

$$U = U_{\max} \frac{(8x_0 - x_1)}{6x_0}$$

stationärer Drallsatz

$$\int_S (\vec{r} \times \rho \vec{u}) \vec{u} \cdot \vec{n} dS = \vec{M}_{R \rightarrow F}$$

$$\vec{r} \times \rho \vec{u} = (x_1 \vec{e}_1) \times (\rho U \vec{e}_2)$$
$$= -\rho x_1 U \vec{e}_3$$

$$\vec{M}_{R \rightarrow F} = b \int_{2x_0}^{8x_0} -\rho x_1 U^2 dx_1 \vec{e}_3$$

$$\vec{M}_{F \rightarrow R} = b \rho \int_{2x_0}^{8x_0} x_1 \frac{U_{\max}^2}{(6x_0)^2} (8x_0 - x_1)^2 dx_1 \vec{e}_3$$
$$= \frac{b \rho U_{\max}^2}{36 x_0^2} \int_{2x_0}^{8x_0} x_1 (8x_0 - x_1)^2 dx_1 \vec{e}_3$$





$$= \rho \frac{U_{max}^2}{36 x_0^2} b \int_{2x_0}^{8x_0} 64 x_0^2 x_1 - 16 x_0 x_1^2 + x_1^3 dx_1$$

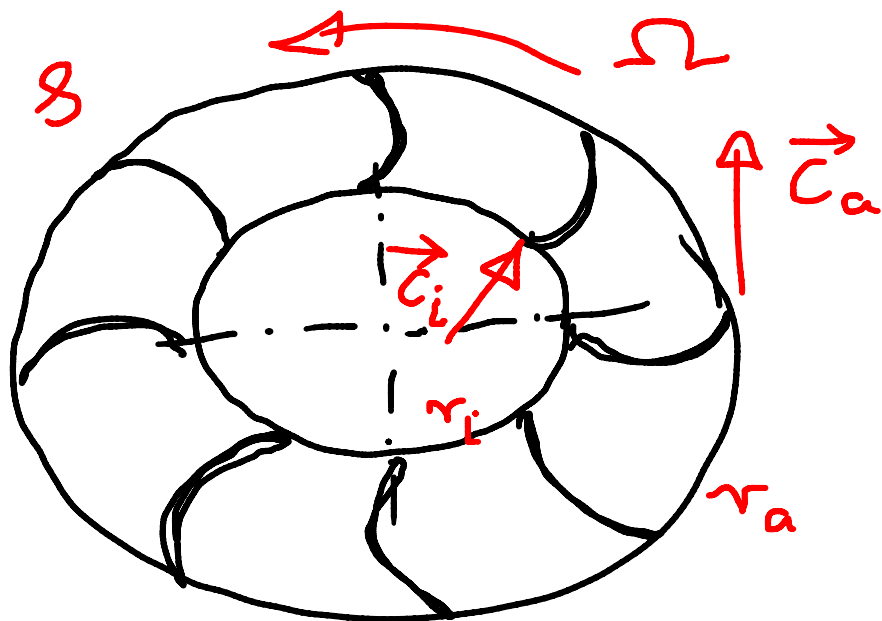
$$= \rho \frac{U_{max}^2}{36 x_0^2} b \left[64 x_0^2 \frac{x_1^2}{2} - \frac{16}{3} x_0 x_1^3 + \frac{x_1^4}{4} \right]_{2x_0}^{8x_0} =$$

$$\underline{\underline{7 \rho b U_{max}^2 x_0^2}}$$

$$\underline{\underline{Q = \int_S \vec{u} \cdot \vec{n} dS = b \int_{2x_0}^{8x_0} \frac{U_{max}}{6x_0} (8x_0 - x_1) dx_1}}$$

$$= \frac{U_{max}}{2} \cdot b \cdot 6x_0 = 3bx_0 U_{max}$$

$$\underline{\underline{U_{max} = \frac{Q}{3bx_0}}}$$



$$\int_S \vec{r} \times s \vec{c} \underbrace{\vec{c} \cdot \vec{n}}_{dQ} dS$$

$$\vec{e}_r \times \vec{e}_\theta = \vec{e}_z$$

$$r_i \vec{e}_r \times s (c_{ir} \vec{e}_r + c_{i\theta} \vec{e}_\theta) = s r_i c_{i\theta} \vec{e}_z$$

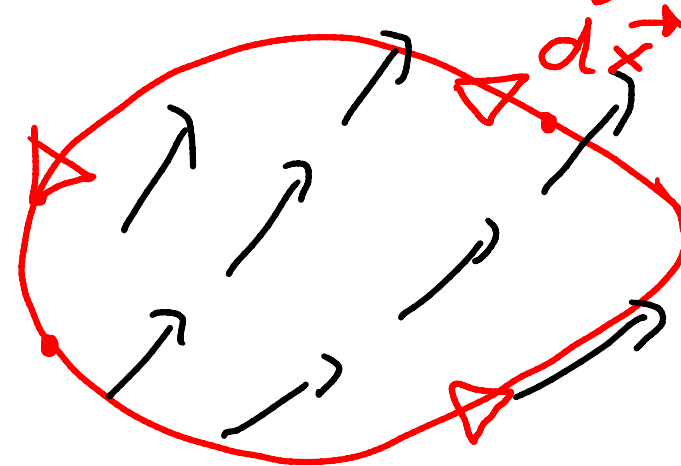
$$Q = 2\pi r_i c_{i\theta}$$

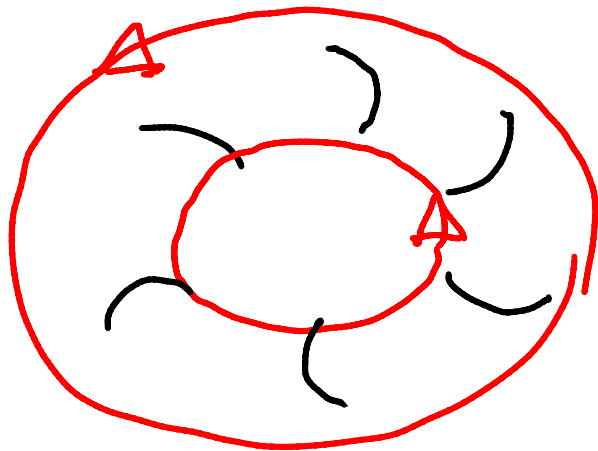


$$M_z = \rho r_a c_{na} (+Q) + \rho r_i c_{ni} (-Q)$$
$$= \dot{m} (r_a c_{na} - r_i c_{ni})$$

Eulersche Turbinen-
gleichung

$$\Gamma = \oint \vec{C} \cdot d\vec{x}$$





$$T_i = \oint \vec{c}_i \cdot d\vec{x}$$

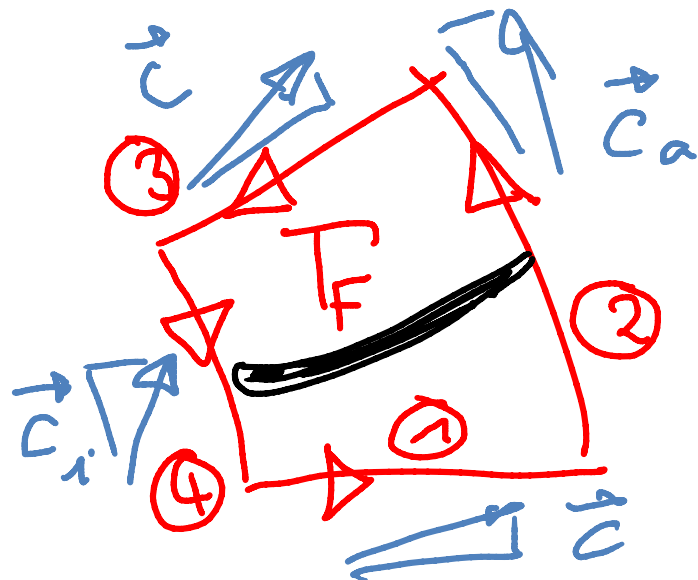
$$= c_{mi} 2\pi r_i$$

$$T_a = \oint \vec{c}_a \cdot d\vec{x} =$$

$$c_{ma} 2\pi r_a$$

$$M_z = m \left(r_a c_{ma} - r_i c_{mi} \right)$$

$$= \frac{m}{2\pi} [T_a - T_i]$$



$$\vec{T}_F = \int_{r_i}^{r_a} c_r dr$$

$$+ \int_0^{2\pi/n} c_{na} r d\varphi$$

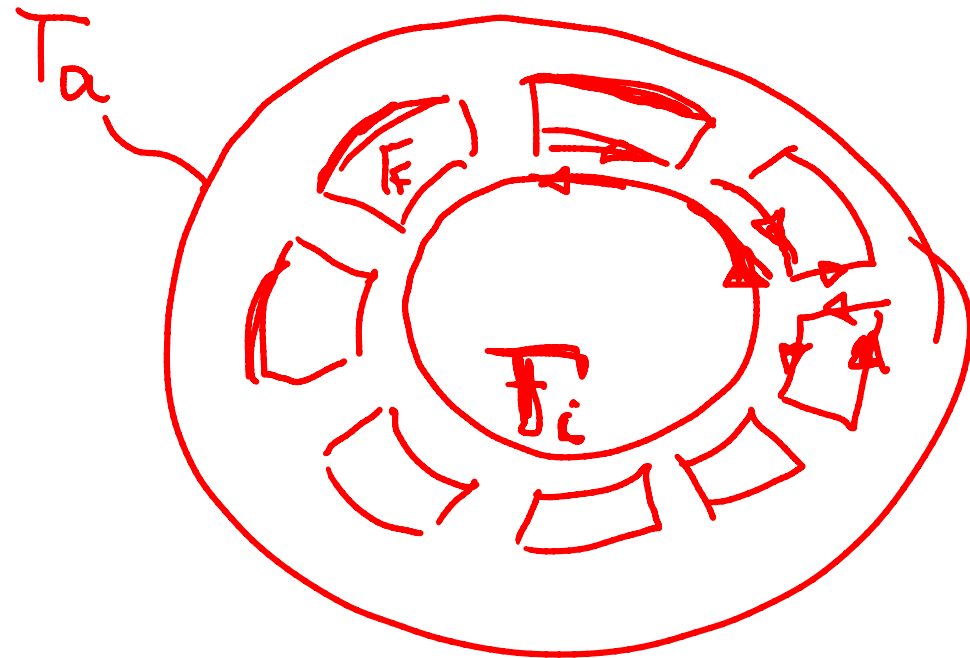
$$+ \int_{r_i}^{r_a} c_r dr$$

$$+ \int_0^{2\pi/n} c_{ni} r d\varphi =$$

$$\int_7^1 1 dx = [x]_7^1 = 1 - 7 = -6$$

$$\vec{T}_F = \frac{2\pi}{n} (c_{na} r_a - c_{ni} r_i)$$

$$T_a = T_i + n T_F$$



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Einführung in die
Hydrodynamik
Vorrechenübung