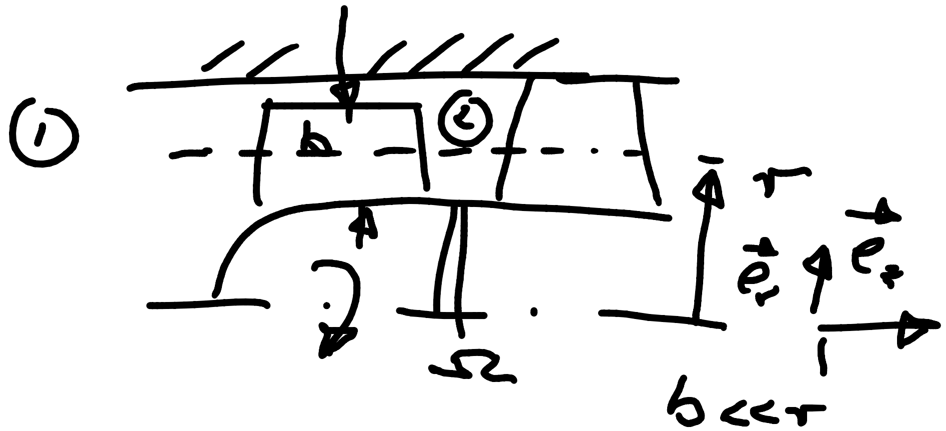
Drallsatz an Turbomaschine.

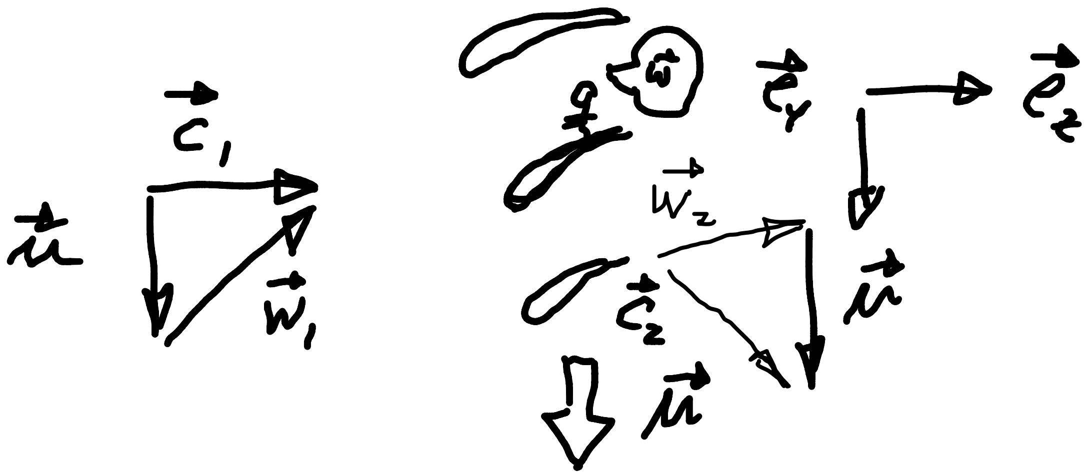
$$M_z = \dot{m}\,(r_2 c_{u2} - r_1 c_{u1})$$

Axiale Turbomaschine

$$\vec{c} = \vec{w} + \vec{u}$$

$$\vec{u} = r\Omega\,\vec{e}_\varphi$$





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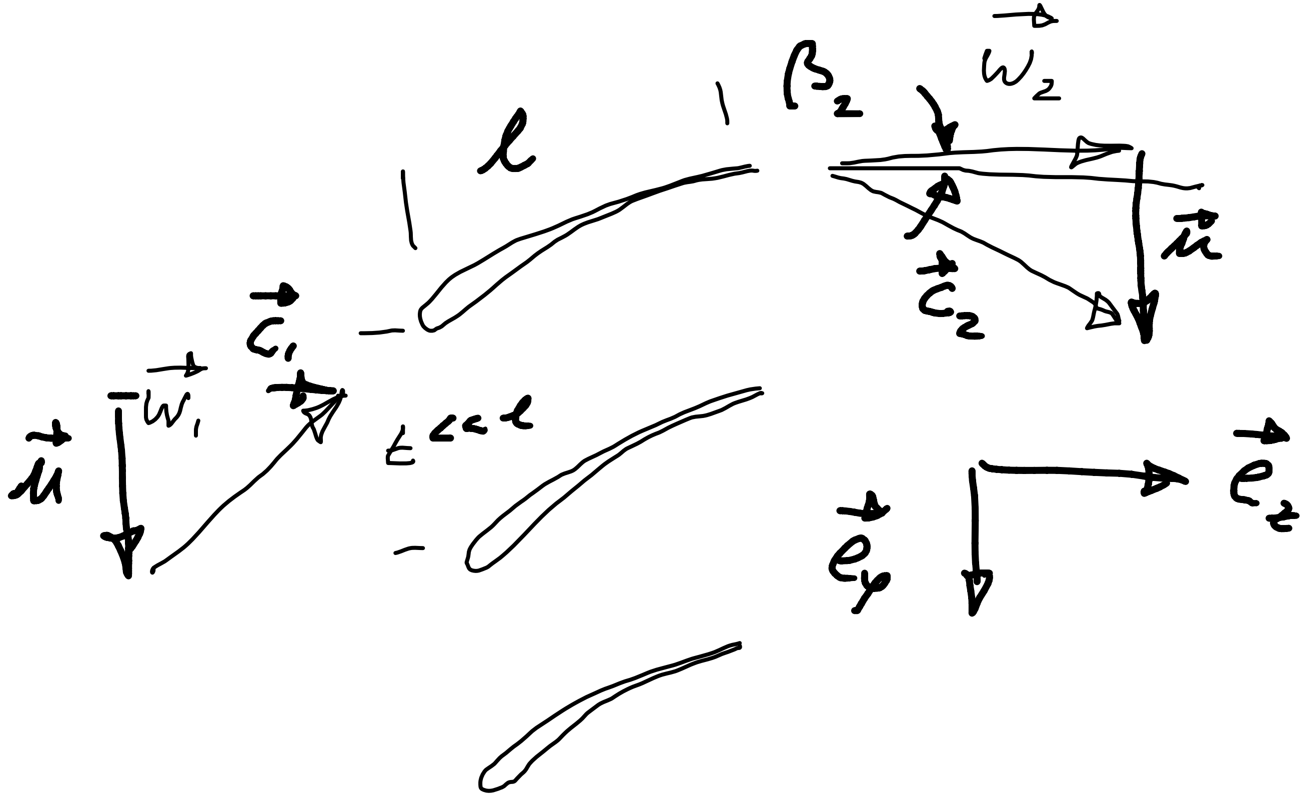

1 i.d.R. Drallfreie Zuströmung $r c_{u1} \equiv 0.$

2 $\vec{c}_2 \cdot \vec{e}_\varphi = (\vec{w}_2 + \vec{u}) \cdot \vec{e}_\varphi = w_{u2} + r\Omega$

$$= -w_{z2} \cot \beta_2 + r\Omega$$

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$$M_z = \dot m \big(r_2 c_{u2} - \underbrace{r_1 c_{u1}}_{=0} \big)$$

$$= \dot m \left(r^2 \Omega - r w_{z2} \cot \beta_2 \right) \quad \Big| \cdot \Omega$$

$$P = \dot m \left((r\Omega)^2 - (r\Omega)\, w_{z2} \cot \beta_2 \right).$$

$$w_{z2} = \frac{\dot m}{\rho A}$$

$$P = \dot m (r\Omega)^2 \left[1 - \frac{\dot m}{r \Omega \rho A} \cot \beta_2 \right].$$

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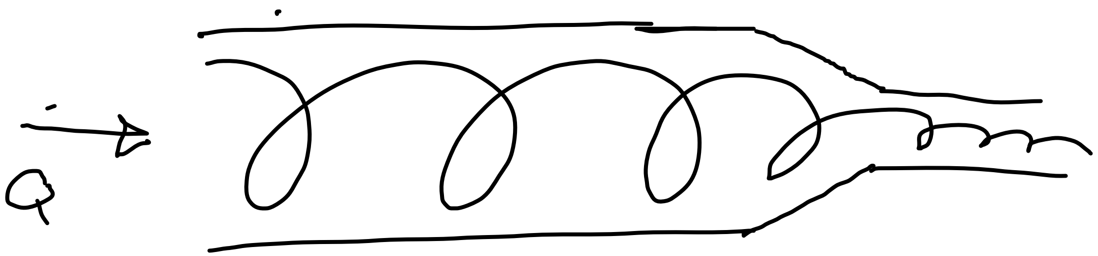
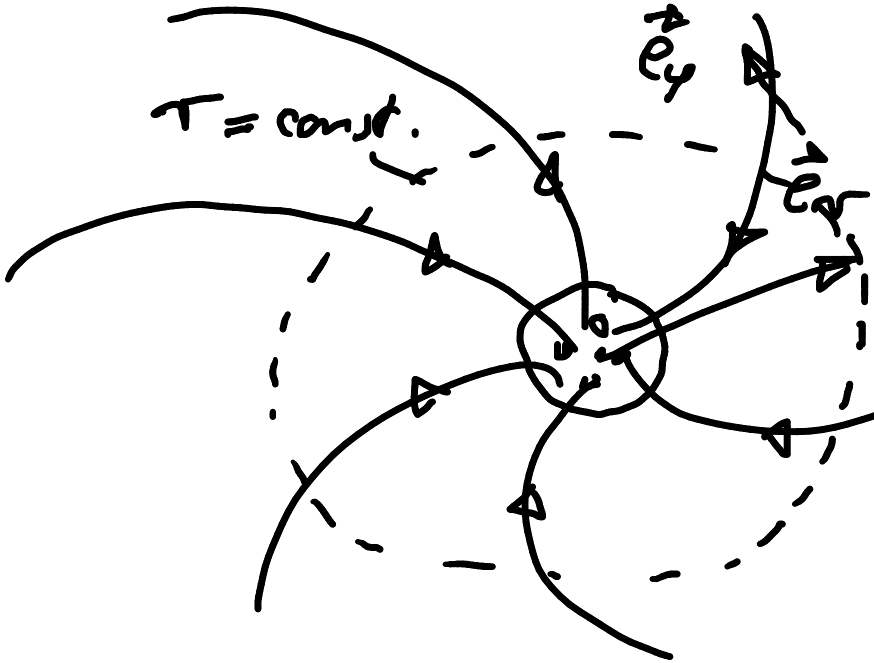
$$\Gamma_z = 0 \leadsto \underbrace{r\, c_u = \text{const.}}$$





$$\vec{u} = -\frac{Q}{2\pi r}\,\vec{e}_r + \underbrace{\frac{\Gamma}{2\pi r}}_{c_u}\,\vec{e}_\varphi$$

$$\frac{d\vec{x}}{ds} = \frac{\vec{u}}{|\vec{u}|}$$

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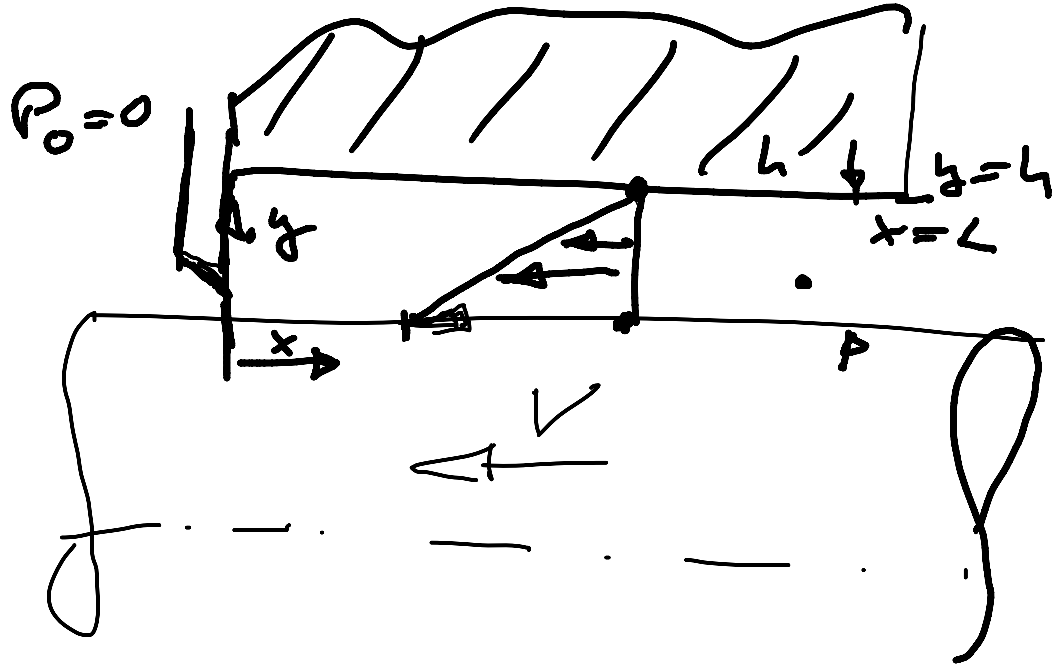
Schichtenströmungen

Pöe

öe



$$Q := \left| \int_0^h u \, dy \right|$$

$$= \frac{1}{2} V h \neq 0.$$

$p(x=0$

$$\left.\begin{aligned} u(0) &= -V \\ u(y=h) &= 0 \end{aligned}\right\} \text{Haftbedingung}$$

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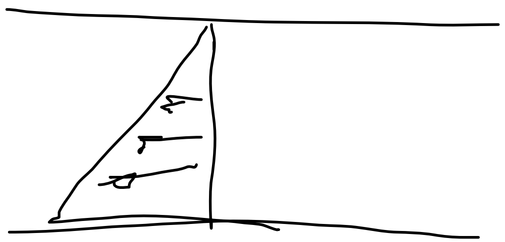
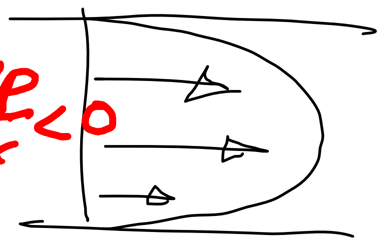
Schleppströmung + Druckgetriebene Strömung





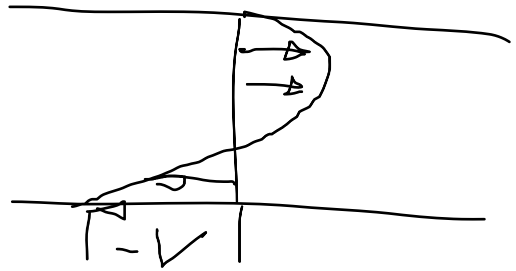
$\frac{dp}{dx} < 0$

$\longmapsto x$

Couette Strömung

Poiseuille Strömung

=

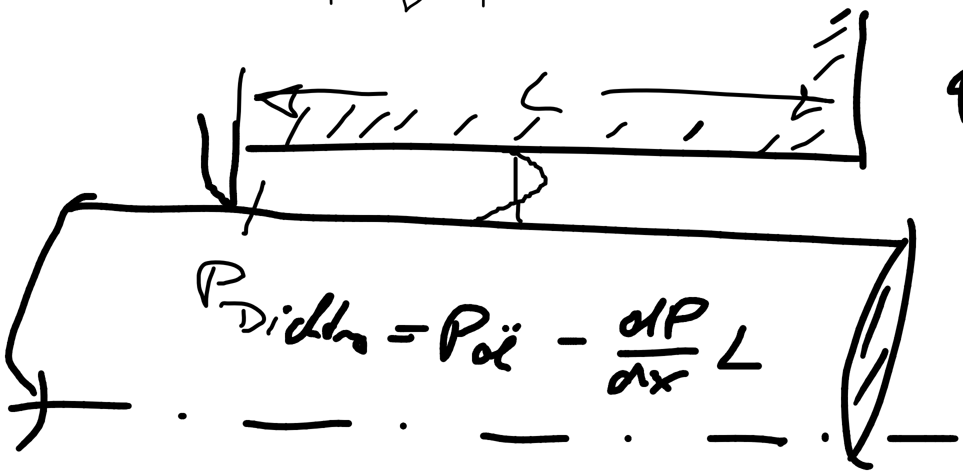


$Q \overset{!}{=} 0$

$\rightsquigarrow \frac{dp}{dx}$

p_i

p_a



$p_{Dichtung} = p_a - \frac{dp}{dx} L$

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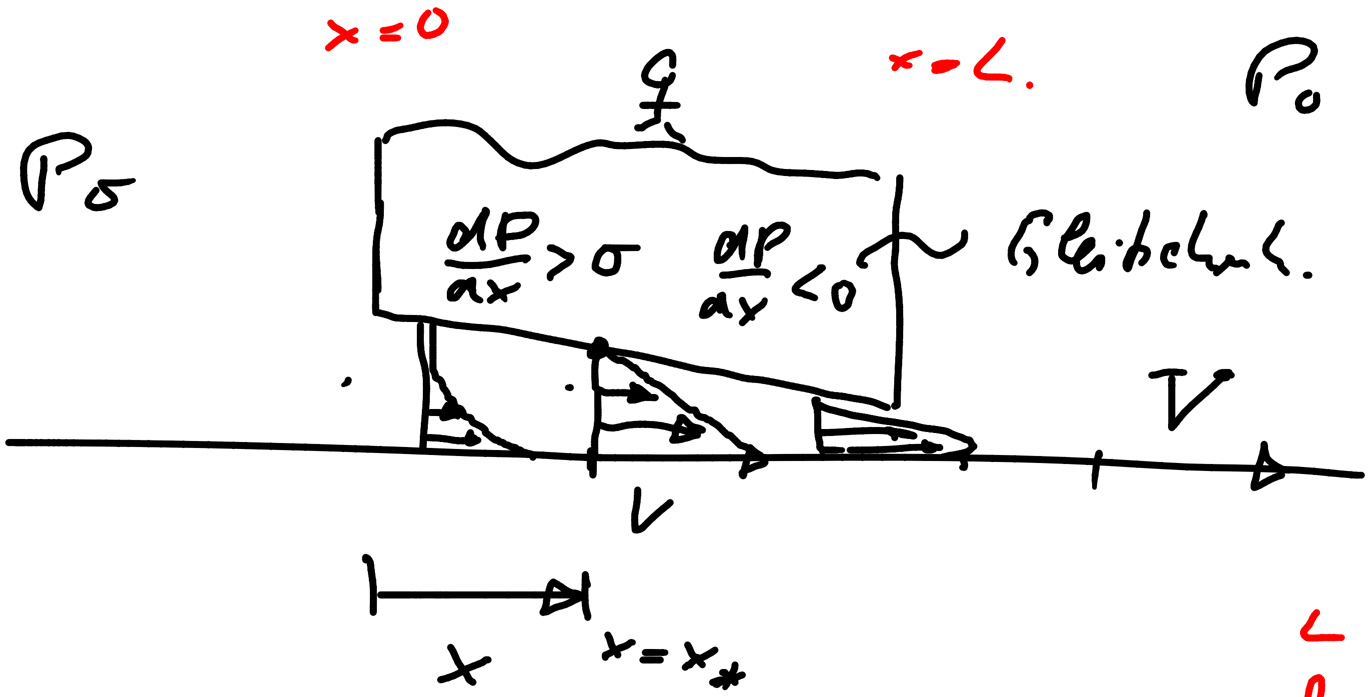
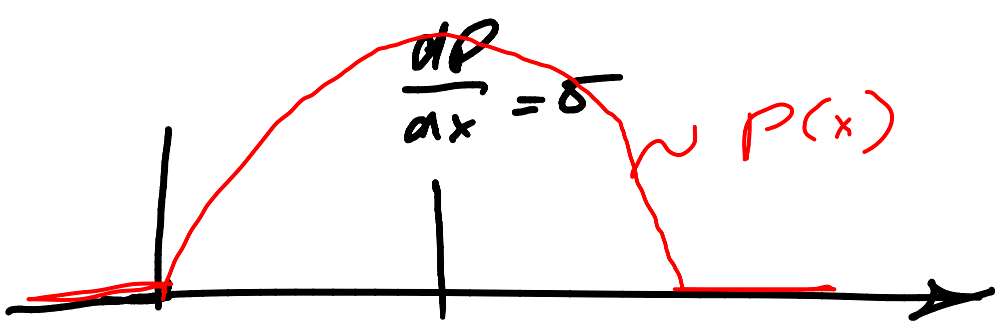
Hydrodynamische Lager.



$$F = \int_0^L p(x)\,dx$$



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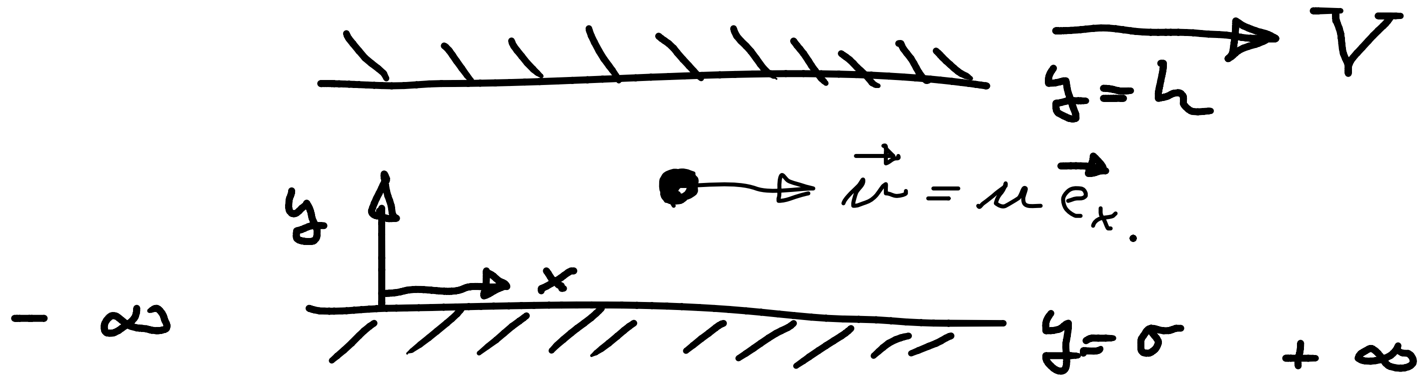
Ebene Fall + stationär Strömung



Impulsbilanz für ein Teilchen

$$\underbrace{\rho \frac{D\vec{u}}{Dt}}_{\equiv 0 \text{ bei stationärer Schichtströmung.}} = -\nabla p + \rho \vec{k} + \underbrace{\nabla \cdot (\eta \nabla \vec{u})}_{\nabla \cdot \underline{T}}.$$

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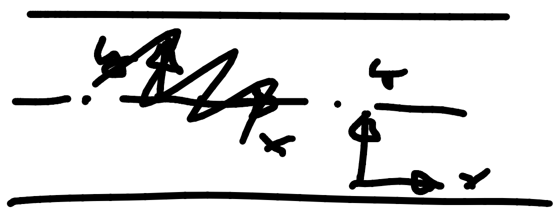
x-Komponente

$$\sigma = \underbrace{-\frac{dp}{dx}}_{K} + \frac{d}{dy}(\tau_{xy})$$



$$-K = \frac{d}{dy}(\tau_{xy})$$

$$C_1 - K y = \tau_{xy}$$

C_1 ist die Integrationskonstante.

$\tau_{xy} = \eta \frac{du}{dy}$ für ein newtonsches Medium

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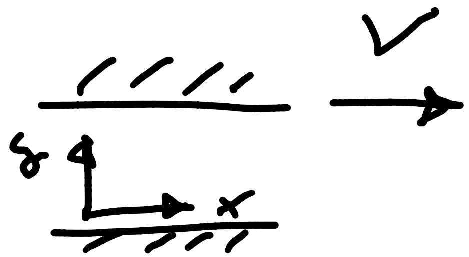
$$C_1 - Ky = \eta \frac{du}{dy}$$



$$C_1 y - \frac{1}{2} K y^2 + C_2 = \eta\, u(y)$$

$$u(0) \overset{!}{=} 0 \rightsquigarrow C_2 = 0.$$

$$u(h) \overset{!}{=} V \rightsquigarrow C_1 h - \frac{1}{2} K h^2 = \eta V$$

$$C_1 = \frac{\eta V}{h} + \frac{1}{2} K h.$$

$$u(y) = V \frac{y}{h} + \frac{1}{2} K h y - \frac{1}{2} K y^2$$
$$= V \frac{y}{h} + \frac{1}{2} K h^2 \left(\frac{y}{h} - \left(\frac{y}{h}\right)^2 \right).$$

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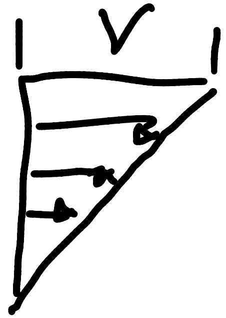
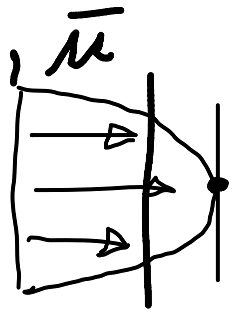
$$u(y) = V\frac{y}{h} + \frac{1}{2}\frac{Kh^2}{2}\left(\frac{y}{h}-\left(\frac{y}{h}\right)^2\right)$$





$$\frac{3}{2}\bar{u} = u_{max.}$$

$$0 = -\frac{dp}{dx} + \eta\frac{d^2u}{dy^2}$$ lineare DGL.

$\rightsquigarrow$ Superposition ist erlaubt.

$$\bar{u} = \frac{1}{h}\int_0^h \frac{1}{2}\frac{Kh^2}{2}\left(\frac{y}{h}-\left(\frac{y}{h}\right)^2\right)dy$$

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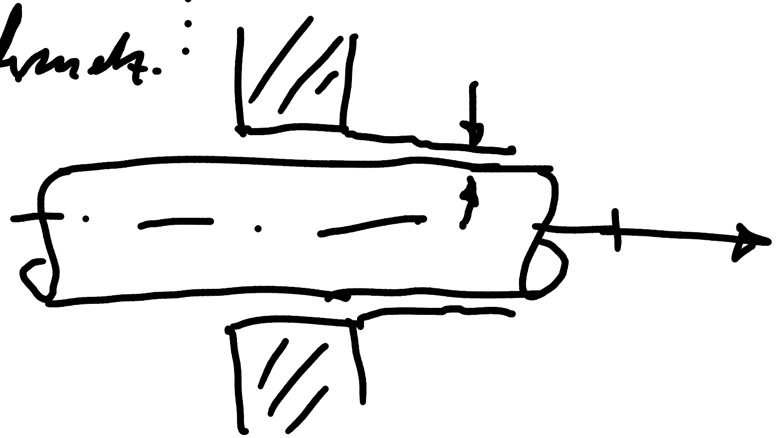
Ebene Schichtenströmung : Gleitlager

: Dichtungen

: Mikroströmung.

:

Schmutz.



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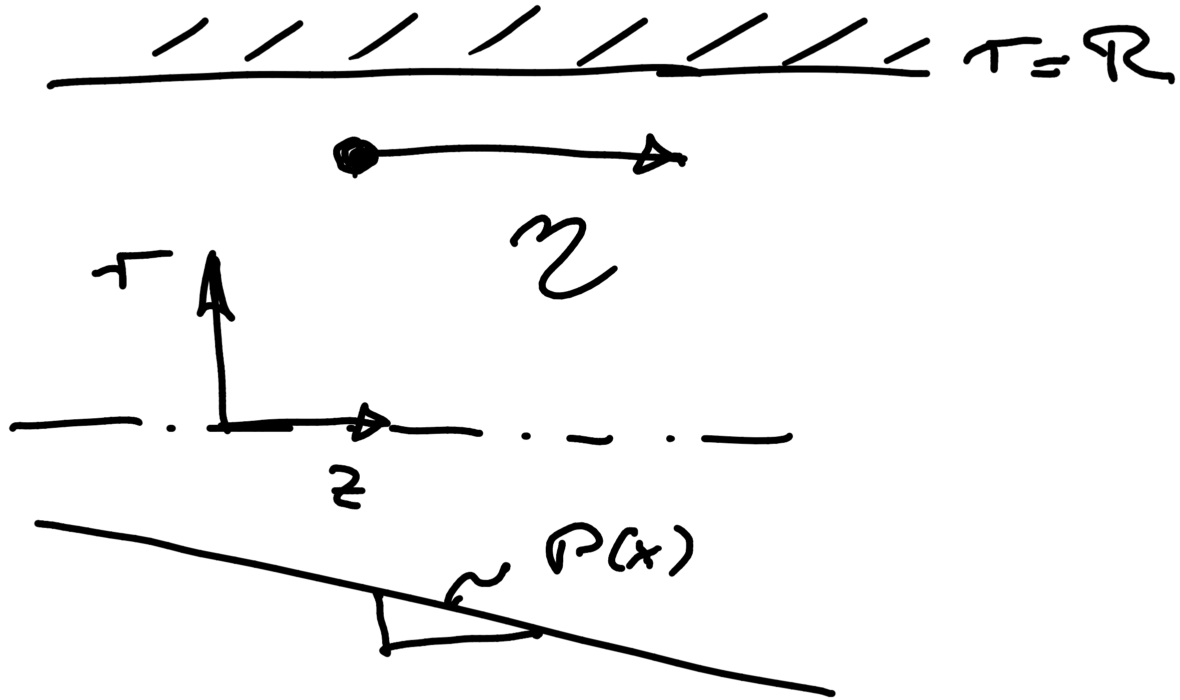
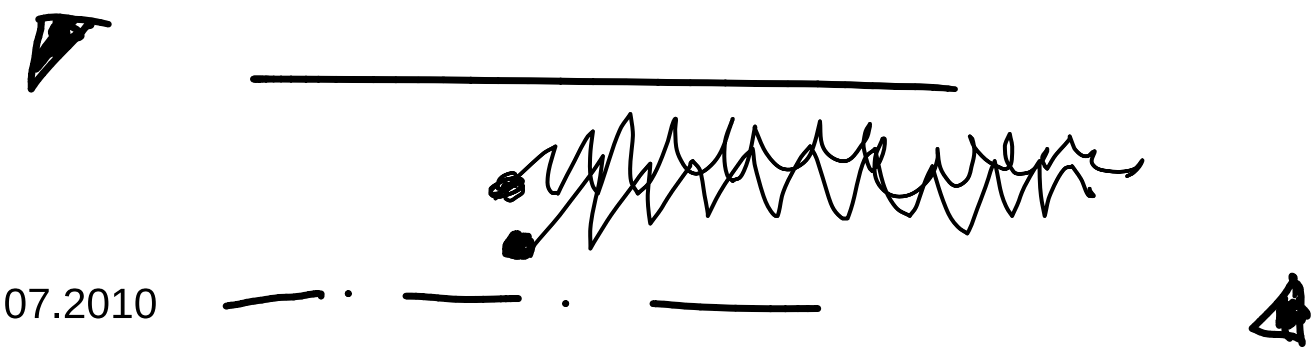
Schichtenströmung in Zylinderkoordinaten.



$$K = -\frac{dp}{dz}$$



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$$\underbrace{\rho \frac{D\vec{u}}{Dt}}_{0} = -\nabla p + \eta \Delta \vec{u}. \qquad \Delta = \nabla \cdot \nabla = \nabla^2$$

$\vec{u} = u_z \vec{e}_z$ für laminare Schichtstr.

$$\nabla p = -\frac{dp}{dz}\vec{e}_z = K \vec{e}_z$$

$-\dfrac{K}{\eta} = \Delta u_z$ Poisson-Gleich.

$$= \frac{1}{r}\frac{d}{dr}\left[r \frac{du_z}{dr}\right]$$

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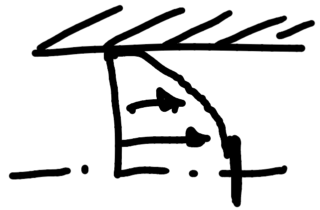
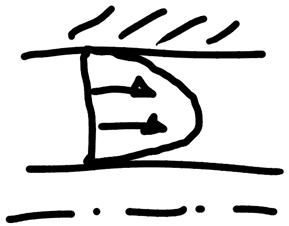
$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$



$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}.$$

Bitte ~~imm~~ bei Kreiszylinder koordinaten nachlesen!

$$-\frac{K}{\eta} = \frac{1}{r}\frac{d}{dr}\left[r\frac{du_z}{dr}\right].$$



$$-\frac{1}{2}\frac{K}{\eta}r + \underbrace{\cancel{\frac{C_1}{r}}}_{\equiv 0.} = \frac{du_z}{dr}.$$

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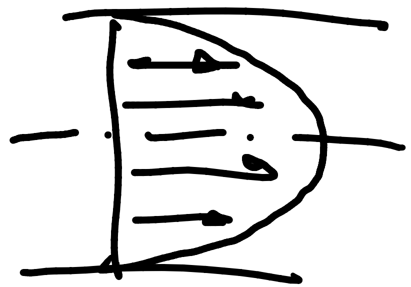
$$-\frac{1}{4}\frac{K r^2}{\eta} + C_2 = u(r)$$

$$u(r=R) \overset{!}{=} 0 \quad \text{Haftbedingung.}$$

$$-\frac{1}{4\eta} K R^2 + C_2 = 0$$

$$u(r) = \frac{K R^2}{4\eta}\left(1-\left(\frac{r}{R}\right)^2\right).$$



$$u_{max} = \frac{K R^2}{4\eta} \qquad \bar{u} = \frac{u_{max}}{2}.$$

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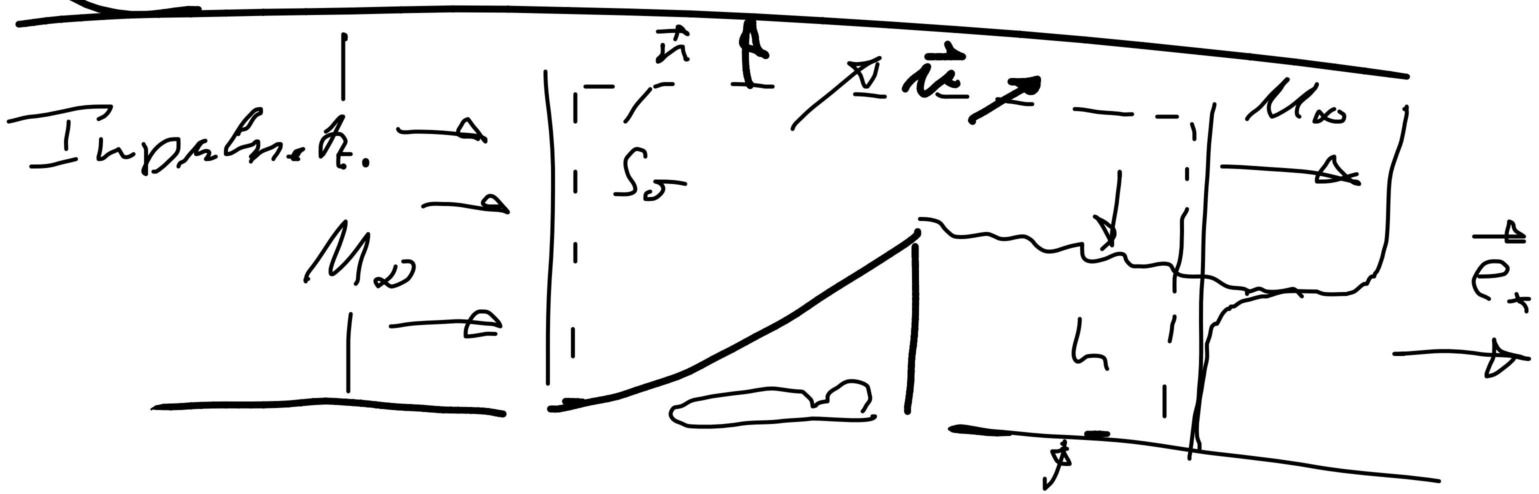
Hydro-statik.

$\nabla p = \vec{f}$. | Archimedes / Zentrifug.

$\rho = \text{const.}$ Schwere Flüssigkeit

Kraft auf Wände.



$$\int_{S_\sigma} \rho \vec{u}\cdot\vec{e}_x \underbrace{\vec{u}\cdot\vec{n}}_{\neq 0}\, dS' = \int \rho u_x u_y \, dS'.$$

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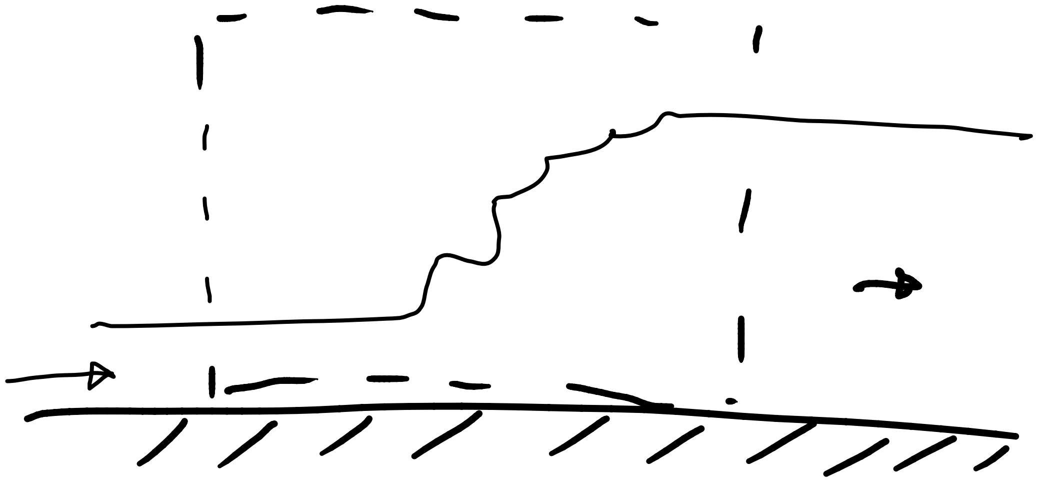

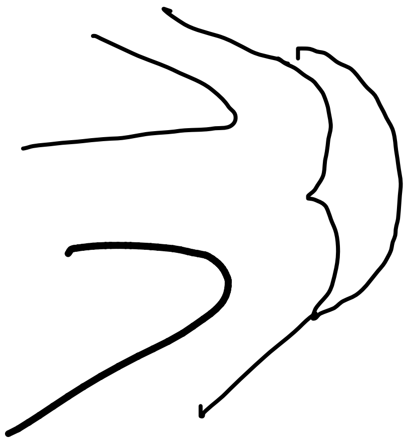

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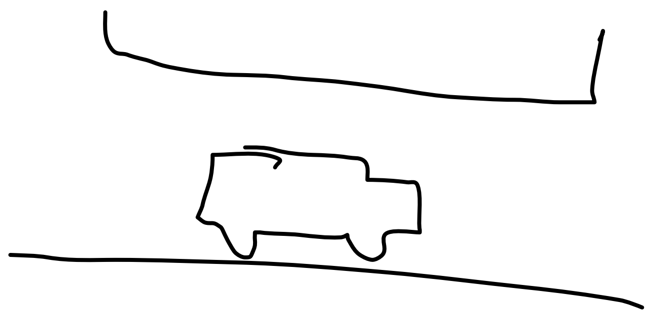
$$\frac{\rho}{2} u_1^2 + p_1 + \rho g z_1 = \frac{\rho}{2} u_2^2 + p_2 + \rho g z_2 + \underbrace{\int_1^2 \rho \dot{u} \, ds}_{} + \Delta p_V$$

$$\frac{\rho}{2} w_1^2 + p_1 - \frac{1}{2} \rho (r\Omega)^2 = \text{const.}$$

$$\Delta p_V = \frac{\rho}{2} (u_1 - u_2)^2$$



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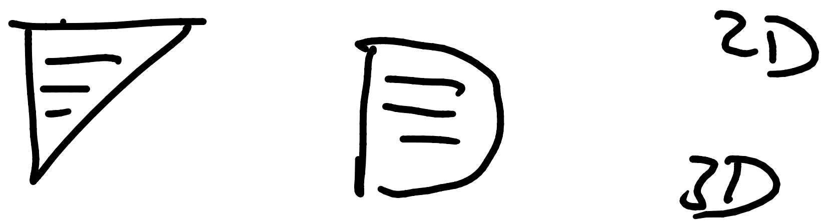
Drallsatz

$$M_z = \dot m \left(r_2 c_{u2} - r_1 c_{u1} \right)$$

$$\vec c = \vec w + \vec u \,.$$



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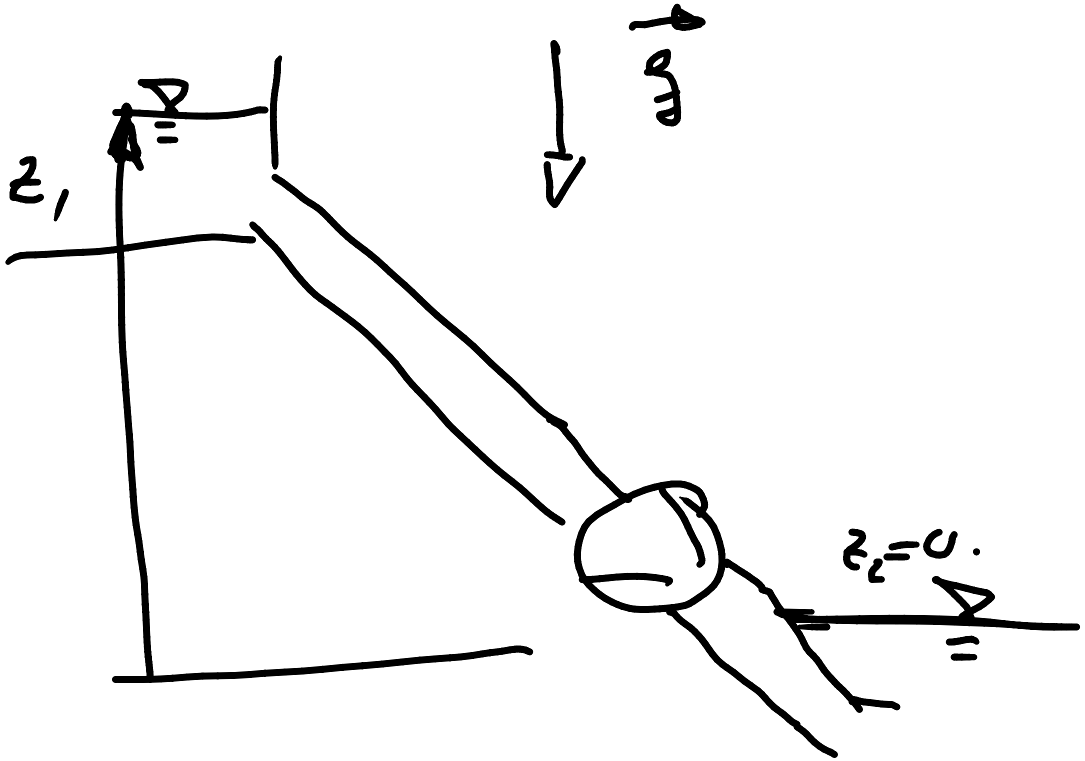
$$p + \frac{\rho}{2} u^2 + \psi = \text{const}$$

$$(2) \quad -\nabla \psi = \vec{f}$$



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