Towards Supernova Simulations with Six Species Neutrino Transport

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Outline

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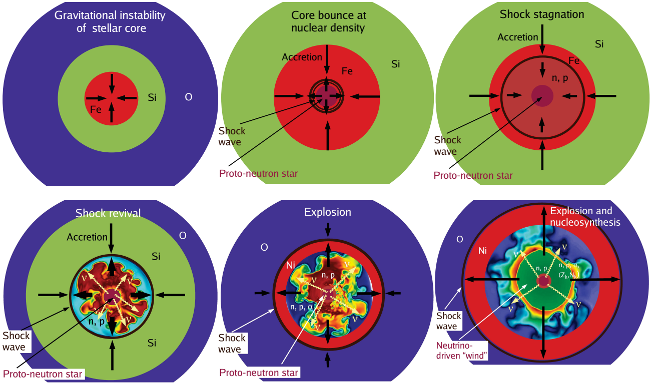
1. Muonization in core-collapse supernova (CCSN)
2. Consequences of the muonization in its modelling:
   - adding muons to the equations of state (EOS)
   - coupling *e* and μ neutrino flavours in the transport (six species neutrinos transport)
3. Implementation in AGILE-BOLTZTRAN
4. Summary and conclusions

The supernova mechanism



H.-Th. Janka, *et al*, PTEP 01A309 (2012)

Supernova conditions shortly after bounce

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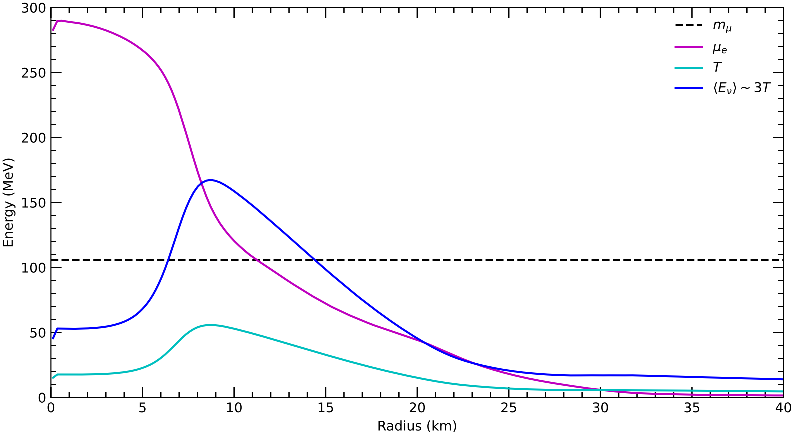

Muon lepton flavour reactions

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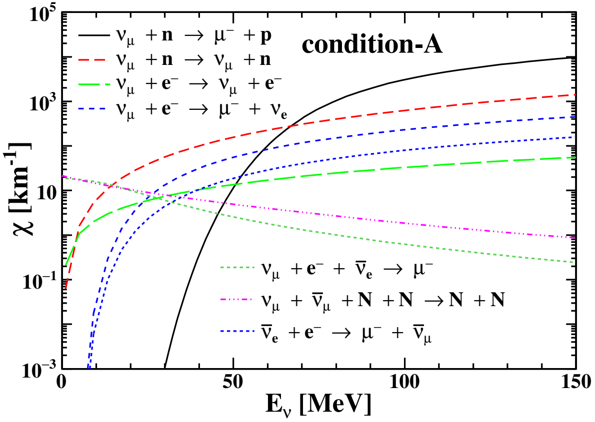
Neutrino production	Muon lepton flavour production: Semileptonic	Muon lepton flavour production: Leptonic
$e^- + e^+ \to \nu + \bar{\nu}$	$\bar{\nu}_\mu + p \to n + \mu^+$	$\bar{\nu}_e + e^- \to \bar{\nu}_\mu + \mu^-$
$N + N \to \nu + \bar{\nu} + N + N$	$\nu_\mu + n \to p + \mu^-$	$\nu_e + e^+ \to \nu_\mu + \mu^+$
	$n \to p + \bar{\nu}_\mu + \mu^-$	$\nu_\mu + e^- \to \nu_e + \mu^-$
		$\bar{\nu}_\mu + e^+ \to \bar{\nu}_e + \mu^+$
		$\nu_\mu + \bar{\nu}_e + e^- \to \mu^-$
All $\nu, \bar{\nu}$ flavours particularly $\nu_\mu, \bar{\nu}_\mu$	Excess of μ^-	

Couple e and μ-neutrino flavours!

Relevance of the neutrino flavour coupling

Full kinematics opacities of G. Guo, *et al.* (2020)



G. Guo, *et al.* (2020)

condition-A:

- $t_{pb} = 400$ ms
- $r \simeq 13.6$ km
- $\rho \simeq 10^{14}\,\mathrm{g\,cm^{-3}}$
- $T \simeq 38.3$ MeV
- $Y_e = 0.11$
- $Y_\mu = 0.04$

Existing work

2D CCSN simulations of R. Bollig, *et al.* (2017)

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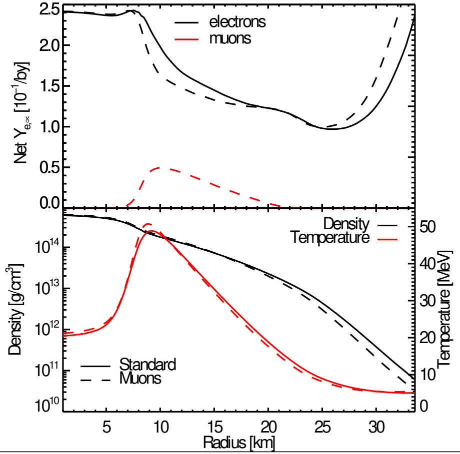

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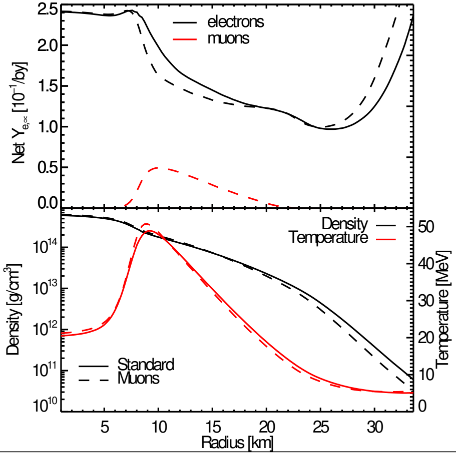

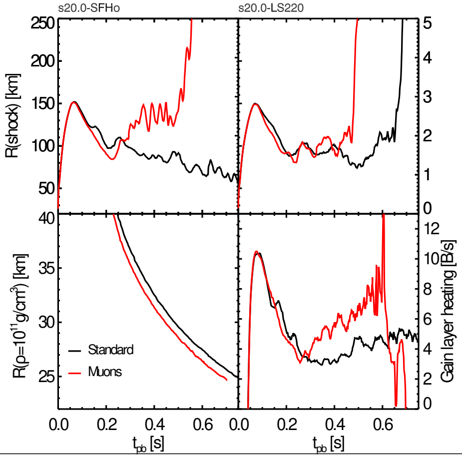

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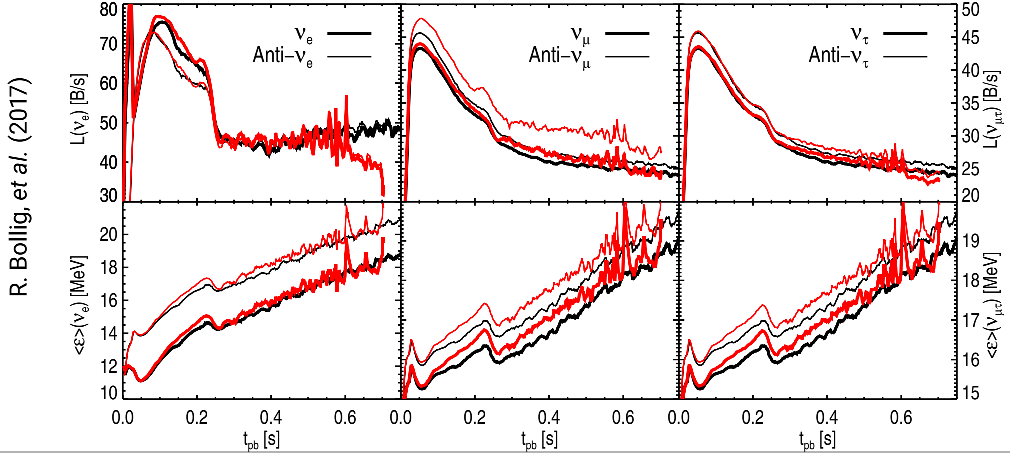

Consistent neutrino lepton flavour coupling in the transport

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- Boltzmann Transport Equation

$$p^{\beta}\frac{\partial f_{\nu_i}}{\partial x^{\beta}} - \Gamma^{\alpha}_{\beta\gamma}p^{\beta}p^{\gamma}\frac{\partial f_{\nu_i}}{\partial p^{\alpha}} = \left(\frac{\mathrm{d}f_{\nu_i}}{\mathrm{d}\tau}\right)_{\text{coll}}, \quad \text{where} \quad \left(\frac{\mathrm{d}f_i}{\mathrm{d}\tau}\right)_{\text{coll}} = F_i(f_{\nu_e}, f_{\bar{\nu}_e}, f_{\nu_\mu}, f_{\bar{\nu}_\mu}, T, Y_e, Y_\mu)$$

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- Energy-momentum conservation

$$\nabla_\alpha T^{\beta\alpha}_{\mathrm{fluid}} = -G^\beta(f_{\nu_e}, f_{\bar{\nu}_e}, f_{\nu_\mu}, f_{\bar{\nu}_\mu}, T, Y_e, Y_\mu)$$

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$$\nabla_\alpha(\rho Y_e u^\alpha) = -m_B L_e(f_{\nu_e}, f_{\bar{\nu}_e}, f_{\nu_\mu}, f_{\bar{\nu}_\mu}, T, Y_e, Y_\mu)$$

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- Muon lepton number conservation

$$\nabla_\alpha(\rho Y_\mu u^\alpha) = -m_B L_\mu(f_{\nu_e}, f_{\bar{\nu}_e}, f_{\nu_\mu}, f_{\bar{\nu}_\mu}, T, Y_e, Y_\mu)$$

where F_i, G^β, L_e, L_μ are the source/shrink rates due to interactions.

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BOLTZTRAN transport equation

A. Mezzacappa, S.W. Bruenn (1993b)

Metric in spherical symmetry:

$$ds^2 = -\alpha^2 dt^2 + \left(\frac{r'}{\Gamma}\right)^2 da^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right).$$

$$\underbrace{\frac{1}{\alpha}\frac{\partial F_\nu}{\partial t}}_{C_t} + \underbrace{\frac{\mu}{\alpha}\frac{\partial}{\partial a}\left(4\pi r^2 \alpha\rho F_\nu\right)}_{D_a} + \underbrace{\Gamma\left(\frac{1}{r} - \frac{1}{\alpha}\frac{\partial\alpha}{\partial r}\right)\frac{\partial}{\partial\mu}\left[\left(1-\mu^2\right)F_\nu\right]}_{D_\mu} + \underbrace{\left(\frac{d\ln\rho}{\alpha dt} + \frac{3u}{r}\right)\frac{\partial}{\partial\mu}\left[\mu\left(1-\mu^2\right)F_\nu\right]}_{O_\mu}$$

$$+ \underbrace{\left[\mu^2\left(\frac{d\ln\rho}{\alpha dt} + \frac{3u}{r}\right) - \frac{u}{r} - \mu\Gamma\frac{1}{\alpha}\frac{\partial\alpha}{\partial r}\right]\frac{1}{E^2}\frac{\partial(E^3 F_\nu)}{\partial E}}_{D_E + O_E} = \underbrace{\left(\frac{\partial F_\nu}{\partial t}\right)^{\mathrm{coll}}}_{C_c},$$

where $F_\nu = F_\nu(a, \mu, E, t) = f_\nu/\rho$.

Finite difference representation of Boltzmann transport equation

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- Finite differencing the transport, energy and lepton number equations, and its linearization,

$$F_{i',j',k',} = F^0_{i',j',k'} + \delta F_{i',j',k'},$$

$$\varepsilon_{i'} = \varepsilon^0_{i'} + \delta\varepsilon_{i'},$$

$$Y_{e,i'} = Y^0_{e,i'} + \delta Y_{e,i'},$$

$$Y_{\mu,i'} = Y^0_{\mu,i'} + \delta Y_{\mu,i'}.$$

where i', j', k' are indices for the mass shell, neutrino angle and energy bins, lead to a system of equations:

$$-\mathbf{C}_i\mathbf{V}_{i-1} + \mathbf{A}_i\mathbf{V}_i - \mathbf{B}_i\mathbf{V}_{i+1} = \mathbf{U}_i,$$

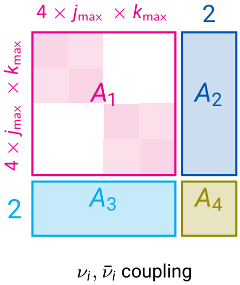
where the solution vector is

$$\mathbf{V}_i = \Big(\delta F^{\nu_e}_{i',1',1'}, \delta F^{\nu_e}_{i',2',1'}, \ldots, \delta F^{\nu_e}_{i',j_{\max},k_{\max}}, \delta F^{\bar\nu_e}_{i',1',1'}, \delta F^{\bar\nu_e}_{i',2',1'}, \ldots, \delta F^{\bar\nu_e}_{i',j_{\max},k_{\max}},$$
$$\delta F^{\nu_\mu}_{i',1',1'}, \delta F^{\nu_\mu}_{i',2',1'}, \ldots, \delta F^{\nu_\mu}_{i',j_{\max},k_{\max}}, \delta F^{\bar\nu_\mu}_{i',1',1'}, \delta F^{\bar\nu_\mu}_{i',2',1'}, \ldots, \delta F^{\bar\nu_\mu}_{i',j_{\max},k_{\max}}, \delta T_{i'}, \delta Y_{e,i'}, \delta Y_{\mu,i'}\Big)^\top$$

Solution of the transport equation

$$-\mathbf{C}_i\mathbf{V}_{i-1} + \mathbf{A}_i\mathbf{V}_i - \mathbf{B}_i\mathbf{V}_{i+1} = \mathbf{U}_i$$

$\mathbf{B}_i$ and $\mathbf{C}_i$ are diagonal representing the coupling of the next and previous shells, and $\mathbf{A}_i = \begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix}$



A_1 is a dense matrix accouting for the coupling in energy, angle and neutrino species.

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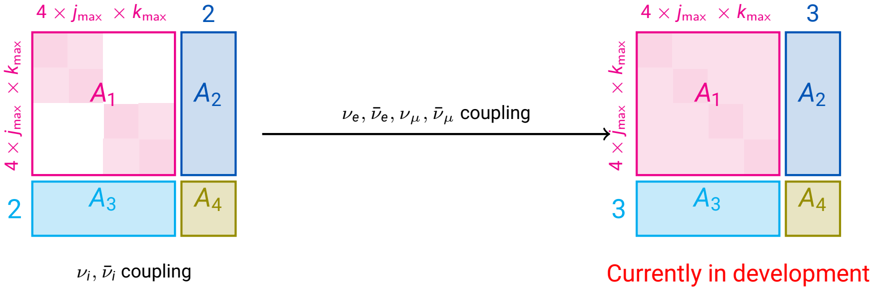

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Solution of the transport equation

- Finally, the system of equations coupling all mass shells can be writen as:

$$\begin{pmatrix} \mathbf{A}_1 & -\mathbf{B}_1 & 0 & 0 & 0 & \cdots \\ -\mathbf{C}_2 & \mathbf{A}_2 & -\mathbf{B}_2 & 0 & 0 & \cdots \\ 0 & -\mathbf{C}_3 & \mathbf{A}_3 & -\mathbf{B}_3 & 0 & \cdots \\ \vdots & 0 & & & & \\ \vdots & \vdots & & & & \end{pmatrix} \begin{pmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \\ \vdots \\ \vdots \end{pmatrix} = \begin{pmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \mathbf{U}_3 \\ \vdots \\ \vdots \end{pmatrix},$$

which is solve in a Newton-Raphson iteration scheme.

Conclusion

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- Post bounce supernova conditions allow muon creation
- Present simulations show appearance of net muon abundance
- 2D simulations show important impact in the explotability and ν-heating
- The strong coupling of ν_e and ν_μ has to be reflected in the transport
- Detailed transport is needed for:
  - ν-oscillations due to angular distribution dependence, in particular fast flavour oscillations
  - Use as benchmark for approximate and moment-based neutrino transport
- We have added a new degree of freedom in the EOS implemented as (ρ, T, Y_e, Y_μ)
- We are currently implementing the coupled transport between e and μ flavour neutrinos