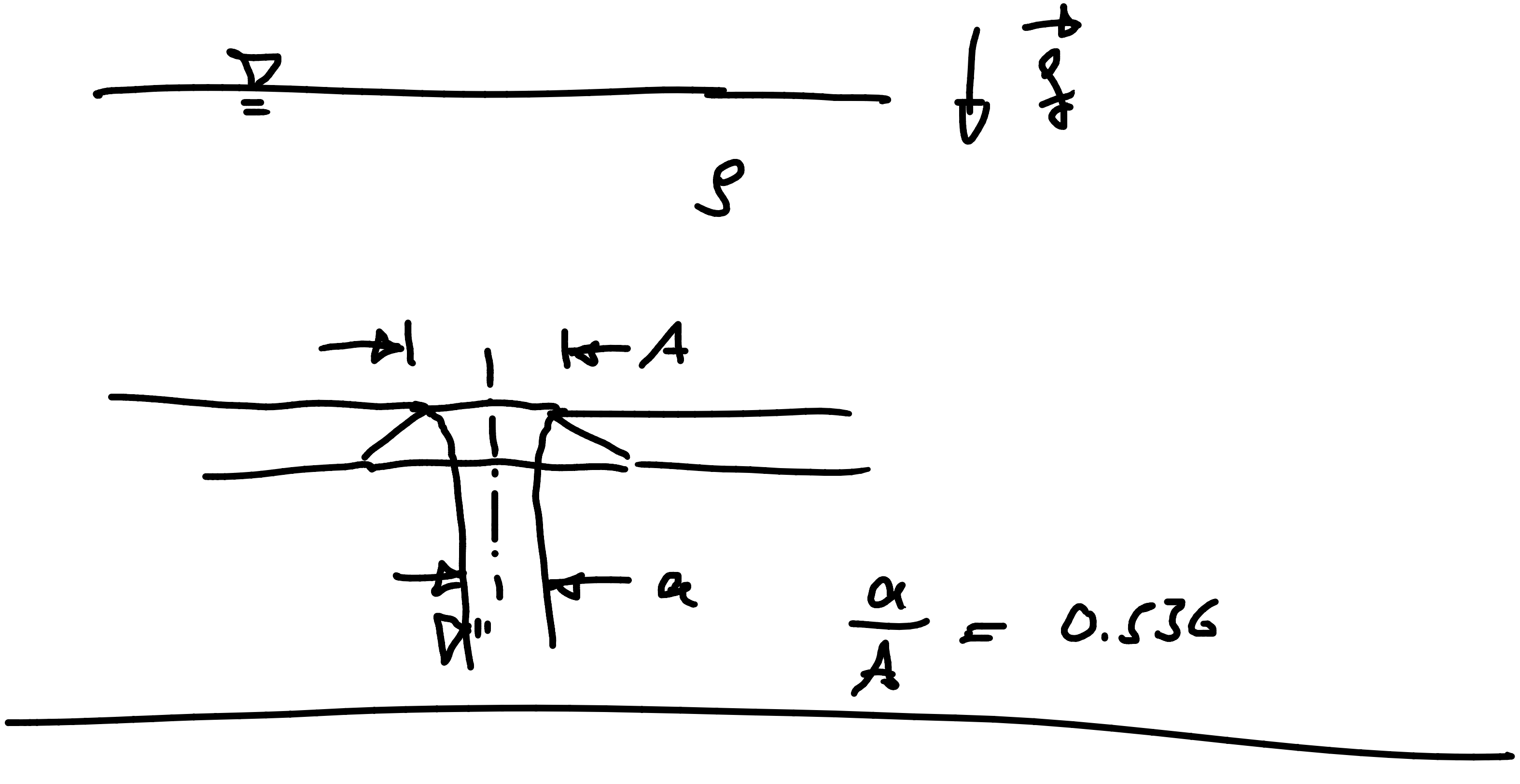
Laminare Schichtenströmung



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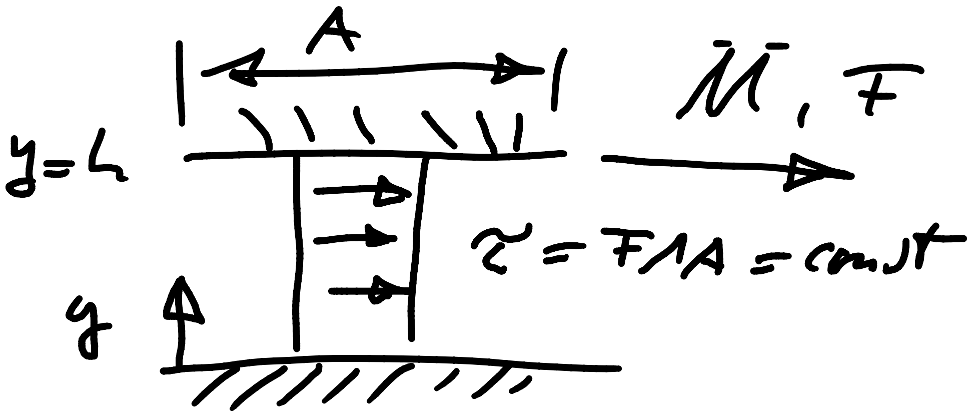
Schichtenströmungen



Schleppströmung od. Couetteströmung

$$\tau = \eta \frac{du}{dy}$$

$$\frac{\tau_w}{\eta} y + C_1 = u(y)$$

$$u(y=0) \leadsto C_1 \equiv 0$$

$$u(y) = \frac{\tau_w}{\eta} y = \frac{\bar{U}}{h} y$$

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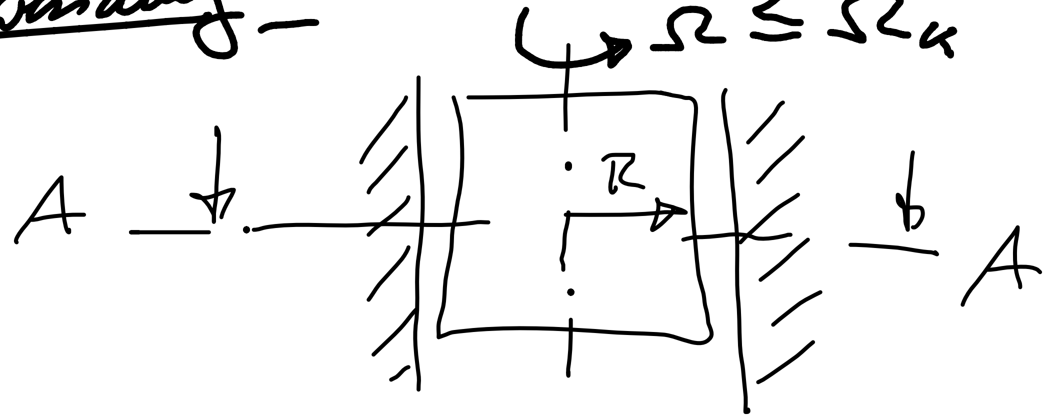
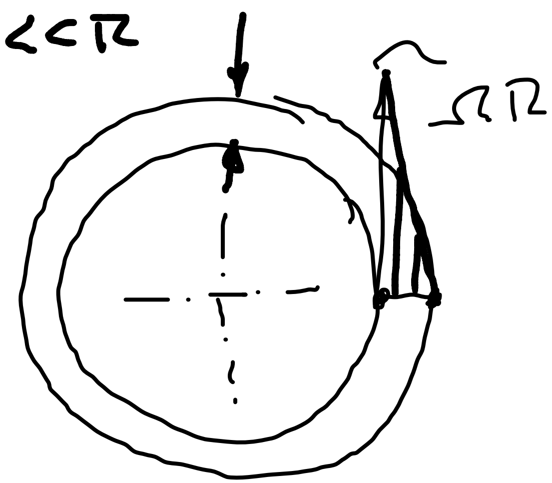
Anwendung:

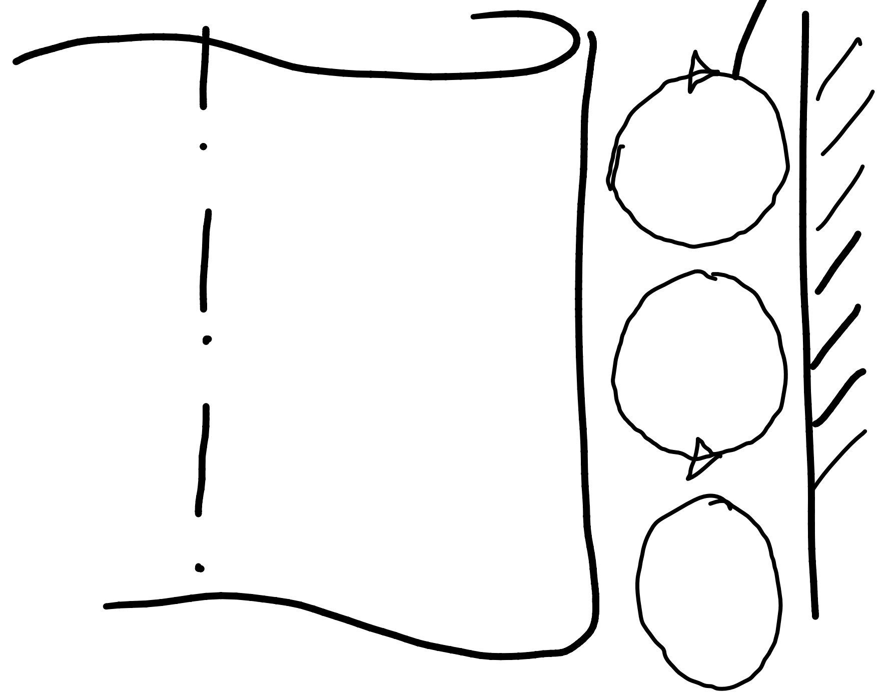




$\Omega \approx \Omega_k$ kritische Drehzahl.

Stabilitätsproblem

Taylor-Wirbel



→ keine Schichtströmung

$\Omega \gg \Omega_k$ → Turbulenz.

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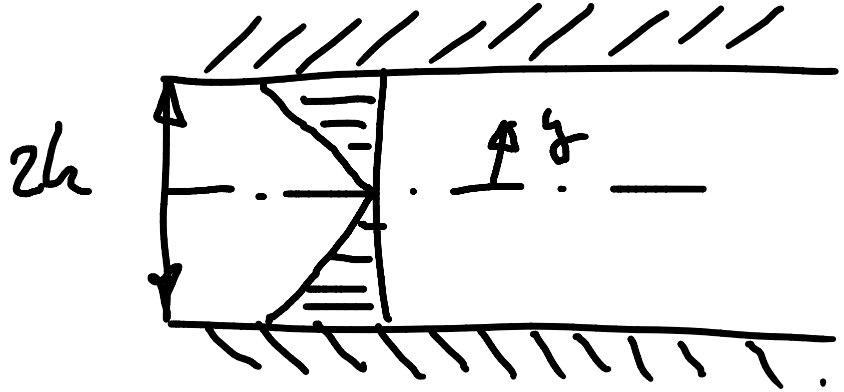
Poiseuille-Strömung



$$\frac{\partial p}{\partial x} = \frac{d}{dy}\tau$$

$$\tau = -\tau_w \frac{y}{h}$$

$$\tau = \eta \frac{du}{dy}$$

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u}{\partial y^2} \quad \text{für} \quad \frac{\partial u}{\partial t} = 0$$

x-Komponente der Bewegungsgleichung für ein Flüssigkeitsteilchen einer Newtonschen Flüssigkeit.

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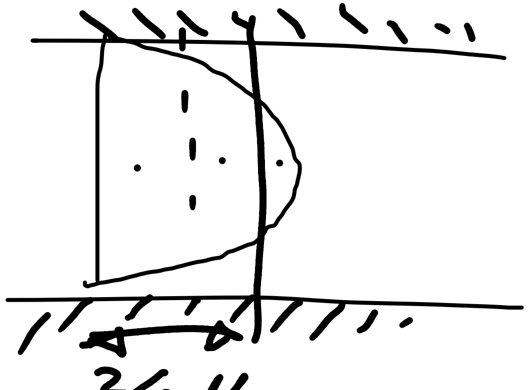
$$u(y) = \frac{K(2h)^2}{2\eta}\left(1 - \frac{y}{2h}\right)\left(\frac{y}{2h}\right).$$

$$\bar{u} = \frac{1}{2h}\int u(y)\,dy = \frac{K(2h)^2}{12\eta}$$

$$K := -\frac{dp}{dx}.$$

$$u_{max} = \frac{3}{2}\bar{u}$$



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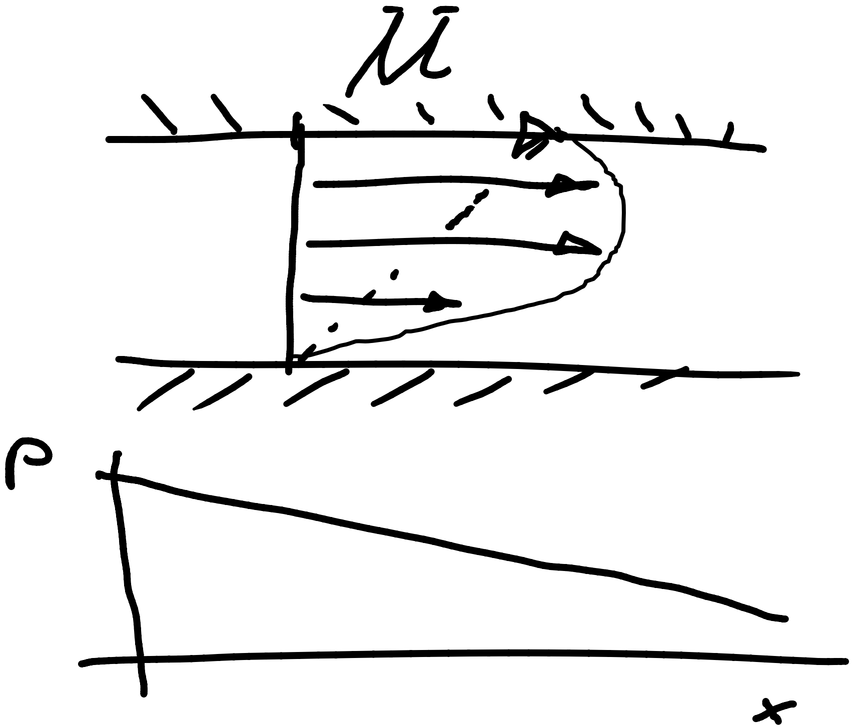
Überlagerung von Druck- und Schleppströmung



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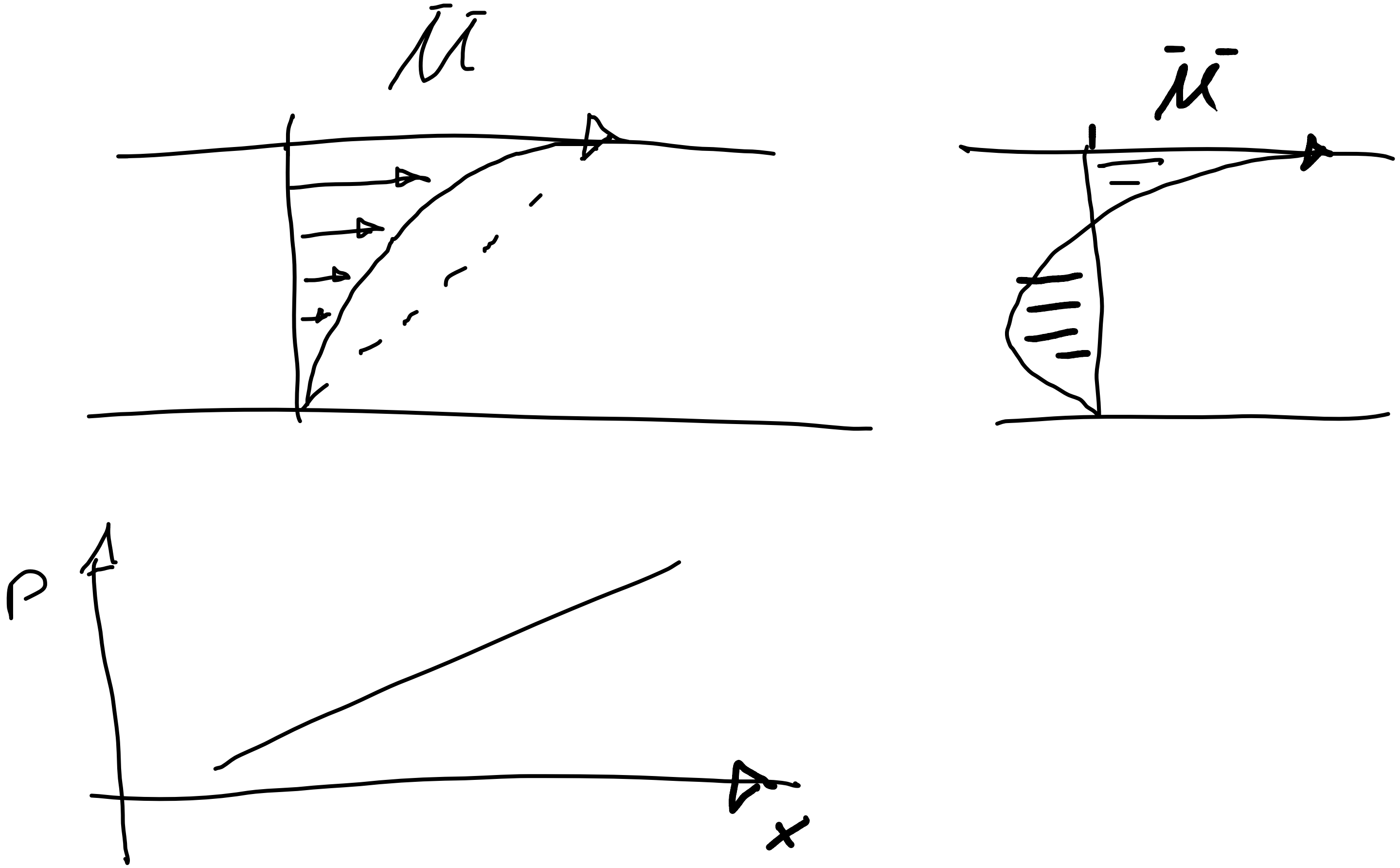

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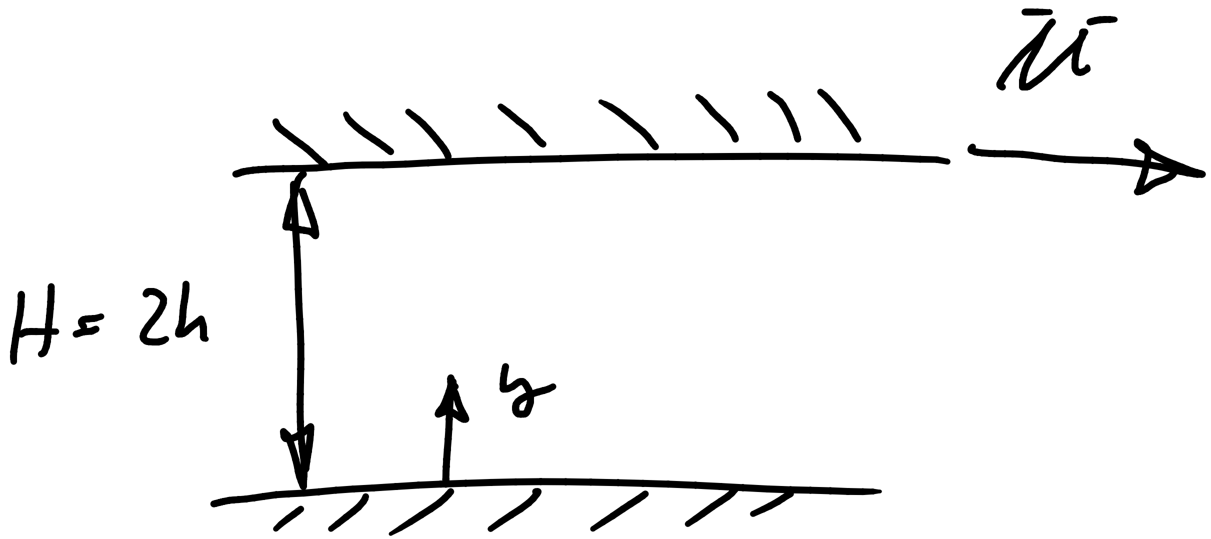
$$u(y) = \frac{y}{H}\bar{U} + \frac{K H^2}{2\eta}\left(1 - \frac{y}{H}\right)\frac{y}{H}$$



$$\boxed{\bar{u} = \frac{1}{2}U + \frac{K H^2}{12\eta}}$$

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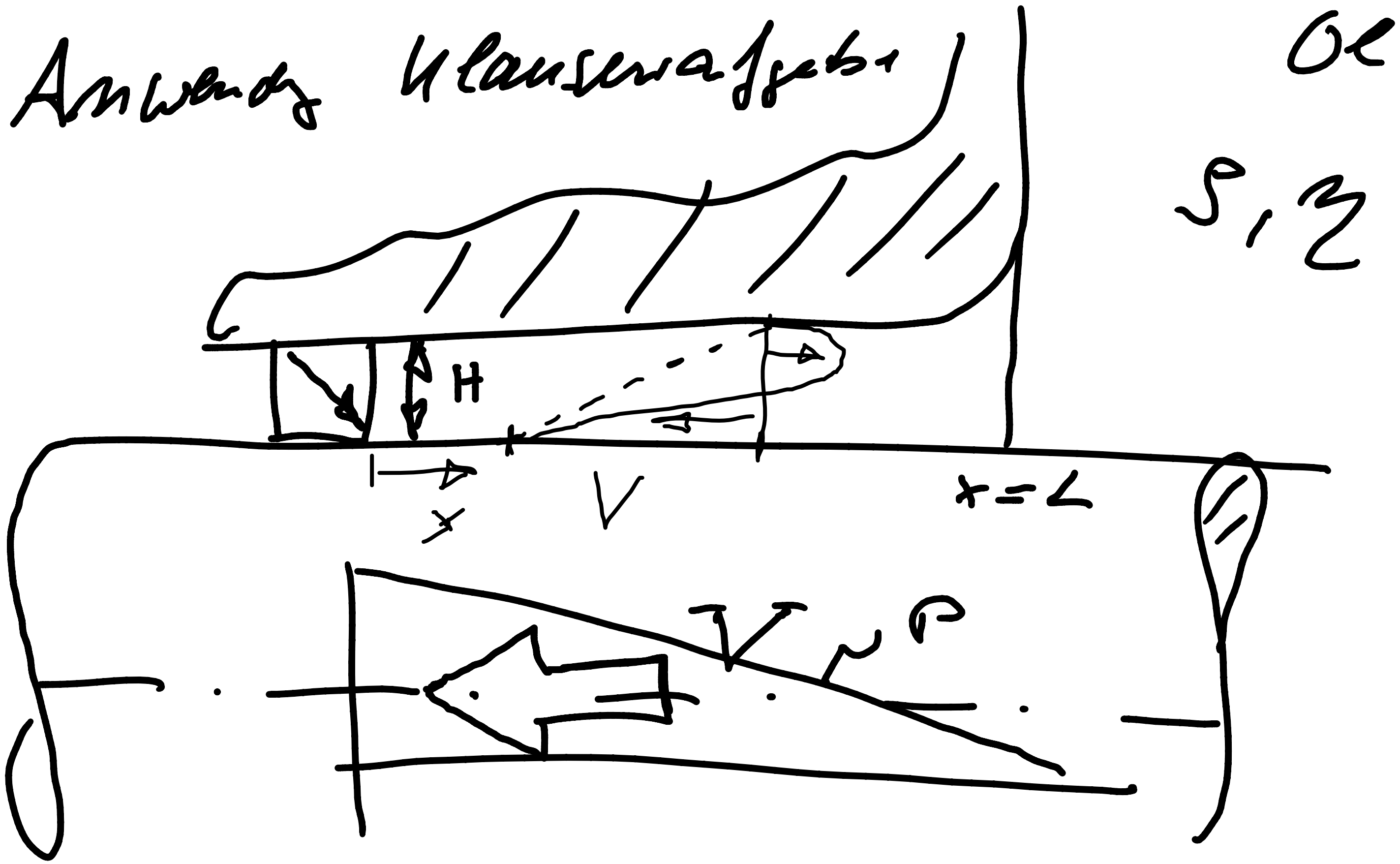
Anwendung Klausuraufgabe



$\bar{u} \equiv 0$ im leckagefreien Fall.

$\leadsto K = -\frac{dp}{dx} = -\frac{V}{2} \ldots\ldots$

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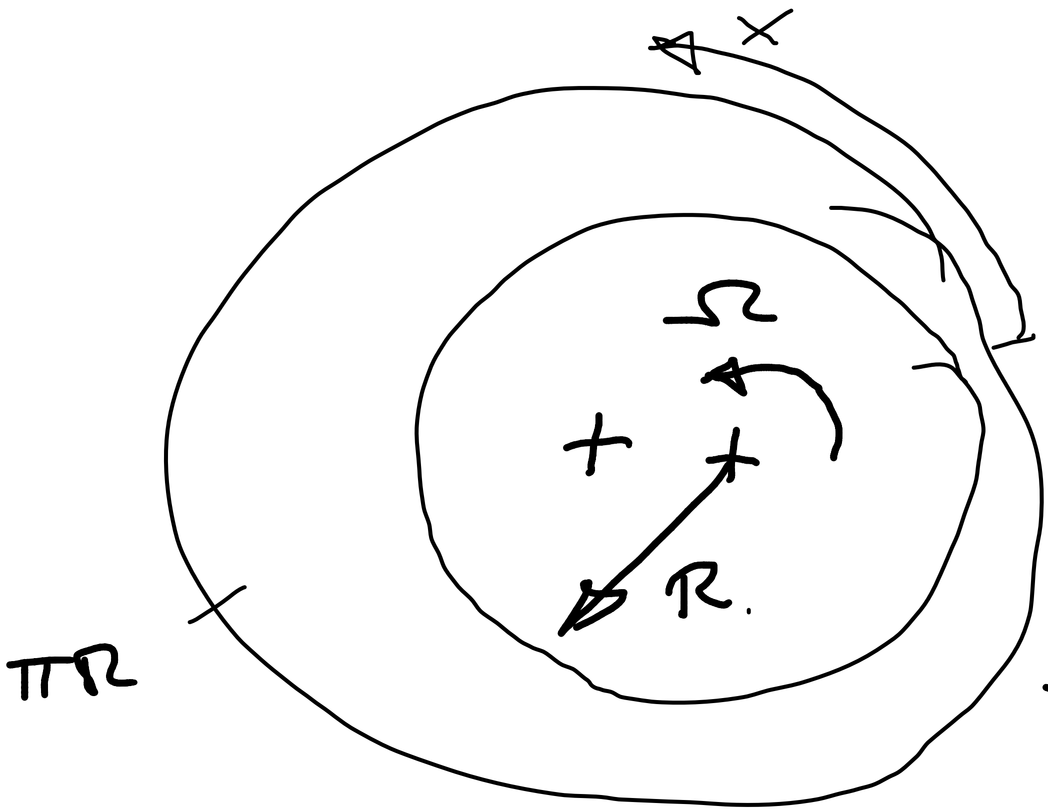
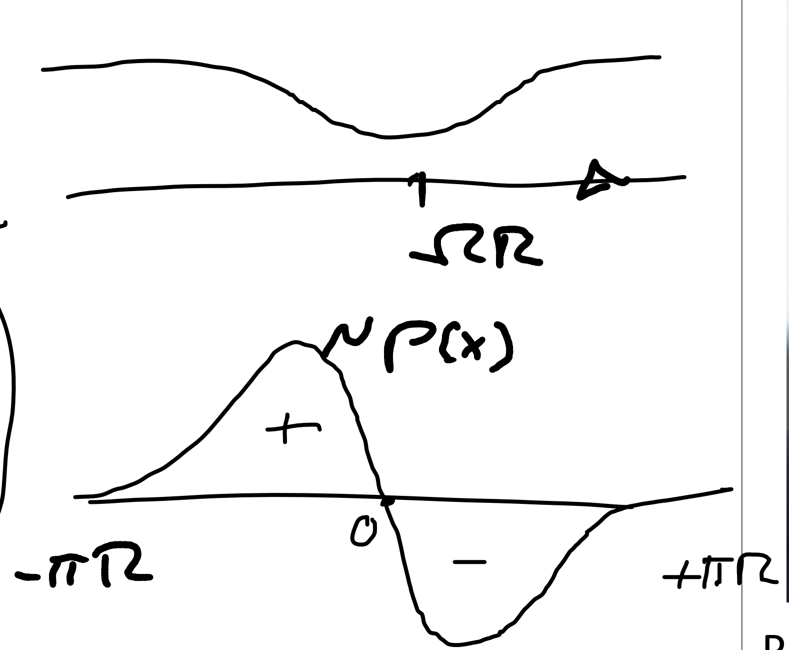
Technisch wichtig:

Hydrodynamische Lager

$\alpha Re \ll 1; \; \alpha = \frac{dh}{dx}$





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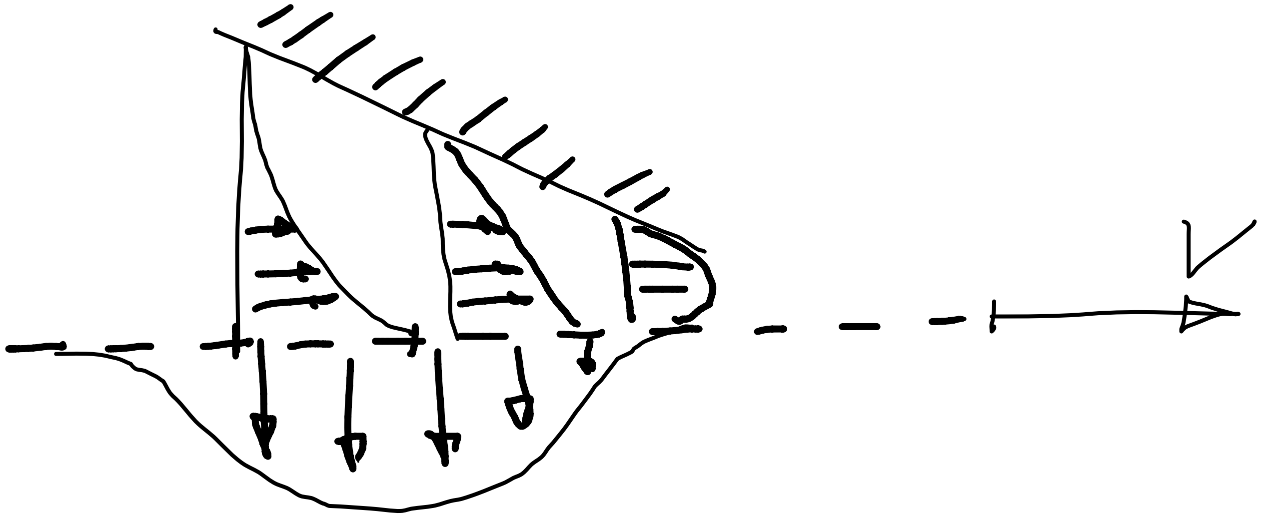

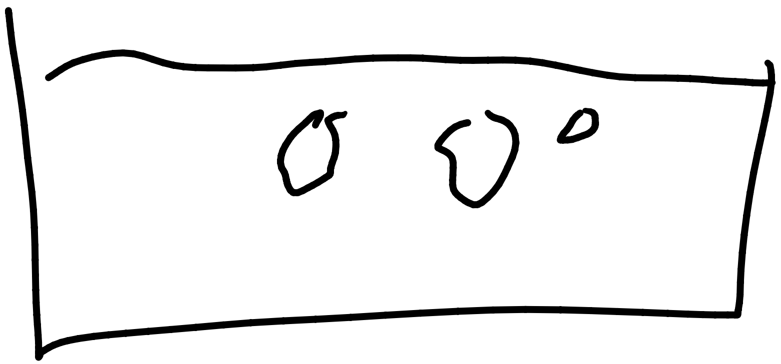

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Strömung durch ein Kreisrohr

$$\vec{u} = u_z \vec{e}_z + u_\varphi \vec{e}_\varphi + u_r \vec{e}_r \quad \rightsquigarrow \quad u_z(r) = ?$$

$u_\varphi \equiv 0$ für eine drallfreie Strömung.

$u_r \equiv 0$ bei undurchlässigen Wänden für die laminare Strömung.

$u_r \neq 0$ bei turbulenter Strömung.

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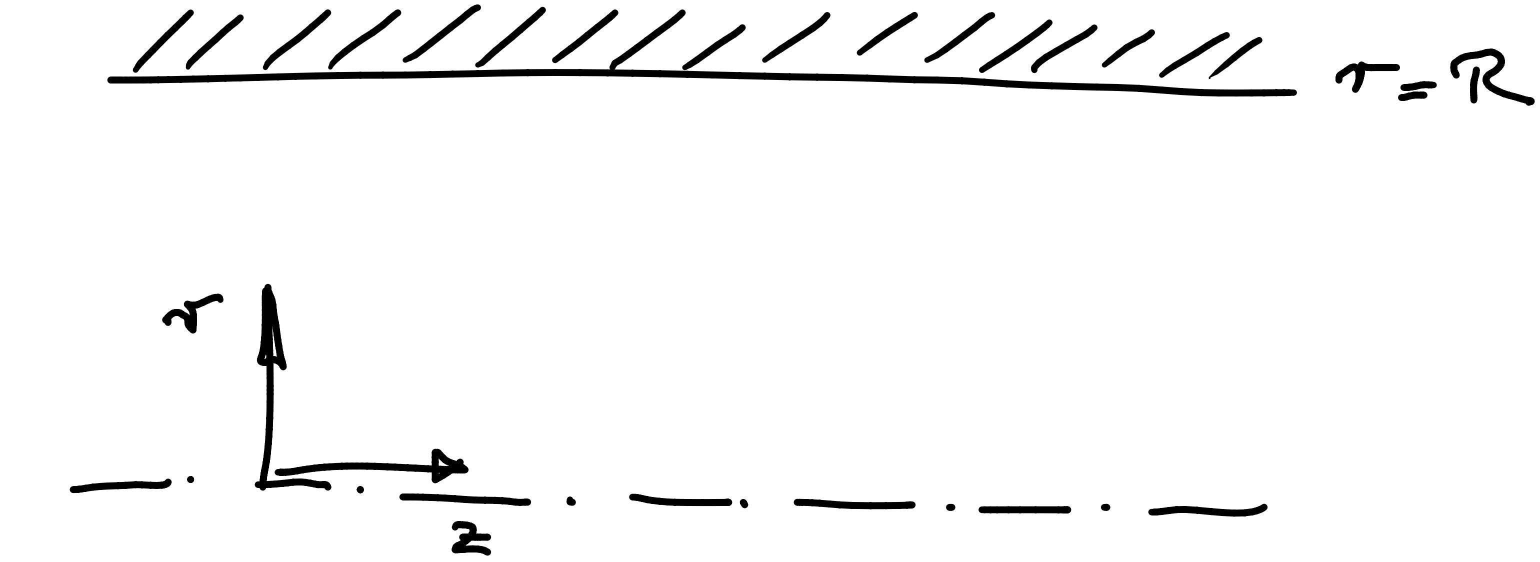

$$\rho \frac{D\vec{u}}{Dt} = -\nabla p + \nabla \cdot \underset{\sim}{P} \quad \Big| \cdot \vec{e}_z \quad \text{Cauchy-Gl.}$$

$$\text{für} \quad \underset{\sim}{T} = -p\underset{\sim}{I} + \underset{\sim}{P}$$

$$\underset{\sim}{P} = \tau_{rr}\vec{e}_r\vec{e}_r + \tau_{r\varphi}\vec{e}_r\vec{e}_\varphi + \underline{\underline{\tau_{rz}\vec{e}_r}}\vec{e}_z + \tau_{r\varphi}\vec{e}_\varphi\vec{e}_r + \tau_{\varphi\varphi}\vec{e}_\varphi\vec{e}_\varphi + \tau_{\varphi z}\vec{e}_\varphi\vec{e}_z + \ldots$$

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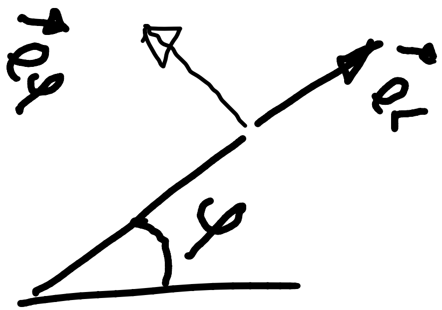
$$\rho \frac{Du_z}{Dt} = -\frac{dp}{dz} + \frac{1}{r}\frac{\partial}{\partial r}(\tau_{rz} r)$$

$$(\nabla \cdot \underset{\sim}{P}) \cdot \vec{e}_z$$

Achtung im krummlinigen Koordinaten
(Zylinderkoordinaten, Kugelkoordinaten)

$$\vec{e}_r \quad \vec{e}_\varphi \quad \vec{e}_z$$



$$\boxed{\frac{\partial \vec{e}_r}{\partial \varphi} \overset{!}{=} \vec{e}_\varphi}$$

⇝ Tipp: Anhang Spurk

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$$\Delta \Phi = \frac{\partial^2 \phi}{\partial z^2} + \underline{\underline{\frac{1}{r}}} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial r^2} + \underline{\underline{\frac{1}{r^2}}} \frac{\partial^2 \phi}{\partial \varphi^2}$$

Dimensionskontrolle!

$\equiv 0$ stationär Schichtstr.

$$\overbrace{\rho \frac{\partial u_z}{\partial t} + \ldots\ldots}^{\equiv 0} = -\frac{dp}{dz} + \frac{1}{r} \frac{d}{dr}\left(\tau_{rz} r\right).$$

$$-\frac{dp}{dz} = \bar{K} + \hat{K} e^{i\Omega t}.$$

$$u_z = \bar{u}_z + \hat{u}_z e^{i\Omega t}$$

$$\tau_{rz} = \bar{\tau}_{rz} + \hat{\tau}_{rz} e^{i\Omega t}$$

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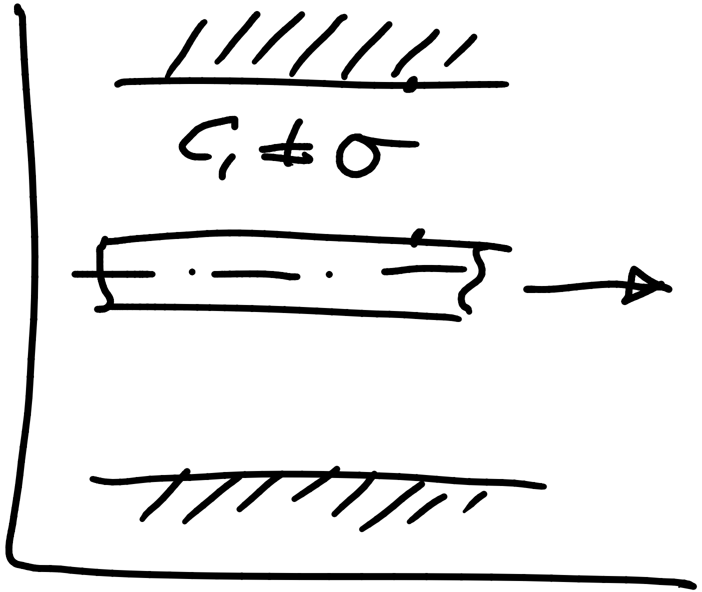
$$\frac{dp}{dz} r = \frac{d}{dr}(r\tau)$$

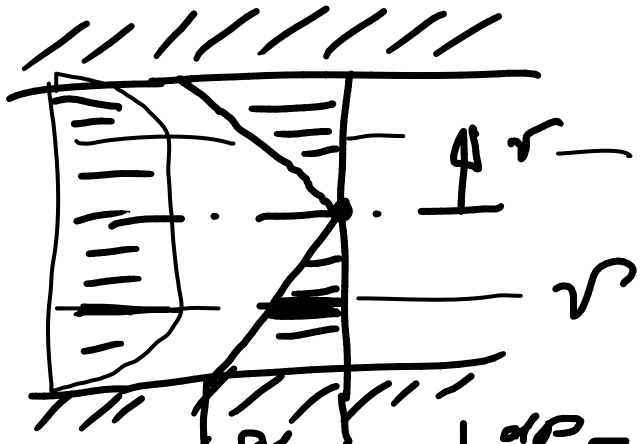


$$\frac{1}{2}\frac{dp}{dz} r^2 + C_1 = \tau r$$

$$\frac{1}{2}\frac{dp}{dz} r + \frac{C_1}{r} = \tau \leadsto C_1 = 0$$

$$\tau = \frac{1}{2}\frac{dp}{dz} r$$

$$\boxed{\tau = -\tau_w \frac{r}{R}}$$



$$\tau_w = -\frac{1}{2}\frac{dp}{dz} R$$

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Für die Newtonsche Flüssigkeit

$$\tau = \eta \frac{du}{dr}$$

$$\tau = -\tau_w \frac{r}{R}$$

$$-\frac{\tau_w}{\eta}\frac{r}{R} = \frac{du}{dr}$$

$$-\frac{\tau_w}{2\eta}\frac{r^2}{R} + C_2 = u(r)$$

$u(r=R) = 0$ Haftbedingung...

$$\rightsquigarrow C_2 = \frac{\tau_w}{2\eta} R$$

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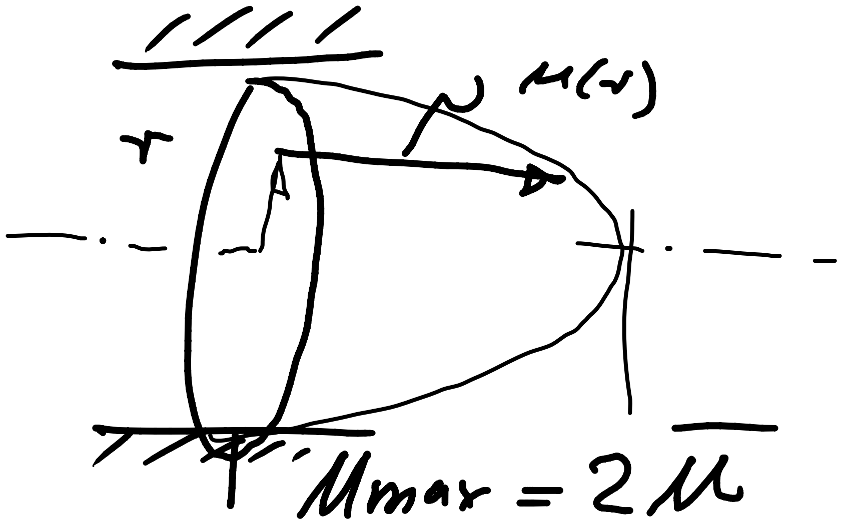
$$u(r) = R\,\frac{\tau_w}{2\eta}\left(1-\left(\frac{r}{R}\right)^2\right).$$



$$u_{max} = 2\,\bar{u}$$

Rotationsparaboloid

Hagen-Poiseuille-Strömung.

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Zusammenhang Volumenstrom und Druckabfall (viskose Durchströmung)

$$\dot V = \int_0^{2\pi} \int_0^{R} u(r)\, r\, d\varphi\, dr$$

$$= \underbrace{\frac{KR^2}{8\eta}}_{\bar u} \underbrace{\pi R^2}_{A}$$

$$\bar u := \frac{\dot V}{A}$$

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$$\Delta p_v := \frac{\varrho}{2}\bar{u}^2 \zeta$$

$$\zeta = \frac{L}{d}\lambda$$

$$:= \frac{\varrho}{2}\bar{u}^2 \frac{L}{d}\lambda$$

λ Druckverlust pro Längeneinheit.

$$d = 2R.$$

$$\boxed{\lambda = \frac{64}{Re}} \qquad Re = \frac{\bar{u}\, d}{\nu}$$

$$\nu = \eta/\varrho$$

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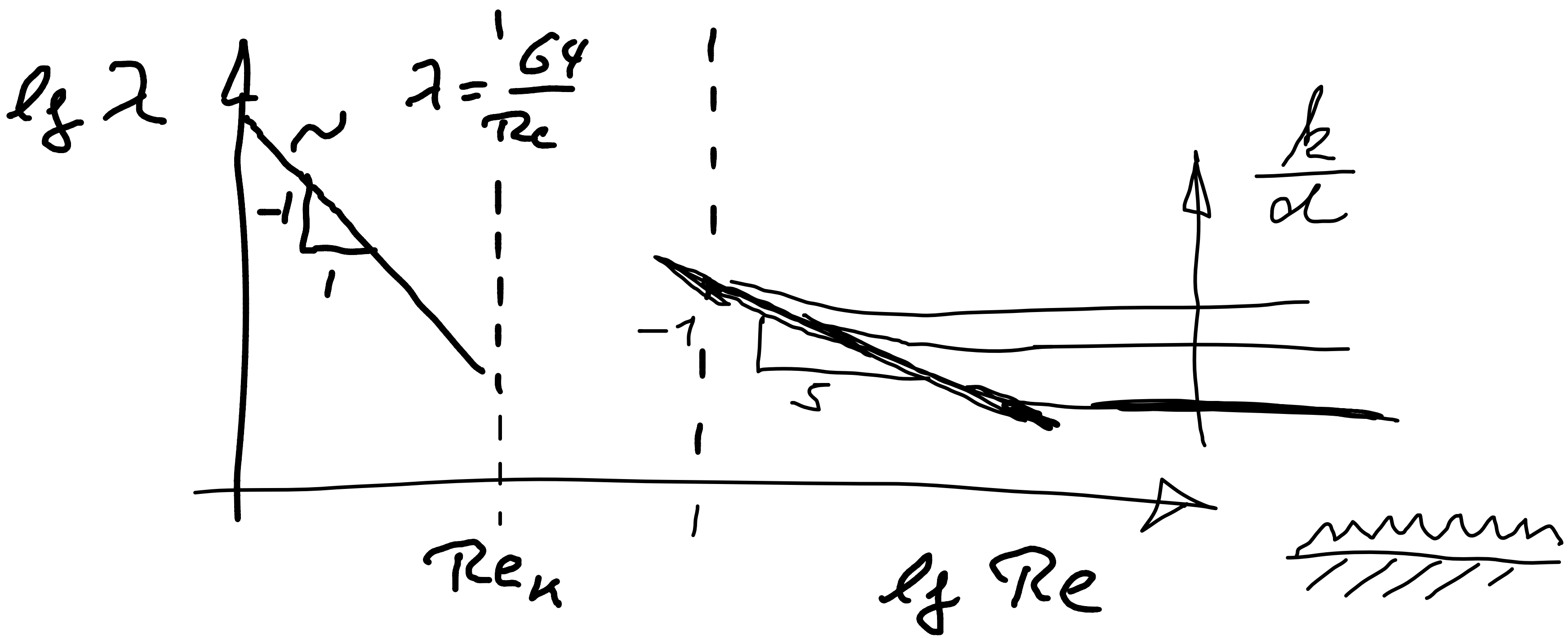

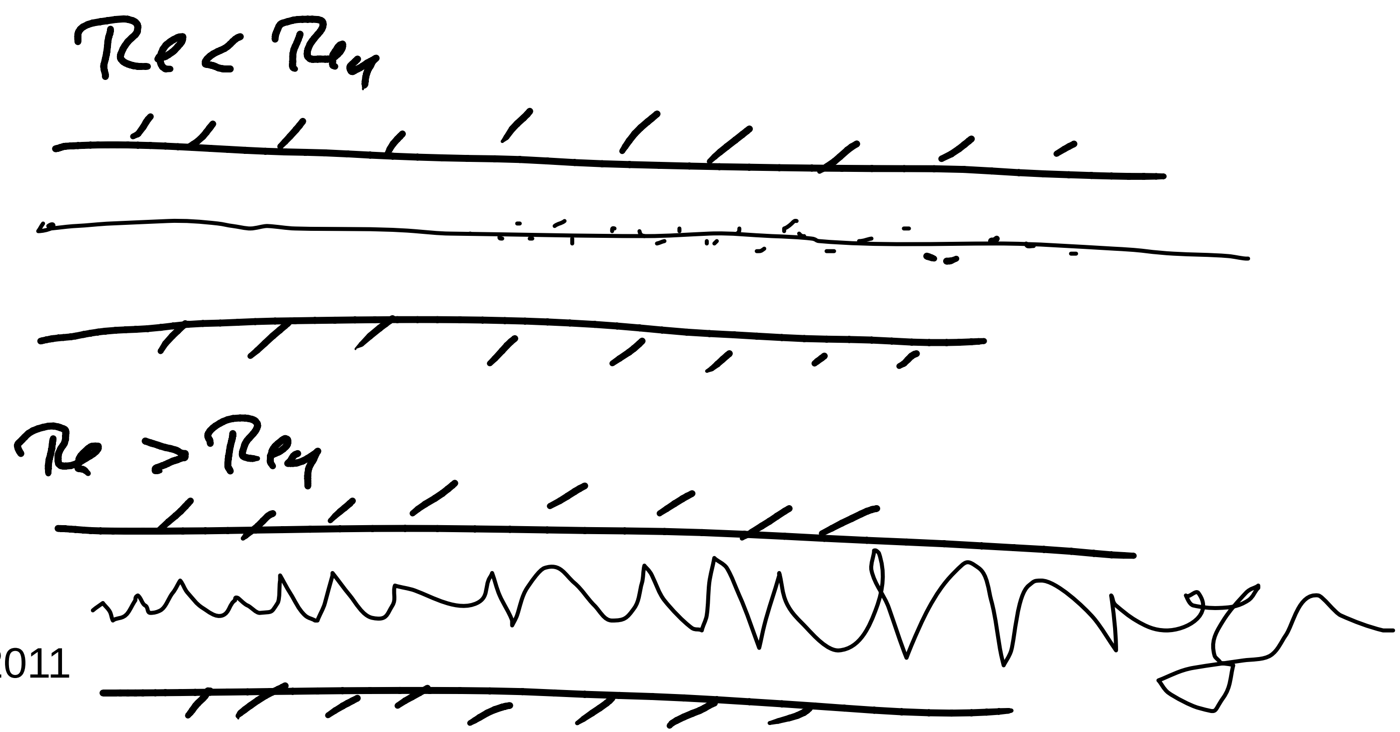

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