$\vec{v} = U\left(1 + u\right)\vec{e}_x + v\,\vec{e}_y$

$v = \frac{\partial h}{\partial t} + U \frac{\partial h}{\partial x}$ an der Stelle $y = 0$.

bis zur Ordnung ε^2 $\qquad \varepsilon = \frac{d}{l}$

Theorie schlanker Körper.

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Arbeit pro Zeiteinheit, die die Front verrichten muss

$$\dot W = \int_0^{\ell} \underbrace{\frac{D}{Dt}\left(m' w\right)}_{F_y} \frac{\partial h}{\partial t}\, dx$$

F_y Seitenkraft pro Längeneinheit

virtuelle Masse $m' = \rho A(x)$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + U\frac{\partial}{\partial x} + \underbrace{O(\varepsilon^2)}$$

$+U\frac{\partial}{\partial x}+$

Erfahrung und Intuition

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$$\dot W = \rho \int_0^L \underbrace{\left(\frac{\partial}{\partial t} + u\,\frac{\partial}{\partial x}\right)}_{\approx \frac{D}{Dt}}\Big(w(x,t)\,A(x)\Big)\frac{\partial h}{\partial t}\,dx$$

$$= \rho \int_0^L \left(\frac{\partial}{\partial t} + u\,\frac{\partial}{\partial x}\right)\left(w(x,t)\,A(x)\,\frac{\partial h}{\partial t}\right) dx \; +$$

$$= \rho \int_0^L \underbrace{\left(\frac{\partial^2 h}{\partial t^2} + u\,\frac{\partial^2 h}{\partial x\,\partial t}\right)}_{= \frac{\partial}{\partial t}\left(\frac{\partial h}{\partial t} + u\,\frac{\partial h}{\partial x}\right) = \boxed{\frac{\partial w}{\partial t}}} \underline{w(x,t)}\,A(x)\,dx$$

$$\frac{1}{2}\frac{\partial}{\partial t}\left(w^2\right)$$

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$$\dot W = \rho \int_0^l \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x}\right)\left\{\frac{\partial h}{\partial t}\, w(x,t)\, A(x)\right\} dx +$$

$$- \rho \int_0^l \frac{1}{2} \frac{\partial}{\partial t}\left(w^2\right) A(x)\, dx .$$

$$\overline{\dot W} = \rho \frac{\partial}{\partial t} \left\{ \int_0^l \frac{\partial h}{\partial t}\, \underset{\hat w(x) \sin \Omega t}{w(x,t)}\, A(x)\, dx - \frac{1}{2} \int_0^l w^2 A(x)\, dx \right\} +$$

$$w(x,t) = \hat w(x) \sin(\Omega t)$$

$$+ \rho u \left[\frac{\partial h}{\partial t}\, w(x,t)\, A(x) \right]_0^l$$

$$\dot W = \rho u A(l) \left.\frac{\partial h}{\partial t}\right|_l w(l,t)$$

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Arbeit pro Zeiteinheit im zeitlichen Mittel

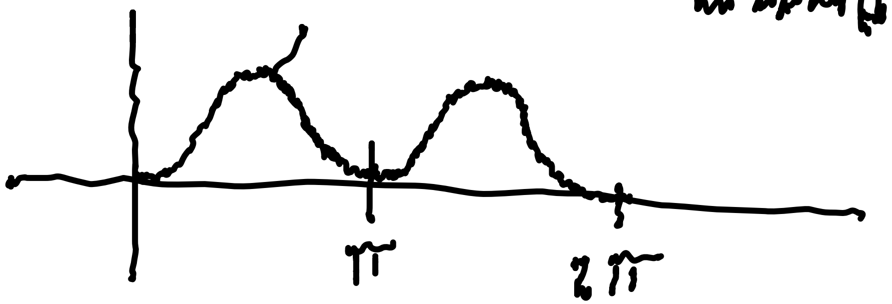
$$\overline{\dot{W}} = \varrho\, U A(\ell)\, \overline{\frac{\partial h}{\partial t}\bigg|_{\ell} w(\ell,t)}\,,$$

da $\dfrac{\partial h}{\partial t}\bigg|_0 = 0$

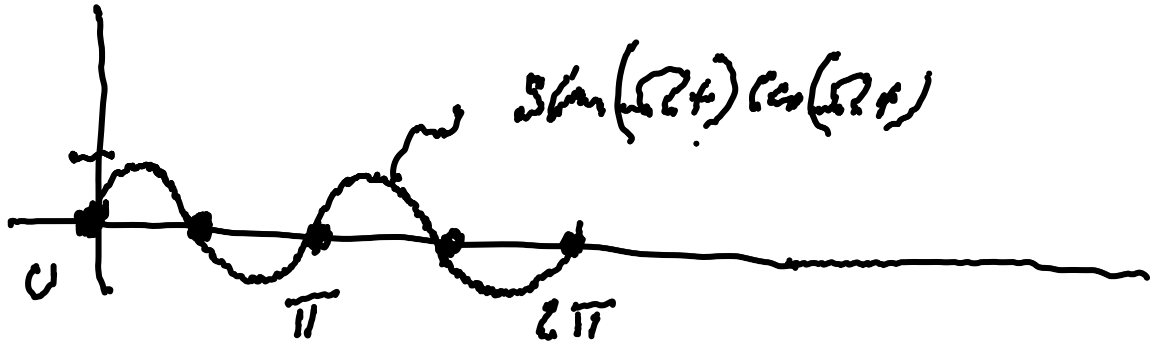
$$\overbrace{\frac{\partial}{\partial t}\int \frac{1}{2} u^2(x)\sin^2(\Omega t)\,dx}= \overbrace{\frac{\partial}{\partial t}\left(\sin^2(\Omega t)\right)}f(x)$$

$$\overline{2\sin(\Omega t)\cos(\Omega t)\,\Omega}$$

$\sin^2 \Omega t$



$\sin(\Omega t)\cos(\Omega t)$



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$f(x,t) = \hat{f}(x)\,\sin(\Omega t)$ stehende Welle.

$f(x,t) = \hat{f}(x)\,\underline{g\left(t - \frac{x}{c}\right)}$ laufende Welle

c = Phasengeschwindigkeit.

$$\frac{\partial}{\partial t}\int f(x,t)\,g(x,t)\,dx = \frac{\partial}{\partial t}\int \hat{f}(x)\,\hat{g}(x)\,\sin(\Omega t)\,\sin(\Omega t - \delta)\,dx$$

$$= \frac{\partial}{\partial t}\sin(\Omega t)\,\sin(\Omega t - \delta)\,F(x)$$

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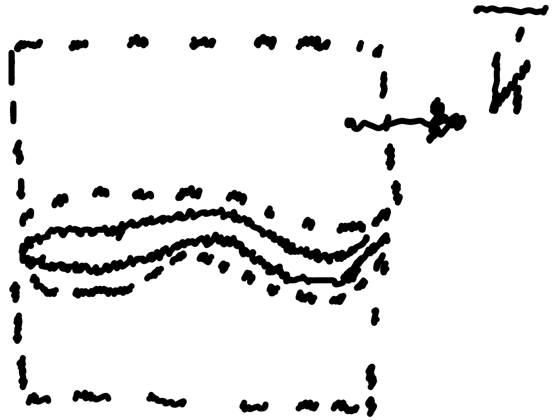
Froude'scher Wirkungsgrad

$$\eta_F = \frac{\overline{F u}}{\overline{\dot W}} = 1 - \frac{\overline{\dot K}}{\overline{\dot W}} \;?\; \checkmark$$

1. Hauptsatz in integraler Form

$$\overline{F u} = \overline{\dot W} - \underset{\parallel}{\overline{\dot K}}$$



Fluß an kinetischer Energie im Nachlauf des Fisches

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$$\overline{\dot{K}} = \overline{\left(\frac{\rho}{2} w^2 A\right)_\ell \, u}$$

Fluss der kinetischen Energie

$$\overline{\dot{W}} = \overline{\frac{\partial h}{\partial t}\left(\rho w A\right)_\ell \, u}$$

$$\eta_F = 1 - \frac{1}{2}\,\frac{\overline{\left(\frac{\partial h}{\partial t} + u\,\frac{\partial h}{\partial x}\right)^2_\ell}}{\overline{\frac{\partial h}{\partial t}\left(\frac{\partial h}{\partial t} + u\,\frac{\partial h}{\partial x}\right)_\ell}}$$

$\left.\frac{\partial h}{\partial x}\right|_\ell$

$h(x,t)$

u

☺ unabhängig von der Dicke, $A(x)$ hat sich gekürzt

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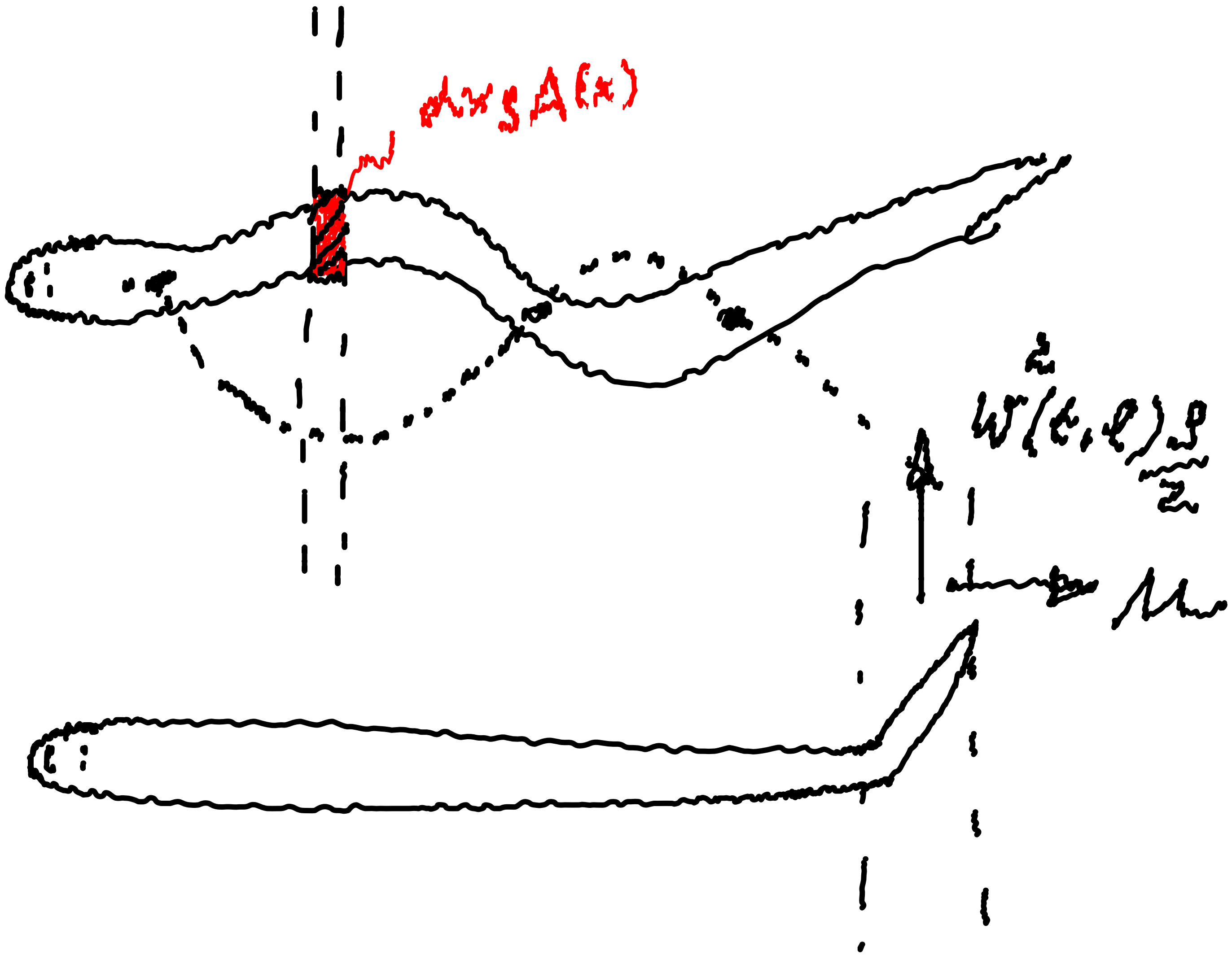

Ziel minimieren die Wucht.

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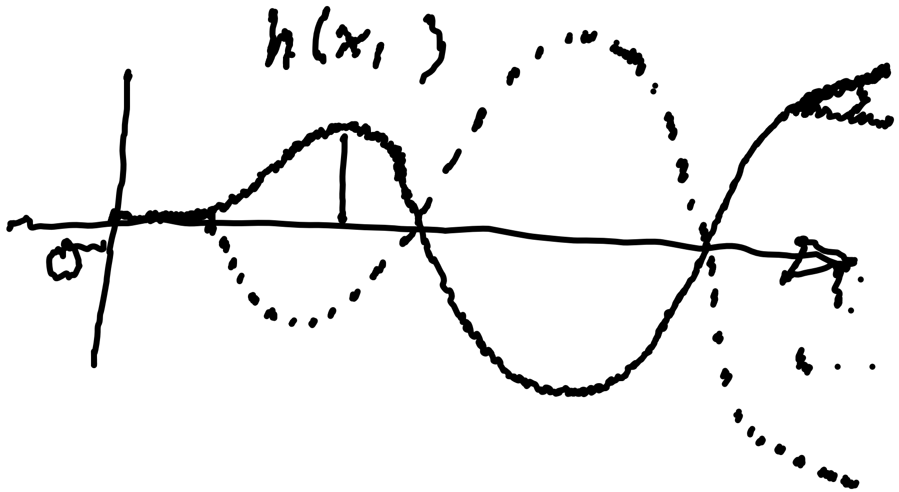
1. Wirkungsgrad bei einer stehenden Welle.

$$\frac{\partial h}{\partial t} = -\Omega H \sin(\Omega t)$$

$$h(x,t) = H(x) \cos(\Omega t)$$

stehende Welle



$$\eta_T = 1 - \frac{\quad}{\left(-\Omega H(\ell) \sin(\Omega t)\right)^2}$$

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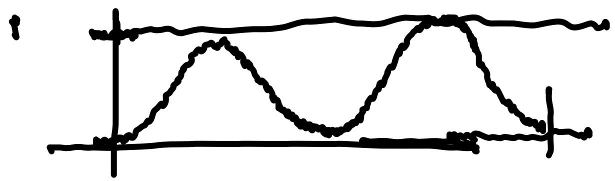
Nenner

$$\frac{\partial h}{\partial x}\left(\frac{\partial h}{\partial t} + u\,\frac{\partial h}{\partial x}\right)_{\ell} =$$

$$\left(H(\ell)\,\Omega\right)^2 \underbrace{\sin^2(\Omega t)}_{\frac{1}{2}} \qquad \underbrace{u\,\Omega\,H(\ell)\,H'(\ell)\,\sin(\Omega t)\cos(\Omega t)}_{0}$$



$$\frac{1}{2\pi}\int_0^{2\pi}\sin^2(\Omega t)\,d(\Omega t) = \frac{\pi}{2\pi} = \frac{1}{2}.$$

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zeitlich

$$\overline{\left(\frac{\partial y}{\partial t} + u\,\frac{\partial y}{\partial x}\right)^2_{x=\ell}} =$$

$$= \overline{\left(-\Omega H(\ell)\sin(\Omega t) + u\,H'(\ell)\cos(\Omega t)\right)^2}$$

$$= +\left(\Omega H(\ell)\right)^2 \frac{1}{2} + \left(u\,H'(\ell)\right)^2 \frac{1}{2}$$

$$\eta_F = 1 - \frac{1}{2}\,\frac{\left(\Omega H(\ell)\right)^2 + \left(u\,H'(\ell)\right)^2}{\left(\Omega H(\ell)\right)^2}.$$

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$$\eta_{opt} = 1 - \frac{1}{2}\,\frac{\left(\frac{\Omega_m H(\ell)}{U_m}\right)^2 + H'(\ell)^2}{\left(\frac{\Omega_m H(\ell)}{U}\right)^2} \leq \frac{1}{2}.$$

Zwei dimensionslose Produkte für die Güte der Welle.

$\frac{\Omega H(\ell)}{U}$ Amplitude

$H'(\ell)$ Pitch.

→ schlechter Wirkungsgrad.

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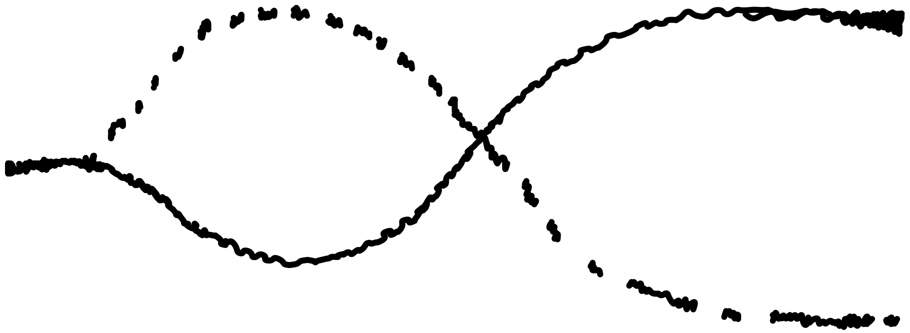
50% [illegible] [illegible] [illegible] für

$H'(\ell) = \sigma$

$H'(\ell) = \sigma$



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2. Fall

Laufende Welle

$dt - \frac{dx}{c} = const$ Charakteristik

bei längerfristig.

$$h(x,t) = f(x)\, g\Big(\overbrace{t - \frac{x}{c}}^{\text{Argument}}\Big) \qquad \text{(Welle)}$$

$f(x)$ Amplitudenfunktion.

g periodische Funktion

c Phasengeschwindigkeit.

$$\frac{\partial h}{\partial x} = h_x = f' g - \frac{1}{c} f g'$$

$$\frac{\partial h}{\partial t} = f g'$$

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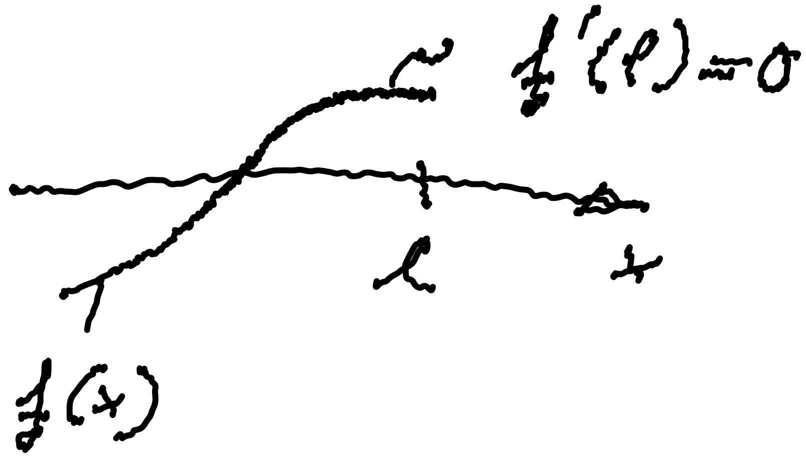
$$\zeta_{ges} = 1 - \frac{f}{g}\,\frac{\left(1-\frac{\ell}{L}\right)^2 f^2\,\overline{g'^2} + \frac{\ell}{L}\,\cancel{f^2\,g'^2}\,\overline{}}{\left(1-\frac{\ell}{L}\right) f^2\,\overline{g'^2}}$$

$f'(\ell) = 0$ Spezialfall kleiner Pickel am Schwanz



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$$\eta_T = 1 - \frac{1}{2}\left(1 - \frac{u}{c}\right) = \frac{1}{2}\left(1 + \frac{u}{c}\right)$$

$\eta_T = 90\%$ für $c = 1{,}25\, u$

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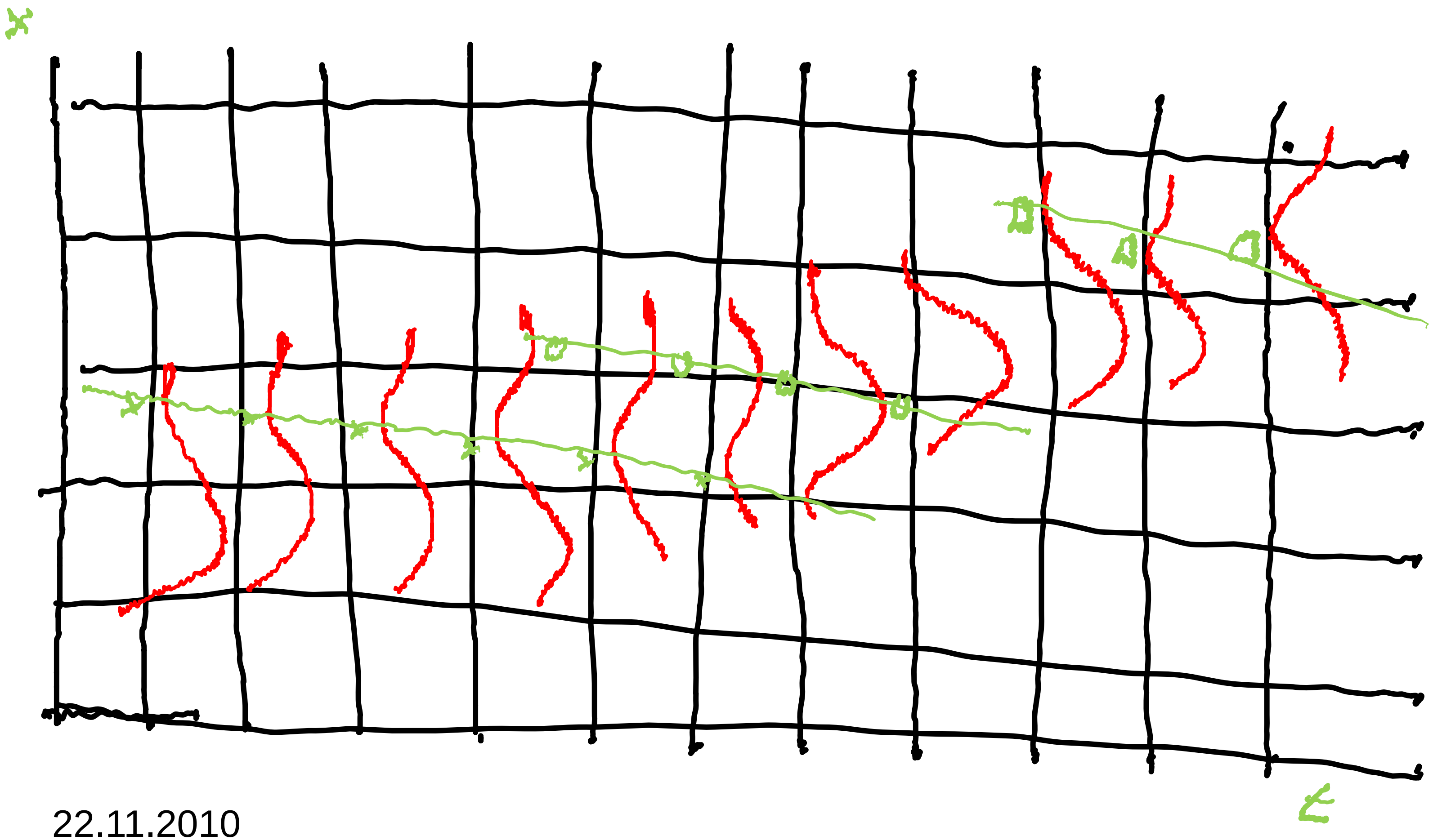
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