Übung: Anwendung Konti-Gleichung

3. VRÜ : Kontinuitätsgleichung

Korrektur

Kiste in Wasser

$$\boxed{p_1 \neq p_0}$$

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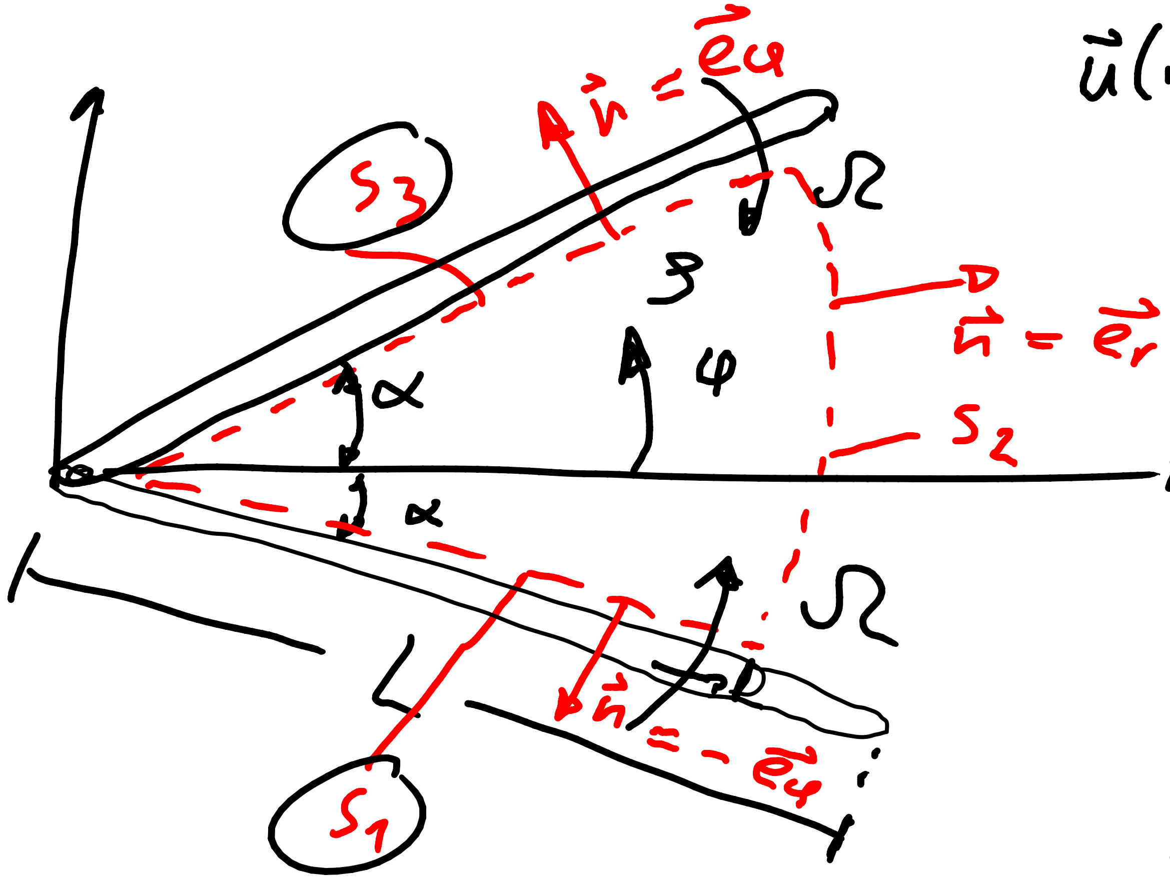

$$\vec{u}(r) = u_w(r)\vec{e}_\varphi$$

gegeben:

$$u_r(r,\varphi) = f(r)\cos\left(\frac{\pi}{2\alpha}\varphi\right)$$

$$L,\ \varrho,\ \Omega,$$

$$\vec{u}(r,\varphi) = \underline{u_r}(r,\varphi)\vec{e}_r + u_\varphi(r,\varphi)\vec{e}_\varphi$$

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1) Wandgeschwindigkeit für beide Platten

$\vec u = \Omega r \vec e_\varphi$

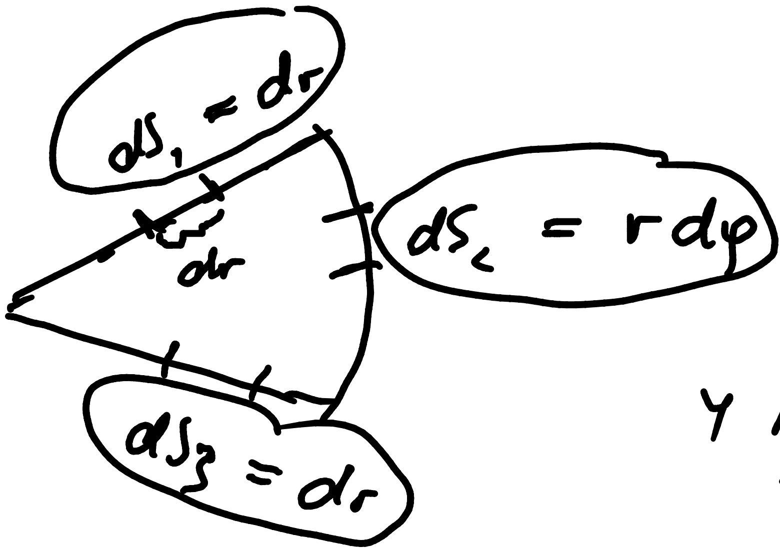
$$\vec u(r) = \vec\Omega \times r\,\vec e_r$$

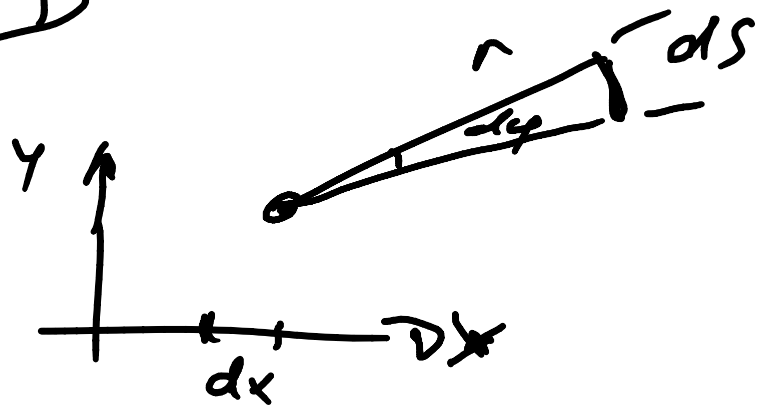
$$\vec u(r) = \pm\Omega\,\vec e_z \times r\,\vec e_r = \underline{\underline{\pm\,\Omega r\,\vec e_\varphi}}$$

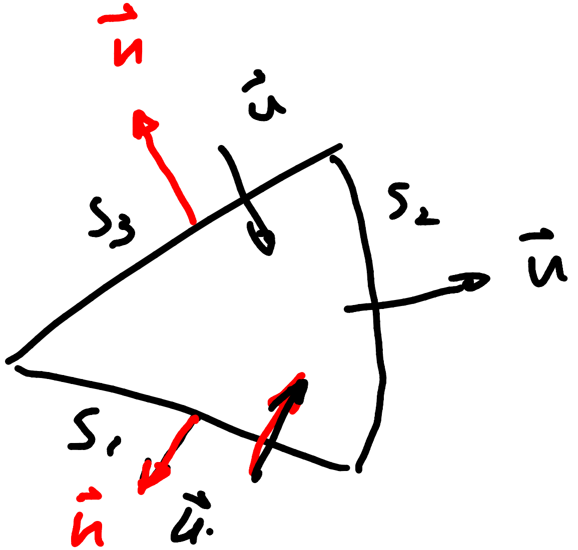
2) $f(r) = ?$ $\qquad \rho = const$

$$\int_V \cancelto{0}{\frac{\partial \rho}{\partial t}}\, dV + \int_S \rho\, \vec{u}\cdot\vec{n}\, dS = 0$$

$$\int_{S_1} \rho\, \vec{u}\cdot\vec{n}\, dS_1 + \int_{S_2} \rho\, \vec{u}\cdot\vec{n}\, dS_2 + \int_{S_3} \rho\, \vec{u}\cdot\vec{n}\, dS_3$$







$$S_1: \quad \vec{u}\cdot\vec{n} = -u_r$$
$$= -\Omega r$$

$$S_3: \quad \vec{u}\cdot\vec{n} = -\Omega r$$

$$S_2: \quad \vec{u}\cdot\vec{n} = f(r)\cos\left(\frac{\pi}{2\alpha}\varphi\right)$$

$$\int_0^r -\Omega r\, dr + \int_{-\alpha}^{+\alpha} f(r)\cos\left(\frac{\pi}{2\alpha}\varphi\right) r\, d\varphi + \dots$$

$$\dots + \int_0^r -\Omega r\, dr = 0$$

$$\vdots$$

$$\Rightarrow -\Omega r^2 + r f(r)\,\frac{2\alpha}{\pi}\left[\sin\left(\frac{\pi}{2\alpha}\varphi\right)\right]_{-\alpha}^{\alpha} = 0$$

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$$f(r) = \frac{\pi}{4\alpha}\,\Omega r$$

1) Kontrollvolumen passend

2) Flächenelemente

3) $\vec{u}\cdot\vec{n}$

4) Einsetzen

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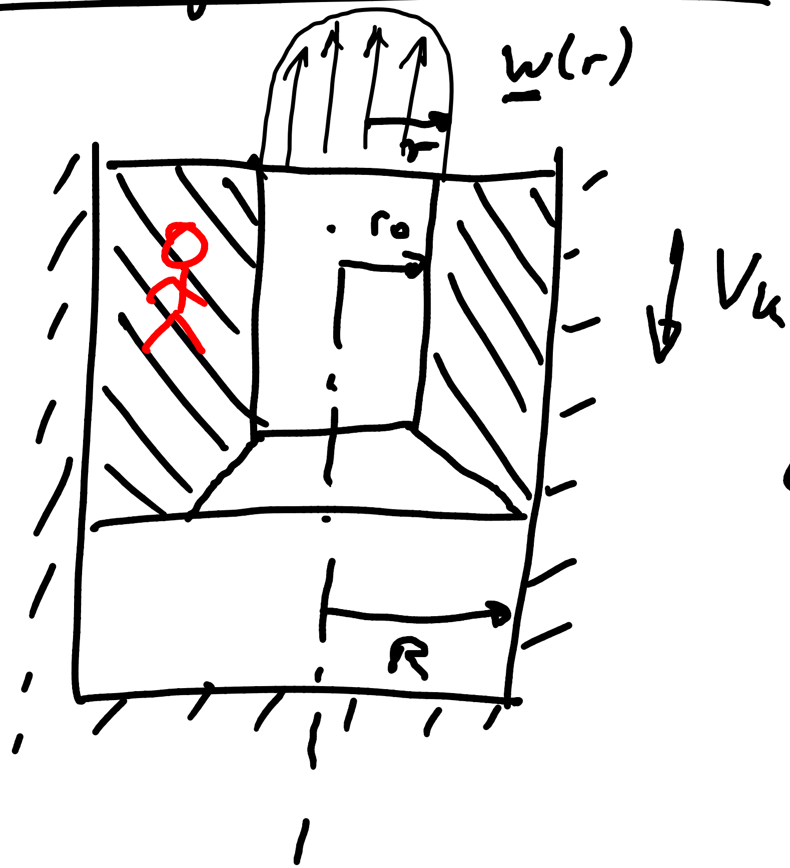
Bewegter Kolben



w : Relativgeschw.

$$w(r) = \underline{W_0}\left\{1 - \left(\frac{r}{r_0}\right)^2\right\}$$

a) Maximalgeschwindigkeit $\underline{W_0}$ im Kolbenfesten Koordinatensystem

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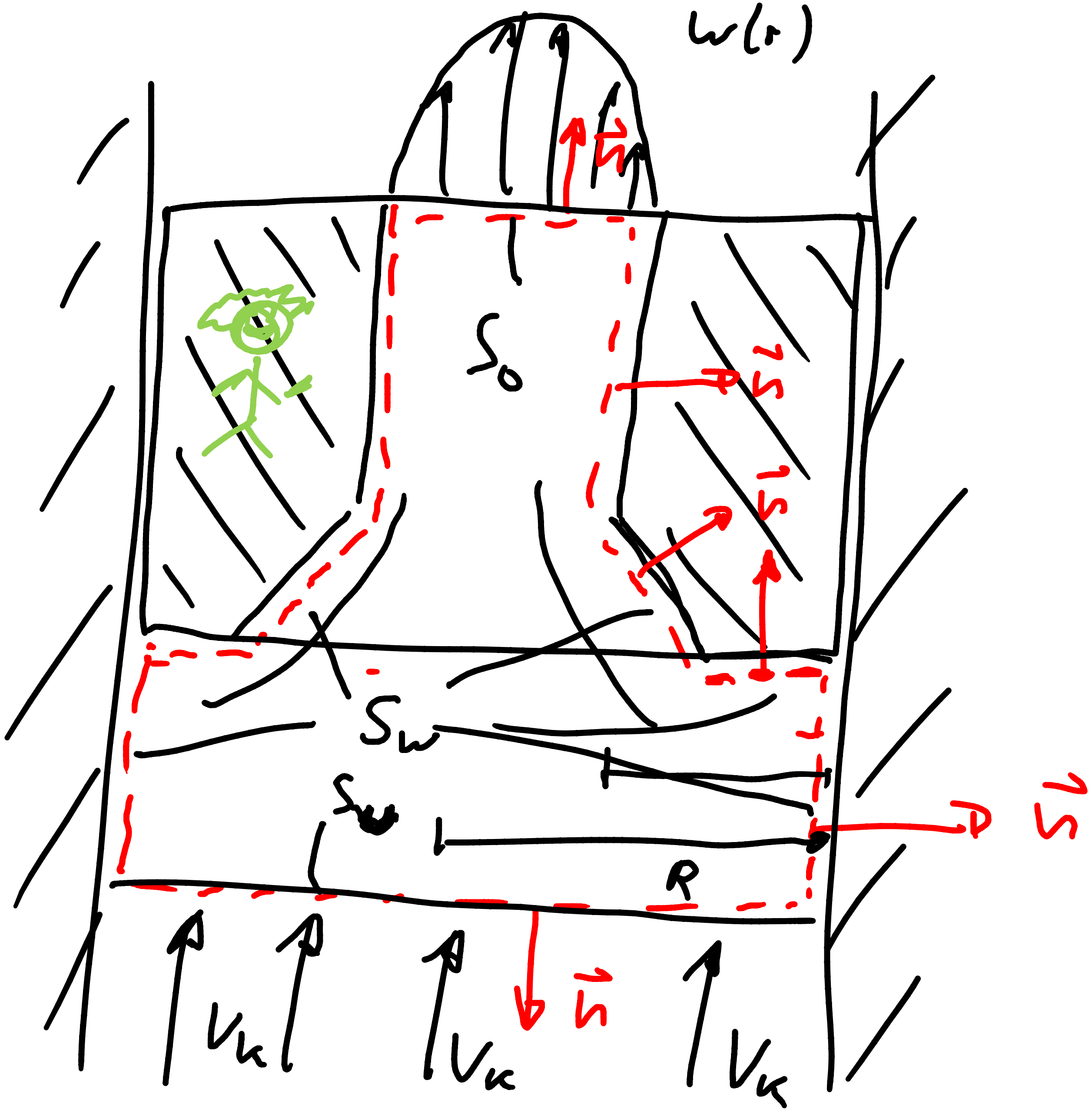

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$$\frac{\partial}{\partial t}\int_V \rho \, dV + \rho \int_S \underline{\vec{w}} \cdot \vec{n} \, dS = 0$$

$$\cancel{\rho} \int_S \vec{w} \cdot \vec{n} \, dS = 0$$

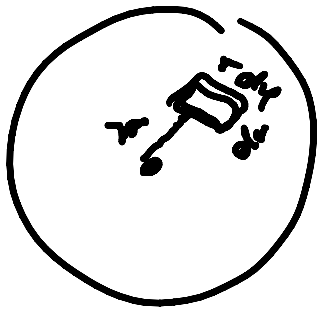
$$\int_{S_u} \vec{w} \cdot \vec{n} \, dS_u + \int_{S_0} \vec{w} \cdot \vec{n} \, dS_0 + \cancelto{0}{\int_{S_W} \vec{w} \cdot \vec{n} \, dS}$$

$$S_W : \vec{w} \cdot \vec{n} = 0$$

S_o:

$$\vec{w}\cdot\vec{n} = w(r)$$

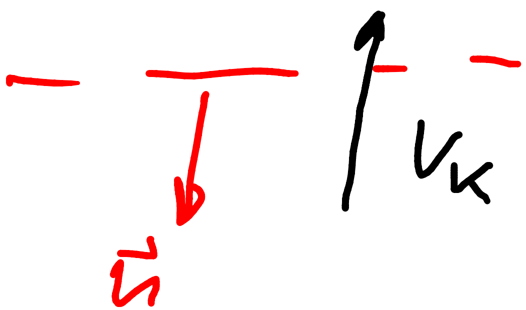
$$dS_o = r\,dr\,d\varphi$$



S_u:

$$dS_u : r\,dr\,d\varphi$$

$$\vec{w}\cdot\vec{n} = -V_K$$



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$$W_0 \int_{\varphi_0}^{2\pi} \int_{r=0}^{r_0} \left\{ 1 - \left(\frac{r}{r_0} \right)^2 \right\} r \, dr \, d\varphi - \ldots$$

$$= \ldots \int_{\varphi=0}^{2\pi} \int_{r=0}^{R} V_K \, r \, dr \, d\varphi = 0$$

$$\Rightarrow \quad \pi R^2 V_K = W_0 2\pi \int_{r=0}^{r_0} \left\{ 1 - \left(\frac{r}{r_0} \right)^2 \right\} r \, dr$$

$$\vdots$$

$$\Rightarrow \quad W_0 = 2 V_K \left(\frac{R}{r_0} \right)^2$$

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