Neutrinoless $\beta\beta$ decay: Nuclear matrix elements and connections to other observables

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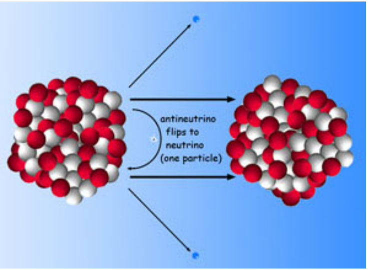
Lawrence Livermore National Laboratory — **E. A. Coello Pérez**

Nuclear matrix elements for new-physics searches

Neutrinos, dark matter studied in experiments using nuclei

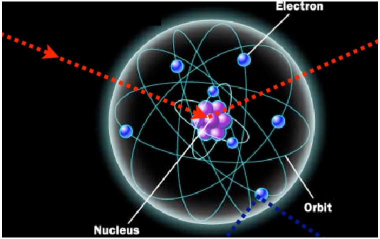
Nuclear structure physics
encoded in nuclear matrix elements
key to plan, fully exploit experiments



$$0\nu\beta\beta\text{: } \left(T_{1/2}^{0\nu\beta\beta}\right)^{-1} \propto g_A^4 \left|M^{0\nu\beta\beta}\right|^2 m_{\beta\beta}^2$$

$$\text{Dark matter: } \frac{d\sigma_{\chi\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\, \zeta_i\, \mathcal{F}_i\right|^2$$

$$\text{CE}\nu\text{NS: } \frac{d\sigma_{\nu\mathcal{N}}}{d\boldsymbol{q}^2} \propto \left|\sum_i c_i\, \zeta_i\, \mathcal{F}_i\right|^2$$



$M^{0\nu\beta\beta}$: Nuclear matrix element
$\mathcal{F}_i$: Nuclear structure factor

Creation of matter in nuclei: $0\nu\beta\beta$ decay

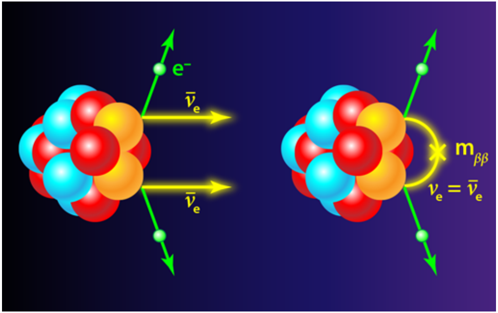
Lepton number conserved
in all processes observed:

single β decay,
$\beta\beta$ decay with 2ν emission...

Uncharged massive particles
like Majorana neutrinos (ν)
allow lepton number violation:

neutrinoless $\beta\beta$ decay
two matter particles ($2e^-$'s) created

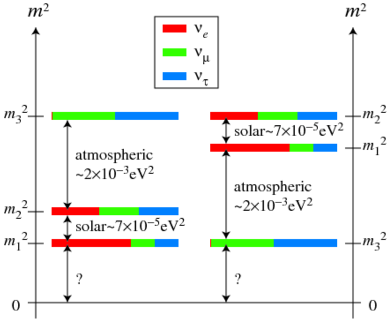
Agostini, Benato, Detwiler, JM, Vissani, arXiv:2202.01787

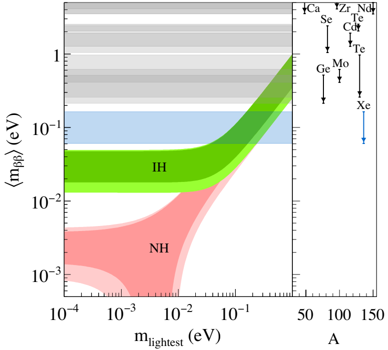


Next generation experiments: inverted hierarchy

Decay rate sensitive to neutrino masses, hierarchy $m_{\beta\beta} = |\sum U_{ek}^2 m_k|$

$$T_{1/2}^{0\nu\beta\beta}\left(0^+ \to 0^+\right)^{-1} = G_{0\nu}\, g_A^4 \left|M^{0\nu\beta\beta}\right|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

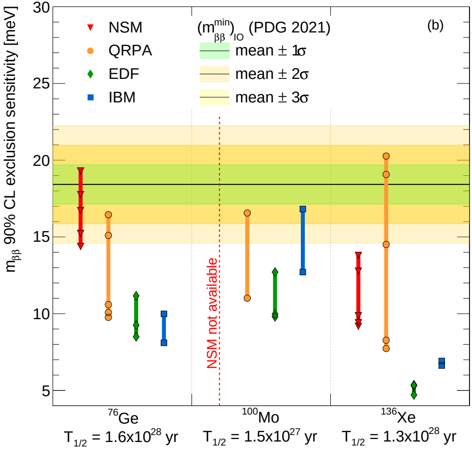




Matrix elements assess if next generation experiments fully explore "inverted hierarchy"

KamLAND-Zen, PRL117 082503(2016)

Uncertainty in physics reach of $0\nu\beta\beta$ experiments



Agostini, Benato, Detwiler, JM, Vissani
Phys. Rev. C 104 L042501 (2021)

$$T_{1/2}^{0\nu\beta\beta\ -1} = G_{0\nu}\, g_A^4 \left|M^{0\nu\beta\beta}\right|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

Nuclear matrix element theoretical uncertainty critical to anticipate $m_{\beta\beta}$ sensitivity of future experiments

Current uncertainty prevents to foresee if next-generation experiments will cover “inverted” neutrino-mass ordering

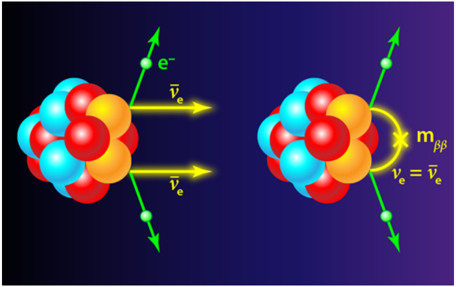
Uncertainty needs to be reduced!

Nuclear Matrix Elements / Structure Factors

Nuclear matrix elements needed in low-energy new-physics searches

$$\langle \text{Final} \,|\mathcal{L}_{\text{leptons}-\text{nucleons}}|\, \text{Initial} \rangle = \langle \text{Final} \,|\int dx\, j^{\mu}(x) J_{\mu}(x) \,|\, \text{Initial} \rangle$$

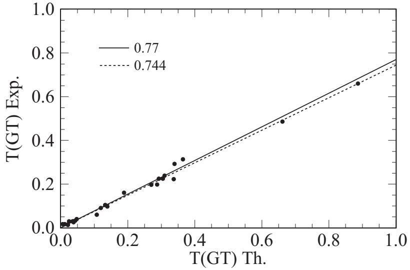
- Nuclear structure calculation of the initial and final states:
  Shell model, QRPA, IBM,
  Energy-density functional
  Ab initio many-body theory
  QMC, Coupled-cluster, IMSRG...
  EFT for heavy nuclei



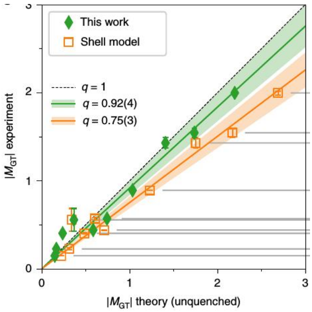
- Lepton-nucleus interaction:
  Hadronic current in nucleus:
  phenomenological,
  effective theory of QCD

β–decay Gamow-Teller transitions: “quenching”

β decays (e^- capture): nuclear shell model vs ab initio



Martinez-Pinedo et al. PRC53 2602(1996)



Gysbers et al. Nature Phys. 15 428 (2019)

$$\langle F| \sum_i [g_A\, \boldsymbol{\sigma}_i \tau_i^-]^{\text{eff}}\, |I\rangle\,, \quad [\boldsymbol{\sigma}_i \tau]^{\text{eff}} \approx 0.7 \boldsymbol{\sigma}_i \tau$$

Shell model: $\boldsymbol{\sigma}_i \tau$ “quenching”

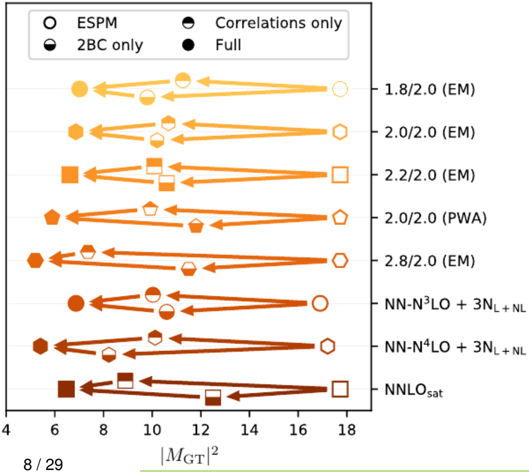
quenching: effects not in model

Ab initio calculations including
meson-exchange currents
and additional nuclear correlations

do not need “quenching”

Origin of β decay "quenching"

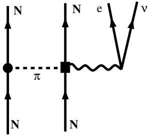
Which are main effects missing in conventional β-decay calculations?
Test case: GT decay of ^{100}Sn



Relatively similar and complementary impact of

- nuclear correlations
- meson-exchange currents

Gysbers et al.
Nature Phys. 15 428 (2019)



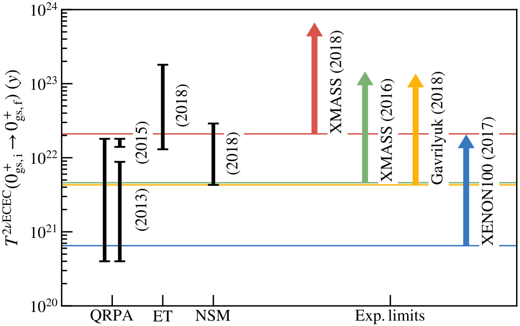
$2\nu\beta\beta$ decay, 2νECEC of ^{124}Xe

Two-neutrino $\beta\beta$ predicted for ^{48}Ca before measurement

Caurier, Poves, Zuker, PLB 252 13(1990)

Recent predictions for 2νECEC ^{124}Xe half-life:

shell model error bar largely dominated by “quenching” uncertainty



Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk
PLB 797 134885 (2019)

Shell model, QRPA and Effective field theory (ET) predictions suggest experimental detection close to XMASS 2018 limit

$2\nu\beta\beta$ decay, 2νECEC of ^{124}Xe

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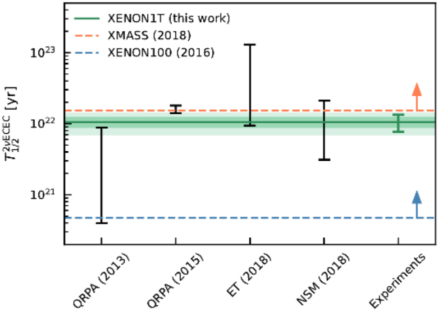


Suhonen
JPG 40 075102 (2013)

Pirinen, Suhonen
PRC 91, 054309 (2015)

Coello Pérez, JM, Schwenk

PLB 797 134885 (2019)

XENON1T
Nature 568 532 (2019)

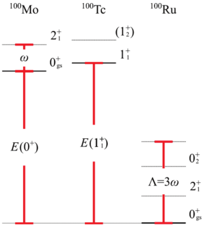
PRC106, 024328 (2022)

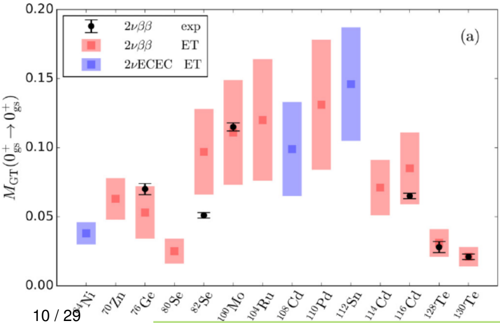
Shell model, QRPA, Effective field theory (ET) good agreement with XENON1T measurement!

Effective field theory of $\beta\beta$ decay

Effective field theory (ET) for $\beta\beta$ decay:
spherical core coupled to one nucleon

Couplings adjusted to experimental data,
uncertainty given by effective theory
(breakdown scale, systematic expansion)



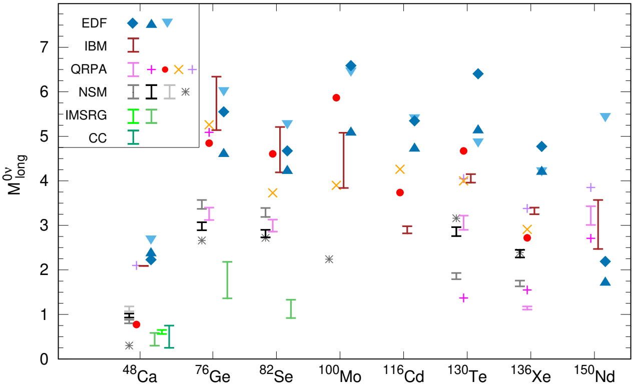


Use β-decay data
to predict $2\nu\beta\beta$ decay
Good agreement, large error
(leading-order in ET)

Coello-Pérez, JM, Schwenk
PRC 98, 045501 (2018)

$0\nu\beta\beta$ decay nuclear matrix elements

Large difference in nuclear matrix element calculations: factor ~ 3



Agostini, Benato, Detwiler, JM, Vissani, arXiv:2202.01787

Ab initio NMEs for ^{48}Ca, ^{76}Ge, ^{82}Se

Yao et al.
PRL124 232501(2020)
Novario et al.
PRL126 182502(2021)
Belley et al.
PRL126 042502(2021)

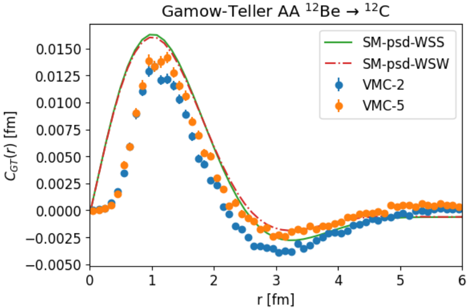
Nuclear shell model NME for ^{100}Mo

Coraggio et al.
PRC105 034312 (2022)

Shell model vs quantum Monte Carlo: correlations

Compare $\beta\beta$ transition densities in
nuclear shell model and quantum Monte Carlo calculations in light nuclei

Generally good agreement at long distances,
short-range correlations missing in shell model



Weiss, Soriano, Lovato, JM, Wiringa, arXiv:2112:08146

Similar findings in Wang et al. PLB 798 134974 (2019)

Generalized contact formalism (GCF)

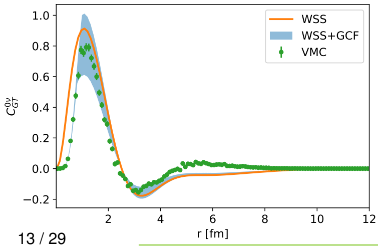
Generalized contact formalism Weiss, Bazak, Barnea PRL 114 012501 (2015)

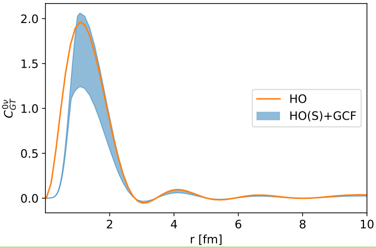
Separation of scales: transition density factorizes for nearby nucleons

$$\Psi \xrightarrow[r_{ij}\to 0]{} \sum_\alpha \varphi^\alpha(\boldsymbol{r}_{ij}) A^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}), \quad \rho_{GT}(r) \xrightarrow[r\to 0]{} -3|\varphi^0(r)|^2 C^0_{pp,nn}(f,i)$$

Contact $C^0(f,i) = \frac{A(A-1)}{2}\langle A^\alpha(f)|A^\beta(i)\rangle$ model dependent
but ratio $C^0_{pp,nn}(X)/C^0_{pp,nn}(Y)$ relatively model independent:

Combine ab initio Quantum Monte Carlo in light nuclei: short distance
with nuclear shell model in light and heavy nuclei: long distance

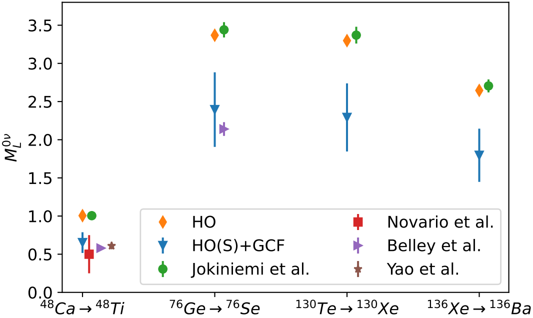




Shell model + GCF $0\nu\beta\beta$-decay matrix elements

GCF builds QMC short-range correlations to shell model densities extended to heavy nuclei where shell model calculations are possible

Weiss et al. arXiv:2112:08146



Short-range correlations included by GCF reduce $0\nu\beta\beta$ NMEs $\sim 30\%$ consistent with ab initio NMEs in ^{48}Ca, ^{76}Ge

Good agreement with ab inition in benchmark NMEs in light nuclei

Light-neutrino exchange: short-range operator

Contact operator contributes to (high-energy) light-neutrino exchange
absorbs cutoff dependence of two-nucleon decay amplitude

$$T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M_{\text{long}}^{0\nu} + M_{\text{short}}^{0\nu}\right)^2 m_{\beta\beta}^2 / m_e^2,$$ Cirigliano et al. PRL120 202001(2018)

$$M_{\text{short}}^{0\nu} \equiv \frac{1.2A^{1/3}\,\text{fm}}{g_A^2} \langle 0_f^+ | \sum_{n.m} \tau_m^- \tau_n^- \, \mathbb{1} \left[\frac{2}{\pi}\int j_0(qr)\, 2g_\nu^{\text{NN}}\, g(p/\Lambda)\, p^2 dp\right] |0_i^+\rangle,$$

$$M_{\text{GT}}^{0\nu} \simeq \frac{1.2A^{1/3}\,\text{fm}}{g_A^2} \langle 0_f^+ | \sum_{n.m} \tau_m^- \tau_n^- \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \left[\frac{2}{\pi}\int j_0(qr)\, \frac{1}{p^2}\, g_A^2\, f^2(p/\Lambda_A)\, p^2 dp\right] |0_i^+\rangle$$

Unknown value (and sign) of the hadronic coupling g_ν^{NN}!

Lattice QCD calculations Davoudi et al. PRL126 152003('21), PRD105 094502('22)
Approximate QCD methods Cirigliano et al. PRL126 172002('21), JHEP05 289('21)

Likely enhances NMEs in heavy nuclei: Wirth et al. PRL127 242502 (2021)
Jokiniemi et al. PLB823 136720 (2021)

Short-range NME: relative impact

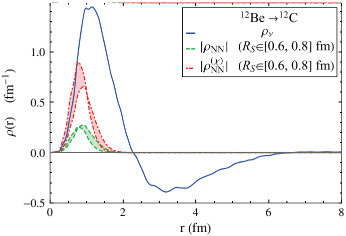
Modified decay rate: $T_{1/2}^{-1} = G_{01}\, g_A^4 \left(M_{\text{long}}^{0\nu} + M_{\text{short}}^{0\nu}\right)^2 \frac{m_{\beta\beta}^2}{m_e^2}$

Assume
$g_\nu^{NN} \sim 1\text{fm}^2$
Cirigliano et al.
PRC100 055504 (2019)

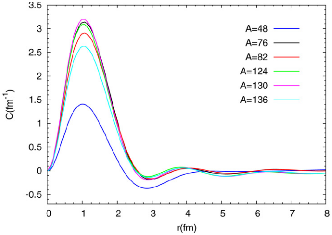
TABLE II. Values of $\mathcal{C}_1 + \mathcal{C}_2$ obtained from the CIB contact interactions in various chiral potentials.

Model	Ref.	R_S (fm)	C_0^{IT} (fm^2)	$(\mathcal{C}_1+\mathcal{C}_2)/2$ (fm^2)	Model	Ref.	Λ (MeV)	$(\mathcal{C}_1+\mathcal{C}_2)/2$ (fm^2)
NV-Ia*	[38]	0.8	0.0158	−1.03	Entem-Machleidt	[34]	500	−0.47
NV-IIa*	[38]	0.8	0.0219	−1.44	Entem-Machleidt	[34]	600	−0.14
NV-Ic	[38]	0.6	0.0219	−1.44	Reinert *et al.*	[39]	450	−0.67
NV-IIc	[38]	0.6	0.0139	−0.91	Reinert *et al.*	[39]	550	−1.01
					NNLO$_{\text{sat}}$	[37]	450	−0.39

$\sim$ 75% correction for QMC ^{12}Be NME
In heavy nuclei, less severe cancellation of dominant $M^{0\nu}$?



Cirigliano et al. PRL120 202001(2018)



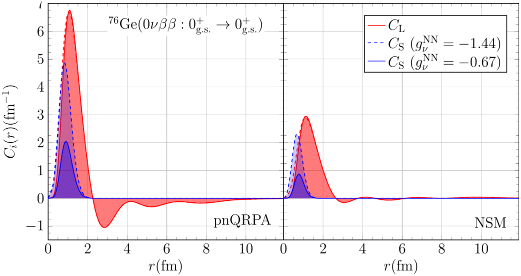
JM et al. NPA818 139 (2009)

Long and short-range NME in heavy nuclei

Relatively stable contribution of new term M_S/M_L:

20% – 50% impact of short-range NME in shell model
30% – 70% impact of short-range NME in QRPA

consistent with 43% effect in IM-GCM for ^{48}Ca
using calculated $nn \to pp + ee$ decay Wirth et al. PRL127 242502 (2021)

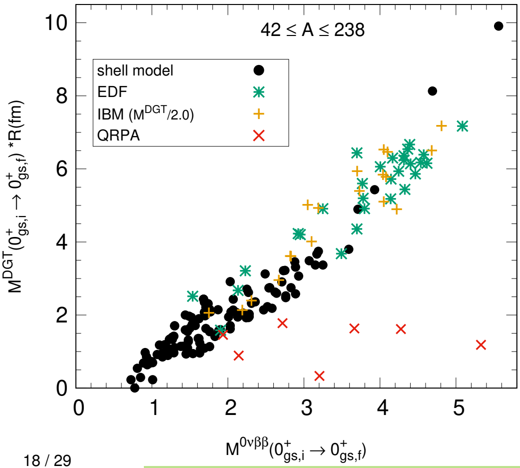


Jokiniemi, Soriano, JM, Phys. Lett. B 823 136720 (2021)

Uncertainty dominated by coupling g_ν^{NN}

Correlation of $0\nu\beta\beta$ decay to DGT transitions

Double GT transition to ground state $M^{\mathrm{DGT}} = \langle \mathrm{F_{gs}} || [\sum_i \boldsymbol{\sigma}_i \tau_i^- \times \sum_j \boldsymbol{\sigma}_j \tau_j^-]^0 || \mathrm{I_{gs}} \rangle|^2$
very good linear correlation with $0\nu\beta\beta$ decay nuclear matrix elements



Double Gamow-Teller correlation with $0\nu\beta\beta$ decay holds across nuclear chart
Shimizu, JM, Yako
PRL120 142502 (2018)

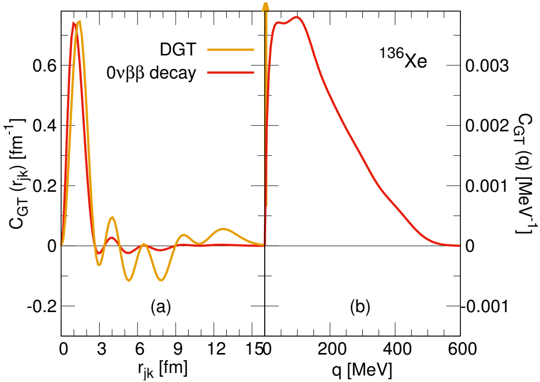
Common to shell model energy-density functionals interacting boson model, ab initio methods (weaker)
Yao et al. PRC106 014315(2022)

Experiments at RIKEN, INFN, RCNP? access DGT transitions

Short-range character of DGT, $0\nu\beta\beta$ decay

Correlation between DGT and $0\nu\beta\beta$ decay matrix elements explained by transition involving low-energy states combined with dominance of short distances between exchanged/decaying neutrons

Bogner et al. PRC86 064304 (2012)



$0\nu\beta\beta$ decay matrix element limited to shorter range

Short-range part dominant in double GT matrix element due to partial cancellation of mid- and long-range parts

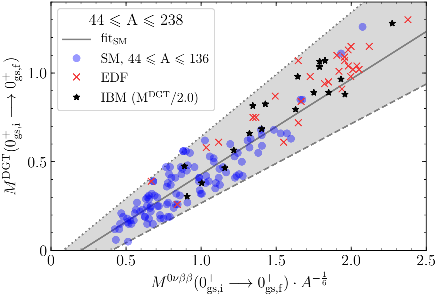
Long-range part dominant in QRPA DGT matrix elements

Shimizu, JM, Yako, PRL120 142502 (2018)

$0\nu\beta\beta$ decay NMEs in EFT of β decay

Effective field theory of β decay can calculate DGT with uncertainties
(similar to calculation of $2\nu\beta\beta$, no energy denominator)
DGT vs 0nbb correlation $\Rightarrow$ predict $0\nu\beta\beta$ NMEs with uncertainties

Because EFT couplings fitted to β decay and GT strengths
shell-model DGT NMEs in correlation need "quenching": $q = 0.42 - 0.65$



As a result, ET $0\nu\beta\beta$ NMEs
^{76}Ge: $M^{0\nu} = 0.2 - 2.4$
^{82}Se: $M^{0\nu} = 0.2 - 2.7$

small NMEs

large uncertainty:
LO in ET, fit, "quenching"

Brase, Coello Pérez, JM, Schwenk
PRC106 034309(2022)

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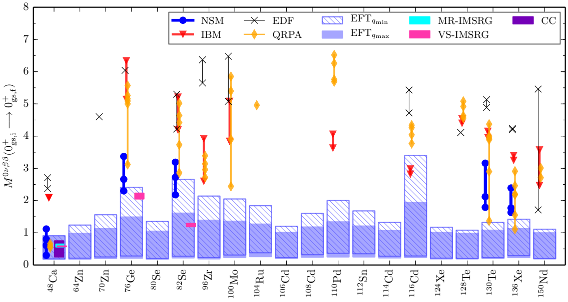
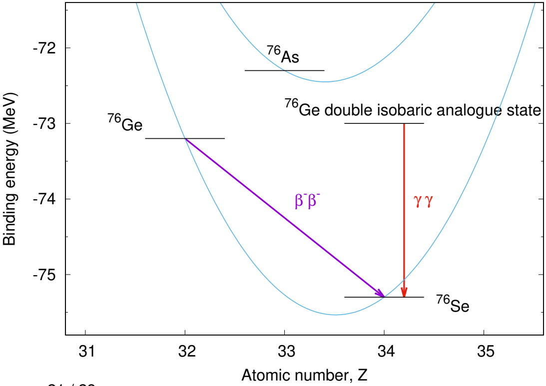


Brase, Coello Pérez, JM, Schwenk, PRC106 034309 (2022)

$\gamma\gamma$ decay of the DIAS of the initial $\beta\beta$ nucleus

Explore correlation between $0\nu\beta\beta$ and $\gamma\gamma$ decays,
focused on double-M1 transitions

$$M^{\gamma\gamma}_{M1\,M1} = \sum_k \frac{\langle 0^+_f | \sum_n (g^l_n \boldsymbol{l}_n + g^s_n \boldsymbol{\sigma}_n)^{IV} | 1^+_k(\text{IAS})\rangle \langle 1^+_k(\text{IAS}) | \sum_m (g^l_m \boldsymbol{l}_m + g^s_m \boldsymbol{\sigma}_m)^{IV} | 0^+_i(\text{DIAS})\rangle}{E_k - (E_i + E_f)/2}$$



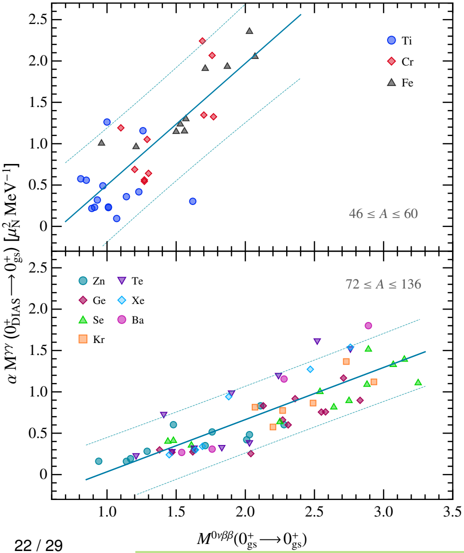
Similar initial and final states
but both in same nucleus
for electromagnetic transition

M1 and GT operators similar,
physics of spin operator
M1 also angular momentum

Different energy denominator

Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Correlation between $M1M1$ and $0\nu\beta\beta$ NMEs



Good correlation between $M1M1$ same-energy photons and $0\nu\beta\beta$ NMEs!

Valid across the nuclear chart for the nuclear shell model

Overall, study $\sim$ 50 transitions several nuclear interactions for each of them

The correlation is slightly different for lighter nuclei: effect of energy denominator

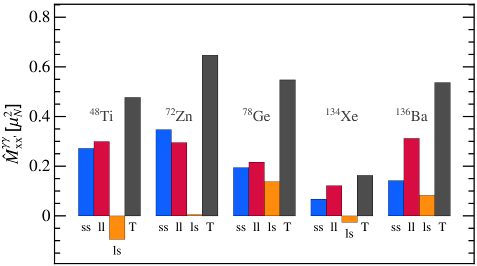
Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Spin, angular momentum decomposition

The numerator NME can be decomposed into

$$\hat{M}_{ss} + \hat{M}_{ll} + \hat{M}_{ls}$$

spin, angular momentum and interference components



Spin, angular momentum terms strikingly similar,
always carry same sign

Interference term
can cancel the other two
but always much smaller

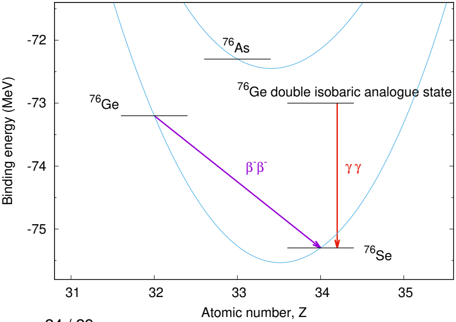
Romeo, JM, Peña-Garay
PLB 827 136965 (2022)

Experimental feasibility of $\gamma\gamma$ decay?

$\gamma\gamma$ decays are very suppressed with respect to γ decays
just like $\beta\beta$ decays are much slower than β decays

$\gamma\gamma$ decays have been observed recently
in competition with γ decays

Waltz et al. Nature 526, 406 (2015), Soderstrom et al. Nat. Comm. 11, 3242 (2020)



Outlook:

Study in detail leading
decay channels for $M1M1$ decay
in DIAS of $\beta\beta$ nuclei

Particle emission,
$M1$, $E1$ decay: $10^{-7} - 10^{-8}$ BR

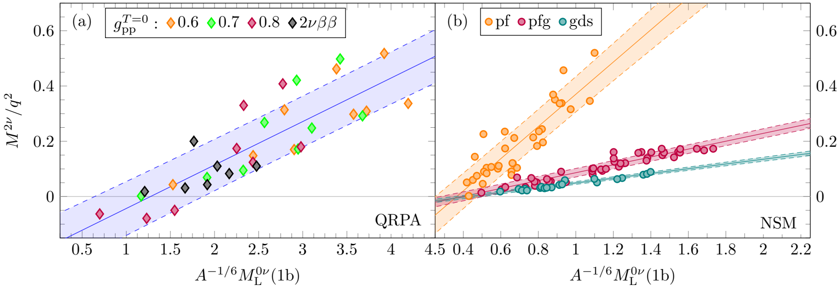
Experimental proposal for ^{48}Ti
by Valiente-Dobón et al.

Valiente-Dobón, Romeo et al., in prep

Correlation of $0\nu\beta\beta$ decay and $2\nu\beta\beta$ decay

Good correlation between 2ν and 0ν modes of $\beta\beta$ decay
in nuclear shell model (systematic calculations of different nuclei)
and QRPA calculations (decays of $\beta\beta$ emitters with different g_{pp} values)

Similar but not common correlation, depends on mass for shell model
$0\nu\beta\beta - 2\nu\beta\beta$ correlation also observed in ^{48}Ca Horoi et al. arXiv:2203.10577



Jokiniemi, Romeo, Soriano, JM, arXiv:2207.05108

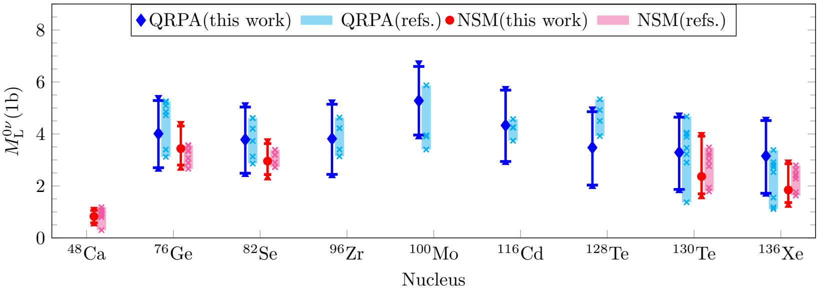
Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs!

$0\nu\beta\beta$ NMEs from $2\nu\beta\beta - 0\nu\beta\beta$ correlation

NMEs consistent with previous nuclear shell model, QRPA results

Theoretical uncertainty involves
systematic calculations covering dozens of nuclei and interactions
error of each calculation (eg quenching) and experimental $2\nu\beta\beta$ error

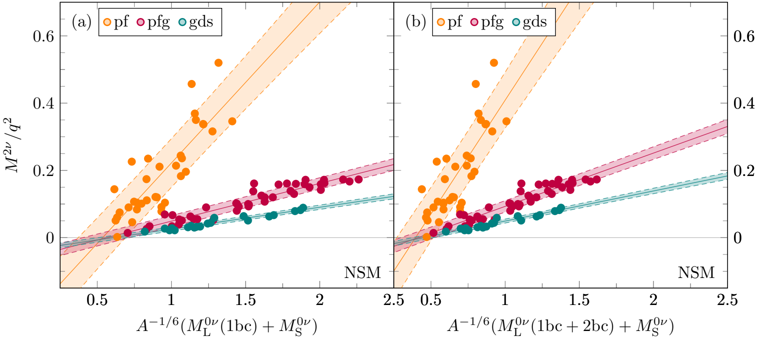
Previous theoretical uncertainty mostly ignored: collection of calculations



Jokiniemi, Romeo, Soriano, JM, arXiv:2207.05108

Correlation of $0\nu\beta\beta$ decay to $2\nu\beta\beta$: general case

A good correlation between $2\nu\beta\beta$ and $0\nu\beta\beta$
also appears when we include to the calculation of $0\nu\beta\beta$ NMEs
2b currents and the short-range nuclear matrix element



Jokiniemi, Romeo, Soriano, JM, arXiv:2207.05108

Use $2\nu\beta\beta$ data to predict $0\nu\beta\beta$ NMEs with 2b currents, short-range NME

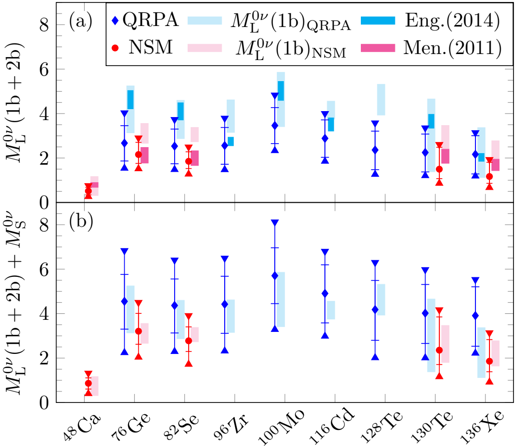
$0\nu\beta\beta$ NMEs from correlation: 2bc, short-range

$0\nu\beta\beta$ NMEs including 2b currents and short-range NME obtained from $0\nu\beta\beta - 2\nu\beta\beta$ correlation and $2\nu\beta\beta$ data

Theoretical uncertainty due to correlation, calculation uncertainties: quenching, 2bc, short-range NME coupling (dominant uncertainty)

First complete estimation of $0\nu\beta\beta$ nuclear matrix elements with theoretical uncertainties

Jokiniemi, Romeo, Soriano, JM, arXiv:2207.05108



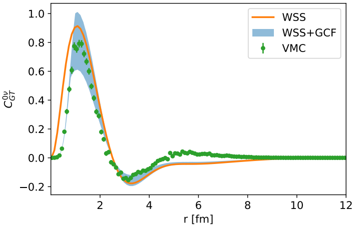
Summary

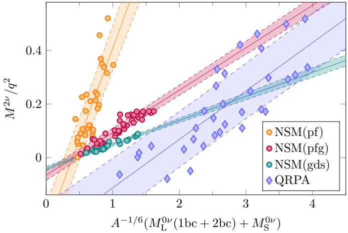
Calculations of $0\nu\beta\beta$ NMEs
challenge nuclear many-body methods,
searches demand reliable NMEs

Ab initio results suggest reduced NMEs
due to nuclear correlations (eg via GCF)
and two-body currents
Likely enhancement by short-range NME

Double Gamow-Teller transitions,
electromagnetic $M1M1$ decay of DIAS
good correlation with $0\nu\beta\beta$ NMEs

Good $0\nu\beta\beta - 2\nu\beta\beta$ correlation
exploit $2\nu\beta\beta$ data to obtain $0\nu\beta\beta$ NMEs
with theoretical uncertainties





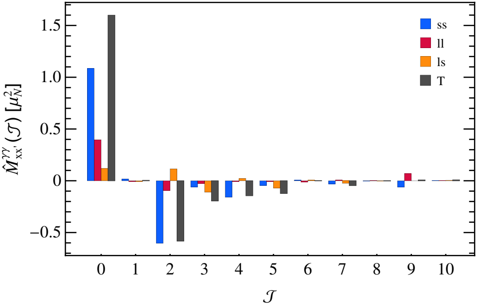
Thank you very much!

Total angular momentum decomposition

The numerator NME can be decomposed into

$$\hat{M}_{ss}(\mathcal{J}) + \hat{M}_{ll}(\mathcal{J}) + \hat{M}_{ls}(\mathcal{J})$$

spin, angular momentum and interference components
and total angular momentum of the nucleons involved in the transition



Dominance of $\mathcal{J} = 0$ terms
for spin and orbital contributions
just like in $0\nu\beta\beta$ decay

Cancellation from $\mathcal{J} > 0$ terms
less pronounced in orbital part

Explains similar behaviour of
spin and orbital components:

$$s_1\, s_2 = \mathcal{S}^2 - 3/2 < 0$$

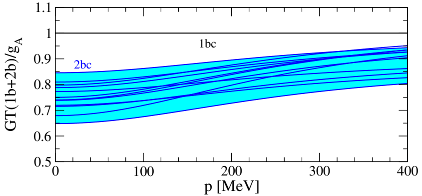
$$l_1\, l_2 = \mathcal{L}^2 - l_1^2 - l_2^2 < 0$$

Romeo et al. PLB 827 136965 (2022)

2b currents in $0\nu\beta\beta$ decay

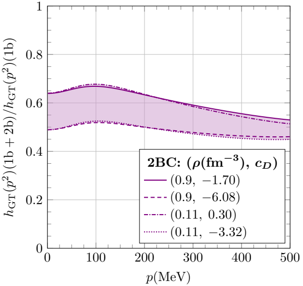
In $0\nu\beta\beta$ decay, two weak currents lead to four-body operator when including the product of two 2b currents: computational challenge

Approximate 2b current as effective 1b current



Quenching reduced to $\sim 20\%$ at $p \sim m_\pi$ for $0\nu\beta\beta$ decay

JM et al. PRL107 062501(2011)



Smaller quenching reduction at $p \sim m_\pi$

Jokiniemi, Romeo, Soriano, JM arXiv:2207.05108

Contact operator for NMEs of heavy nuclei

Calculate $M^{0\nu}_{\text{short}}$ in heavy nuclei used in $0\nu\beta\beta$ searches

Use g_ν^{NN} and Λ values from
charge independence breaking (CIB) contact term of chiral potentials
assume same value for two CIB couplings $\mathcal{C}_1 = \mathcal{C}_2$

$g_\nu^{\text{NN}}(\text{fm}^2)$	Λ (MeV)	
-0.67	450	Reiner et al. Eur. Phys. J. A 54 86 (2018)
-1.01	550	"
-1.44	465	Piarulli et al. Phys. Rev. C 94 054007 (2016)
-0.91	465	"
-1.44	349	"
-1.03	349	"

Consider Gaussian regulators: $h_s = 2g_\nu^{\text{NN}} g(p/\Lambda)$

Perform calculations with the nuclear shell model:
^{48}Ca, ^{76}Ge, ^{82}Se, ^{124}Sn, ^{128}Te, ^{130}Te and ^{136}Xe

and the quasiparticle random-phase approximation method (QRPA):
^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{124}Sn, ^{128}Te, ^{130}Te and ^{136}Xe