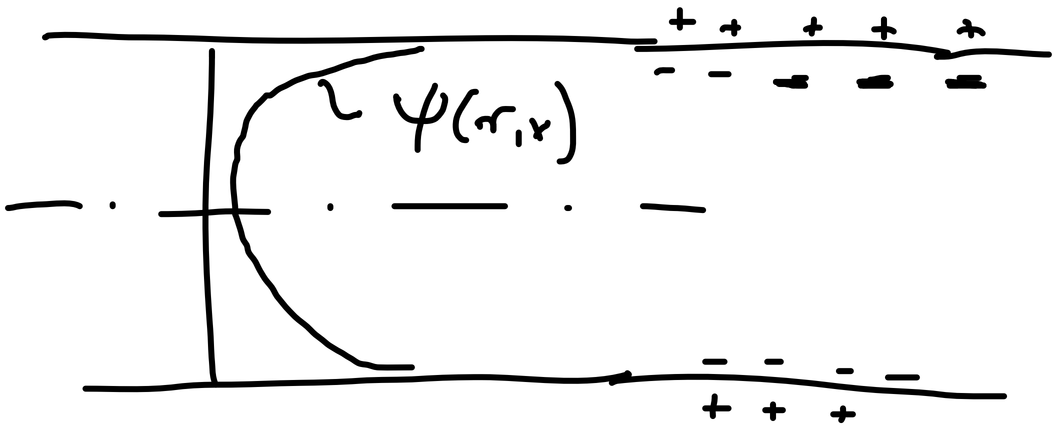
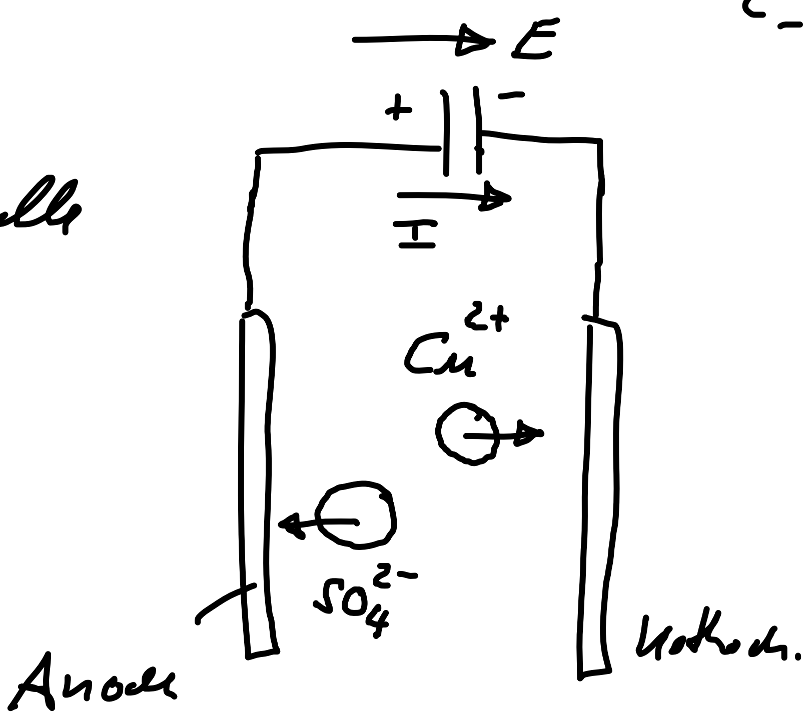


$$\psi(r,x) \longrightarrow c_+(r,x),\ c_-(r,x) \longrightarrow \vec{i}_+ \cdot \vec{e}_r \stackrel{!}{=} 0.$$

$$\vec{i}_- \cdot \vec{e}_r \stackrel{!}{=} 0.$$

Im Gegensatz dazu
Elektrolytische Zelle



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$\vec{i} = f_n(\vec{E})$ Ohmsche Gesetz.

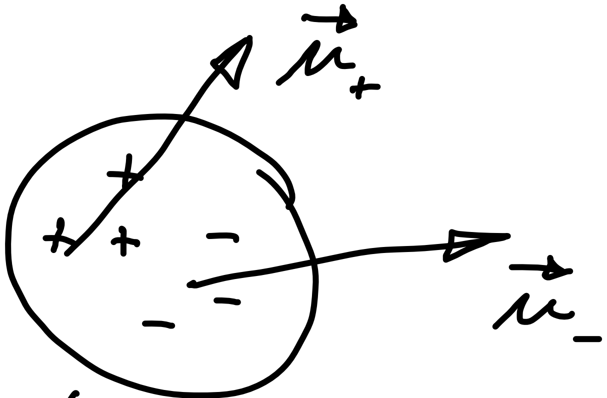
Definition vgl. Fluidtransport

$$\vec{i}_+ := F z \, \vec{j}^*_+ = F z \left(\vec{u}_+ c_+ + \vec{J}^*_+ \right)$$

Diffusion + Coulombsche Anteil.

<u>Molare</u> Stoffstromdichte der k-ten Komponente

$$\vec{j}^*_k := \underbrace{\vec{u}_k c_k}_{\text{Konvektiv}} + \underbrace{\vec{J}^*_k}_{\text{relativ Strom.}}$$



Konzentration der k-ten Komponente

c_k

$$\vec{u}^* = \frac{\sum \vec{u}_k c_k}{\sum c_k} \quad \Big| \quad \vec{u} = \frac{\sum \vec{u}_k \rho_k}{\sum \rho_k}$$

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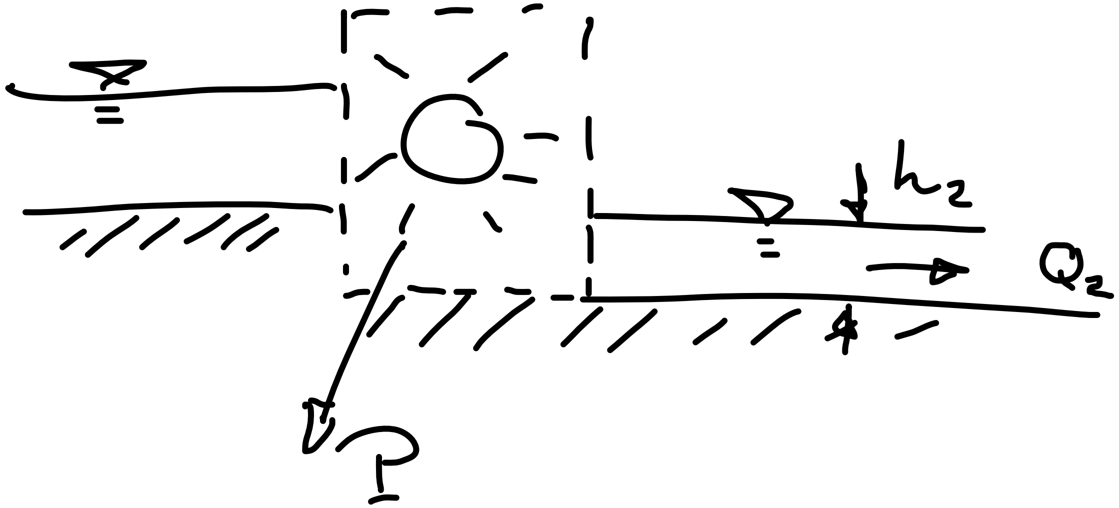

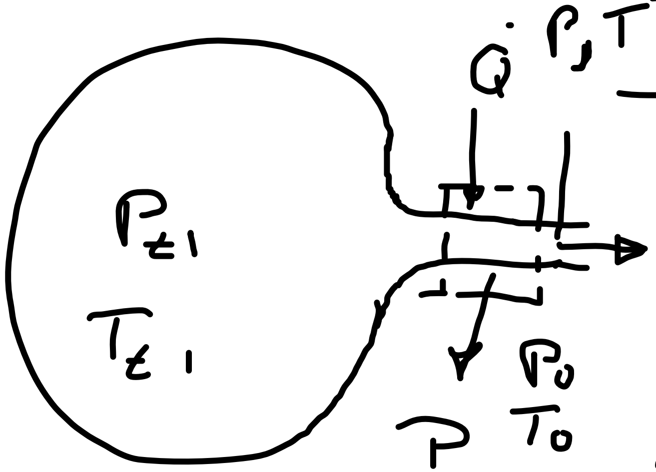

$\boxed{\text{Ernst Becker} \quad \text{Thermodynamik}}$

1. HS. $-P + \dot{Q} = \dot{m}(h - h_4)$ (1)

2. HS. $\dot{m}(s - s_1) = \dfrac{\dot{Q}}{T_0} + \Delta \dot{S}_{irr}$ (2)

$\rightsquigarrow \quad \dot{Q} = \dot{m}(T_0 s - T_0 s_1) - \Delta \dot{S}_{irr} T_0$

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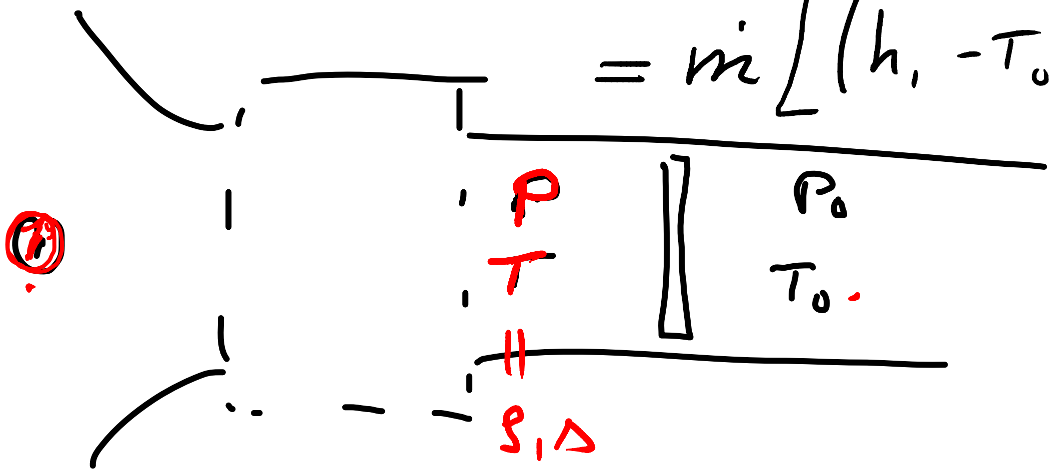
$$P = \dot m\left[(h_1 - T_0 s_1) - (h - T_0 s)\right] - \Delta\dot s_{irr} T_0$$

$$\eta \equiv 1 \rightsquigarrow \Delta\dot s_{irr} \equiv 0 \qquad h - \frac{p}{\rho} = e$$

$$P_{max} = \dot m\left[(h_1 - T_0 s_1) - (h - T_0 s)\right] - \frac{\dot m}{\rho}\left(\overset{-}{p} \overset{+}{+} p_0\right)$$



$$= \dot m\left[(h_1 - T_0 s_1) - e(\rho, s) + T_0 s - \frac{p_0}{\rho}\right]$$

$$\left.\frac{\partial P_{max}}{\partial s}\right|_v \overset{!}{=} 0, \qquad \left.\frac{\partial P_{max}}{\partial v}\right|_s \overset{!}{=} 0 \qquad \Rightarrow \quad p = p_0, \quad T = T_0$$

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$$\boxed{P_{max,opt} = \dot m \left[h_1 - h_0 - T_0 s_1 + T_0 s_0 \right]}$$

$A_0 \rho_0 u_0$

Θ_0

$$\frac{P_{max,opt}}{\dot m} := \text{Exergie}.$$

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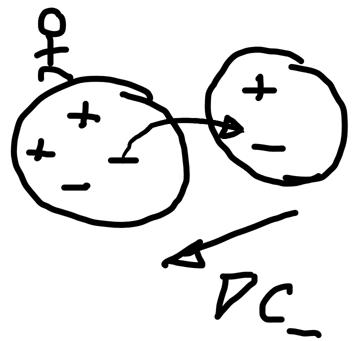
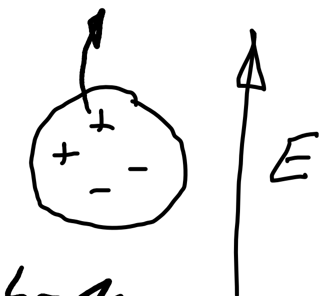
Der gesamte Stoffstromvektor $\vec{j}_+^{\,*}$ setzt sich aus einem konvektiven Anteil $c_+\vec{v}_+^{\,*}$ und einem relativen Anteil zusammen

$$\vec{j}_+^{\,*} = c_+\vec{u}^{\,*} + \vec{J}_+^{\,*}.$$



Materialgesetz für den relativen Stoffstr.

$$\vec{J}_+^{\,*} = \underbrace{-D_+\nabla c_+}_{\text{Ficksches Gesetz}} + v_+ \underbrace{F z c_+}_{\text{Coulombsche Kraft}} \vec{E}$$



$$v_+ := \text{Mobilität} = \frac{\text{Geschwindigkeit}}{\text{Kraft}}.$$

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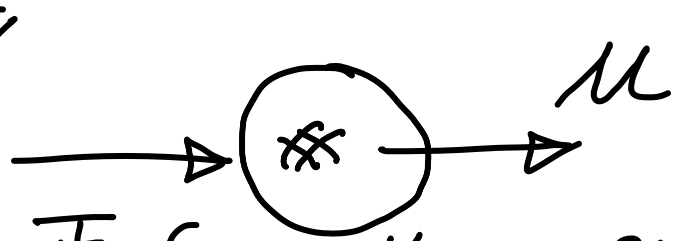

$F = 6\pi \eta a u$ Stokes'sche Widerstand für $Re = \frac{u a \varrho}{\eta} \to 0$.

Mobilität $v = \frac{u}{F} = \frac{1}{6\pi \eta a}$

vgl. Dover Verlag: Theory of Brownian Movement.

$$v_+ = \frac{D_+}{RT} \quad \text{(Nernst-Einstein-Relation)}$$

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$\vec{i}_+ = Fz\, c_+ \vec{u}^* +$	Konvektion	$\vec{i}_- = -Fz\, c_- \vec{u}^* +$
$- Fz\, D_+ \nabla c_+ +$	Diffusion	$+ Fz\, D_- \nabla c_- +$
$+ (Fz)^2 v_+ \vec{E}$	Migration	$-(Fz)^2 v_- \vec{E}$

$$v_+ := \frac{D_+}{RT} \qquad v_- = \frac{D_-}{RT}.$$

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Gesamte Stromvektor

$$\vec{i} = \vec{i}_+ + \vec{i}_-$$

$$= Fz\,\vec{u}^{*}\,(c_+ - c_-) + \qquad \text{Konv.}$$

$$- Fz\,(D_+ \nabla c_+ - D_- \nabla c_-) + \qquad \text{Diff.}$$

$$+ \sigma \vec{E}, \qquad \text{Migration}$$

mit der Abkürzg (Leitwert)

$$\sigma := (Fz)^2 \, \frac{c_+ D_+ - c_- D_-}{RT}$$

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Spezialfall Elektroneutralität

$\vec{i}_+ \cdot \vec{e}_r = 0$

$$0 = -\cancel{D_+}\frac{\partial c_+}{\partial r} - \frac{\cancel{D_+}}{RT} F z\, c_+ \frac{\partial \psi}{\partial r} \quad (1)$$

$\vec{u}^* \cdot \vec{e}_r = 0$

ln

v_+

$\vec{i}_- \cdot \vec{e}_r = 0$

$$0 = -\frac{\partial c_-}{\partial r} + \frac{Fz}{RT}\frac{\partial \psi}{\partial r} c_- \quad (2)$$

⇝ System von partielle DGLn für c_+, c_-

⇝ Lösung ist eine Boltzmannverteilung

$s = kT$.

$$c_{\pm} = c_0 \exp\left(\mp \frac{zF}{RT}\psi\right)$$

Zurück zum Anfang.

Die Ladungen sind Quelle für das elektr. Feld

$$\Delta\psi = -\frac{\rho_e}{\varepsilon}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) = -\frac{Fz}{\varepsilon}\left(c_+ - c_-\right)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right) = \frac{c_0 F z}{\varepsilon}\sinh\left(\frac{zF}{RT}\psi\right)$$

PDGL für ψ

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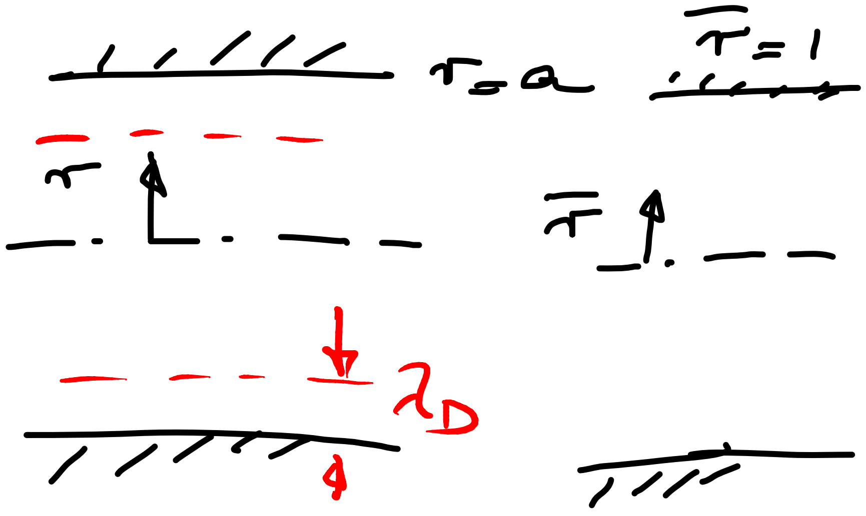
Dimensionslose Darstellung (inspektionelle [illegible])

$$r = \bar{r}\, a$$



$$\psi = \bar{\psi}\, \frac{RT}{zF}$$

$$\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{\psi}}{\partial \bar{r}} \right) = \left(\frac{a}{\lambda_D} \right)^2 \sinh \bar{\psi}$$

$$\lambda_D = \sqrt{\frac{\varepsilon RT}{2 (zF)^2 c_0}}$$

Debyesche Abschirmlänge.

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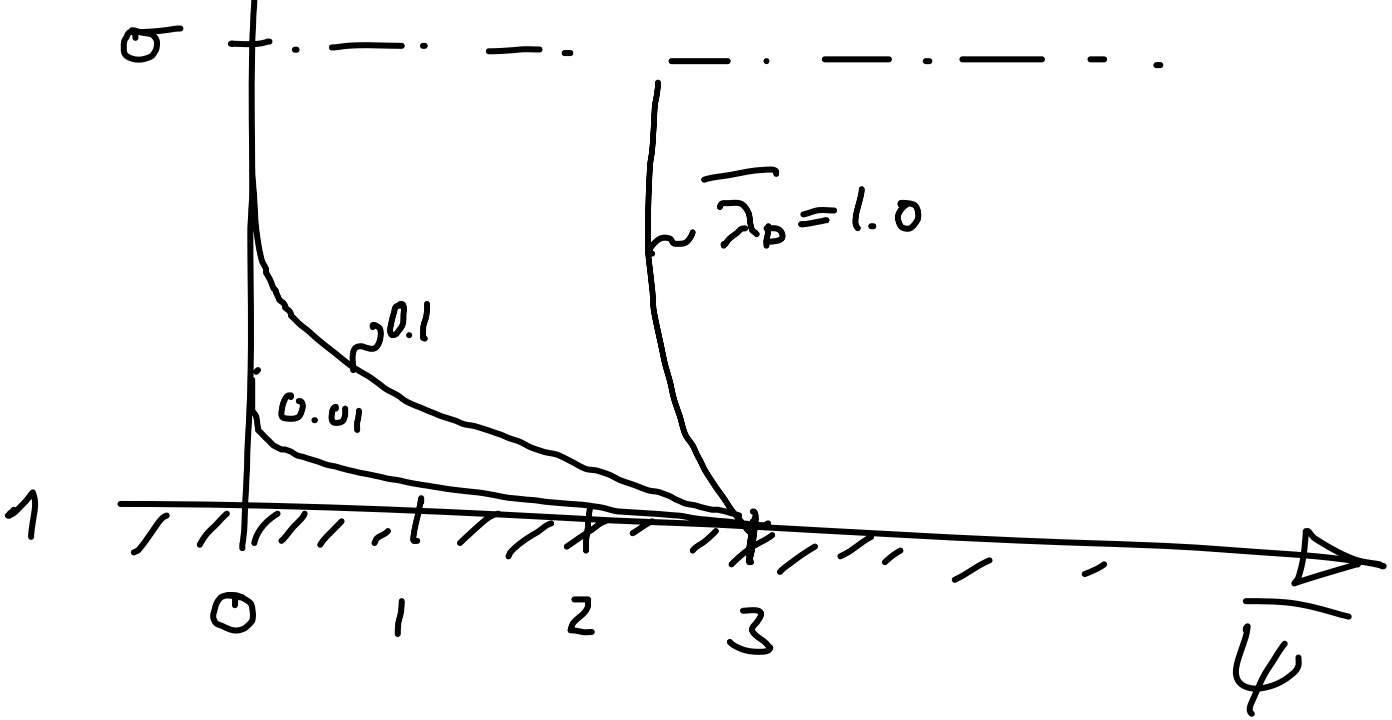
$$\bar{\lambda}_D = \frac{\lambda_D}{a}$$



$$\bar{\lambda}_D^{\,2} \, \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{\psi}}{\partial \bar{r}} \right) = \sinh \bar{\psi} \approx \bar{\psi}$$

für klein $\bar{\psi}$

$\bar{\psi} \ll 1$, d.h. $\psi \ll \frac{RT}{zF} \Rightarrow \sinh \bar{\psi} \approx \bar{\psi}$

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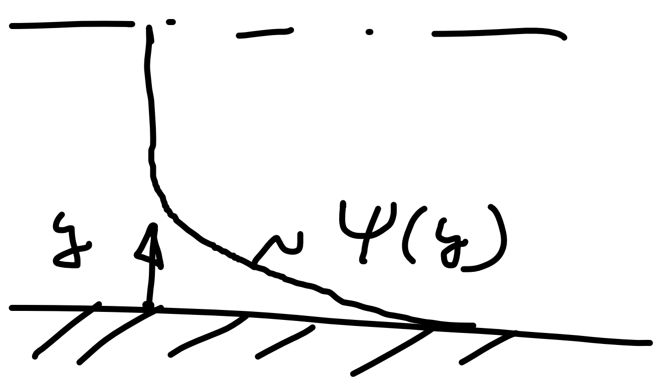
Debey–Hückel Approximation.

$\rightsquigarrow$

$$\psi = \zeta \exp\left(-\frac{y}{\lambda_D}\right)$$



$$u(r) = -\frac{a^2}{4\eta}\frac{dp}{dx}\left(1-\left(\frac{r}{a}\right)^2\right) - \frac{\varepsilon\zeta}{\eta}\left(1-\exp\left(-\frac{a-r}{\lambda_D}\right)\right)E_a$$

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