

1) 05.07 Serie VL

⇒ Klausurvorbereitungsvorlesung

13³⁰ — Ende der Inhalte



TECHNISCHE
UNIVERSITÄT
DARMSTADT



Prof. Dr. Ing. Peter Pelz
Sommersemester 2011
Einführung in die
Hydrodynamik
Vorrechenübung 8

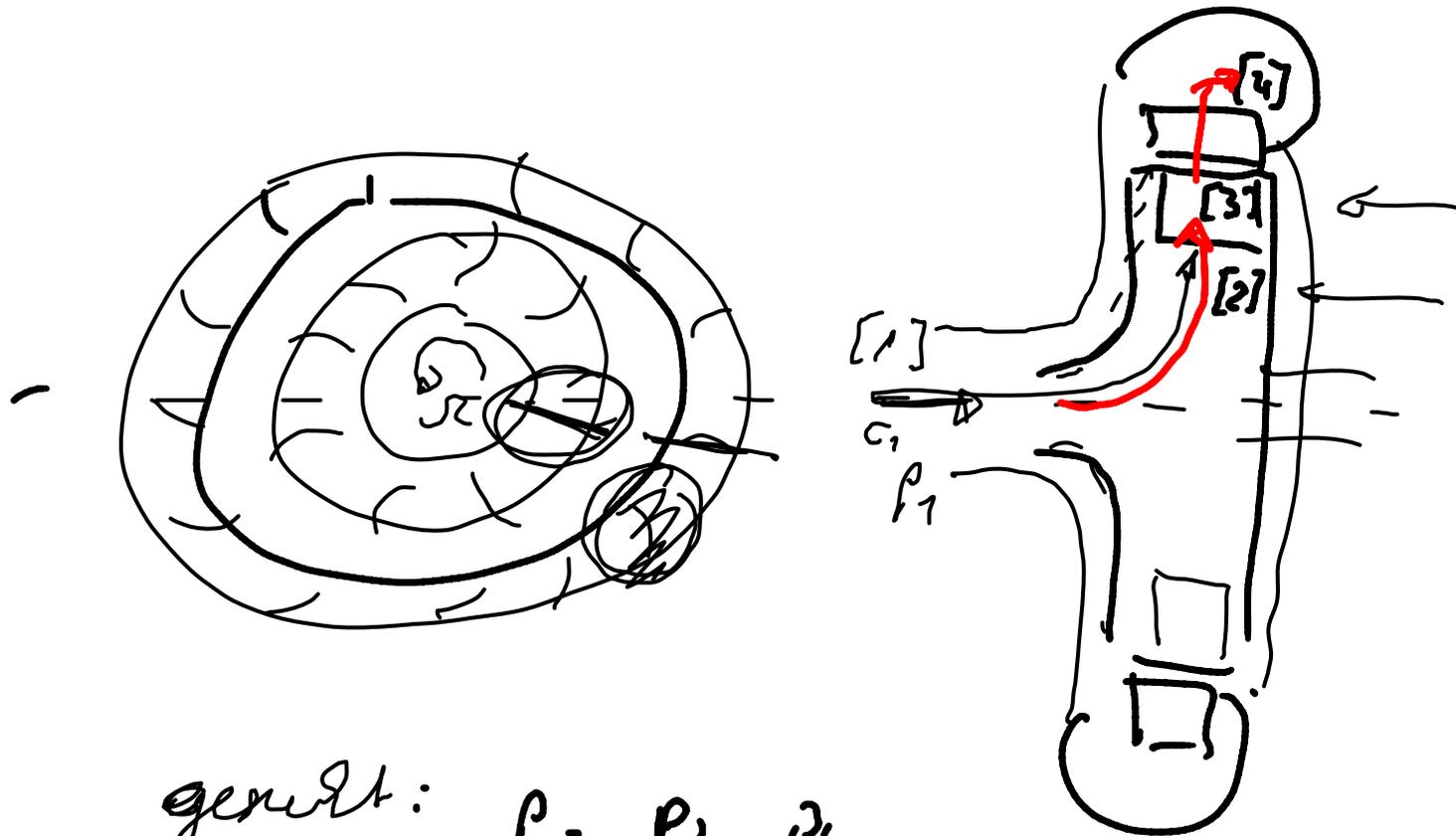


1	19.04. / 24.05	17.05
Potential der Schwerkraft	$\psi = \rho g z$	$\psi = \rho g z$ $= \psi'$
Potential der Zentrifugalkraft	$\psi = -\frac{\rho}{2} r^2 \Omega^2$	$\rightarrow \psi$ $\frac{1}{2} \Omega^2 r^2 \rightarrow \psi'$ $\rightarrow \psi$

Bernoulli $\rho + \frac{\rho}{2} u^2 + \psi = \text{const}$
 $\hookrightarrow \rho g z + \frac{\rho}{2} r^2 \Omega^2$ 81



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gesucht: p_2, p_3, p_4

gegeben $c_1, c_2, c_3, c_4, c_{u3}, R, p_1, \rho$
 R_i

[1] → [2] Bernoulli

$$p_1 + \frac{\rho}{2} c_1^2 = p_2 + \frac{\rho}{2} c_2^2$$

$$\Rightarrow p_2 = p_1 + \frac{\rho}{2} (c_1^2 - c_2^2)$$

$$p_2 + \frac{\rho}{2} \underline{w_2^2} - \frac{\rho}{2} \underline{\Omega^2 R_2^2} = p_3 + \frac{\rho}{2} \underline{w_3^2} - \frac{\rho}{2} \underline{\Omega^2 R_3^2}$$

$$\Rightarrow p_3 = p_2 + \frac{\rho}{2} (\underline{w_2^2} - \underline{w_3^2}) + \frac{\rho}{2} \Omega^2 (R_3^2 - R_2^2)$$

$w_2, w_3 = \dot{r} \Rightarrow$ Geschwindigkeits-
dreiecke

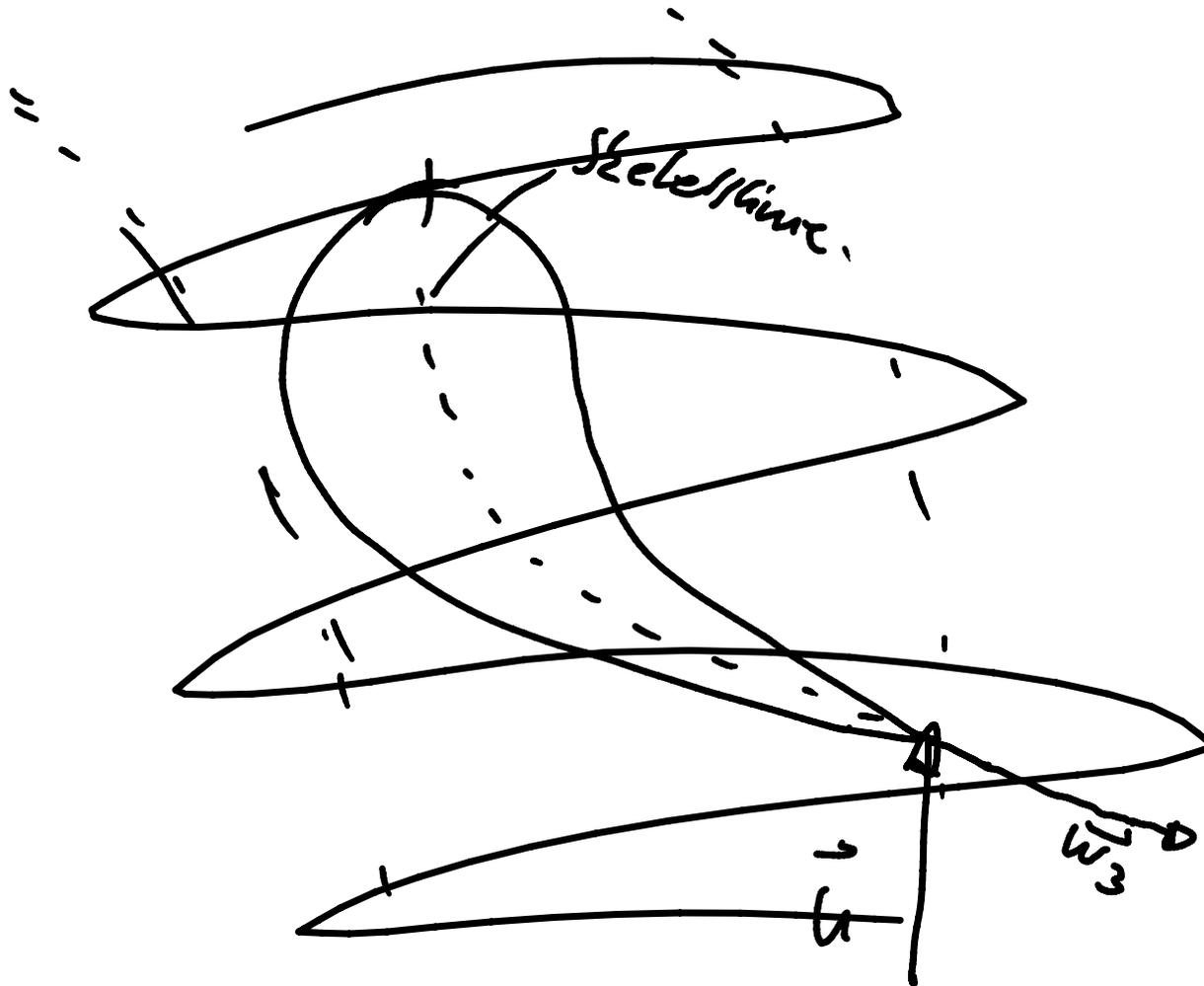


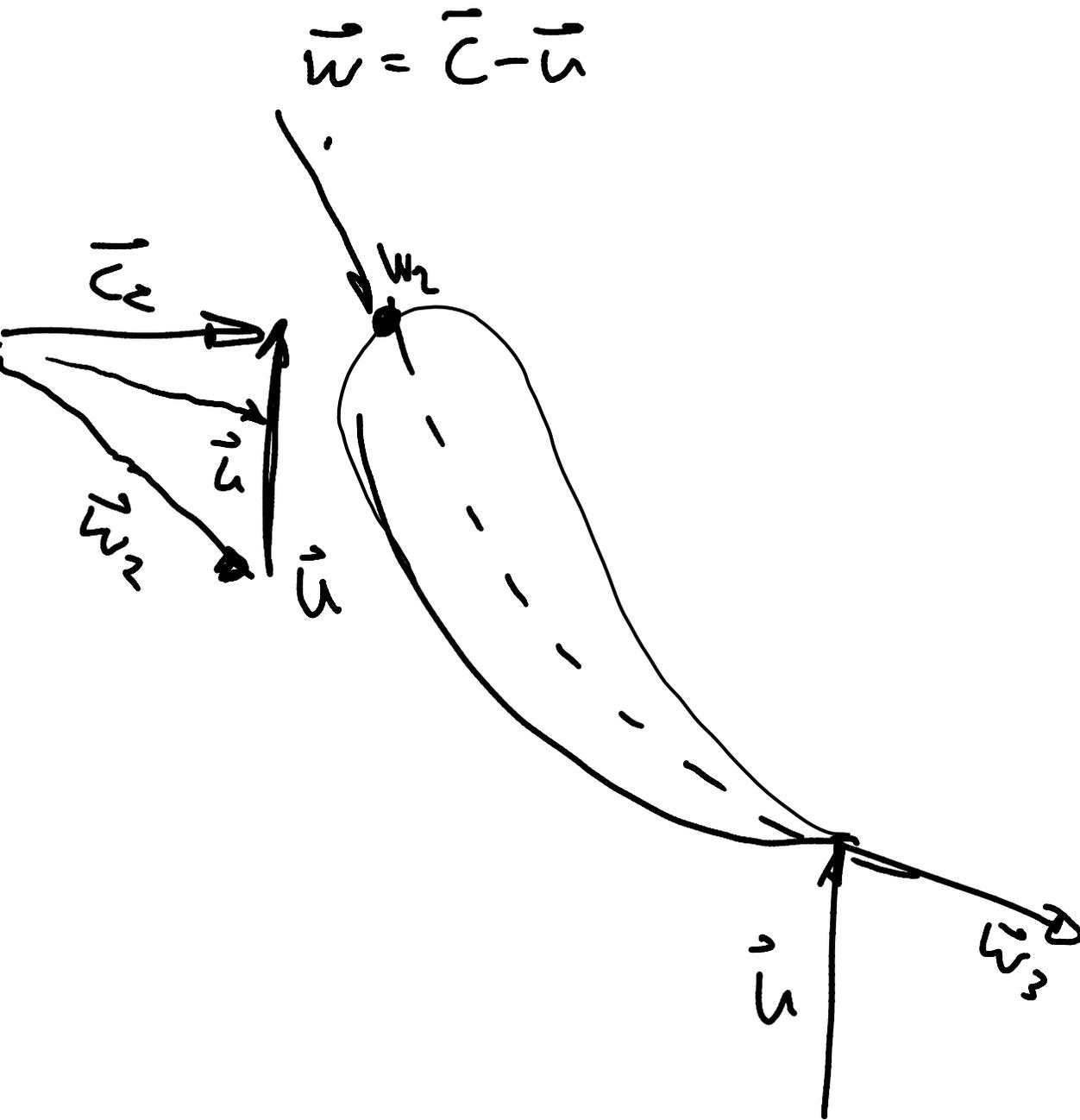
$$\vec{c} = \vec{\omega} + \vec{u}$$

$$\Rightarrow \vec{\omega} = \vec{c} - \vec{u}$$

$$\vec{u} = \vec{\Omega} \times \vec{x}$$

$$= \Omega R \vec{e}_\varphi$$

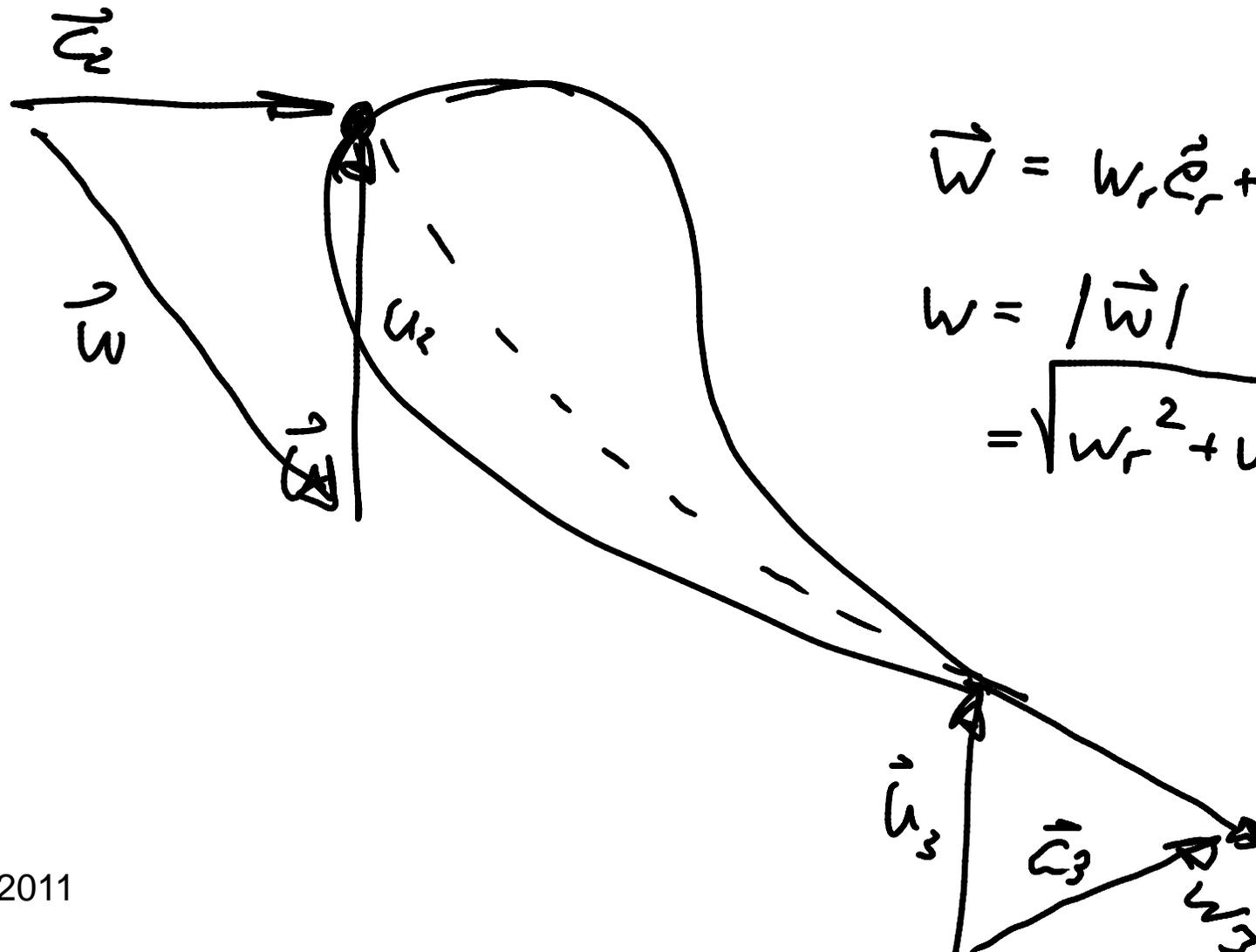




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$$\vec{c} = \vec{w} + \vec{u}$$
$$\Rightarrow \vec{w} = \vec{c} - \vec{u}$$



$$\vec{w} = w_r \vec{e}_r + w_u \vec{e}_\varphi$$
$$w = |\vec{w}|$$
$$= \sqrt{w_r^2 + w_u^2}$$



$$\vec{w} = \vec{c} - \vec{u}$$

$$\begin{pmatrix} w_r \\ w_u \end{pmatrix} = \begin{pmatrix} c_r \\ c_u \end{pmatrix} - \begin{pmatrix} 0 \\ \mathcal{R}R \end{pmatrix} \Rightarrow \begin{aligned} w_r &= c_r \\ w_u &= c_u - \mathcal{R}R \end{aligned}$$

$$\begin{aligned} |\vec{w}| = w &= \sqrt{w_r^2 + w_u^2} \\ &= \sqrt{c_r^2 + (c_u - \mathcal{R}R)^2} \end{aligned}$$

$$w^2 = \underbrace{c_r^2 + c_u^2}_{c^2} - 2c_u \mathcal{R}R + \mathcal{R}^2$$

$$\Rightarrow w^2 = c^2 - 2c_u \mathcal{R}R + \mathcal{R}^2$$

$$[2]: c_{u2} = 0 \Rightarrow w_2^2 = c_2^2 + \Omega^2 R_2^2$$

$$[3]: c_{u3} \checkmark \Rightarrow w_3^2 = c_3^2 - 2c_{u3} \Omega R_3 + \Omega^2 R_3^2$$

\Rightarrow Einsetzen in Bernoulli:

$$P_3 = P_1 + \frac{\rho}{2} (c_1^2 - c_3^2 + 2c_{u3} \Omega R_3)$$



[3] → [4]

$$p_3 + \frac{\rho}{2} c_3^2 = p_4 + \frac{\rho}{2} c_4^2$$

$$\Rightarrow p_4 = \dots$$



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