{\varkappa}{2h}\left(h^2 - y^2\right)$$

Einsetzen der Randbedingungen in die Bewegungsgleichung und Integration

$$\boxed{\frac{\bar{u}}{u_*} = \frac{1}{\kappa} \ln\left(\frac{h+y}{h-y}\right)}$$