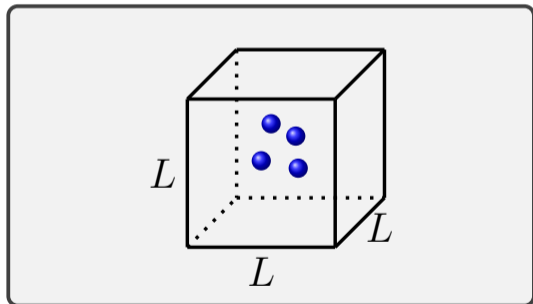
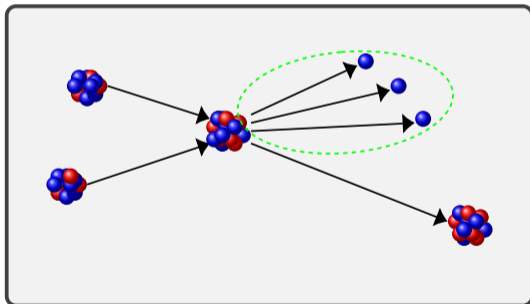


# Energy distribution of the 3n system in pionless effective field theory

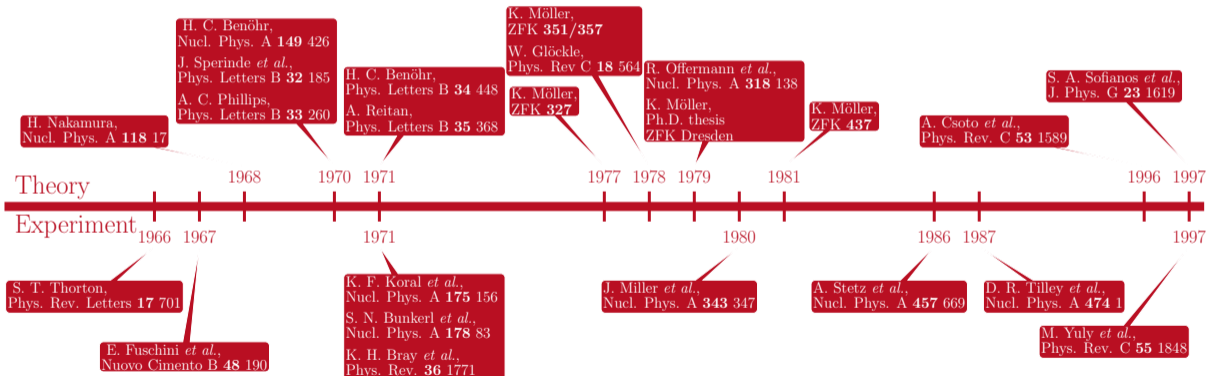


TECHNISCHE  
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DARMSTADT

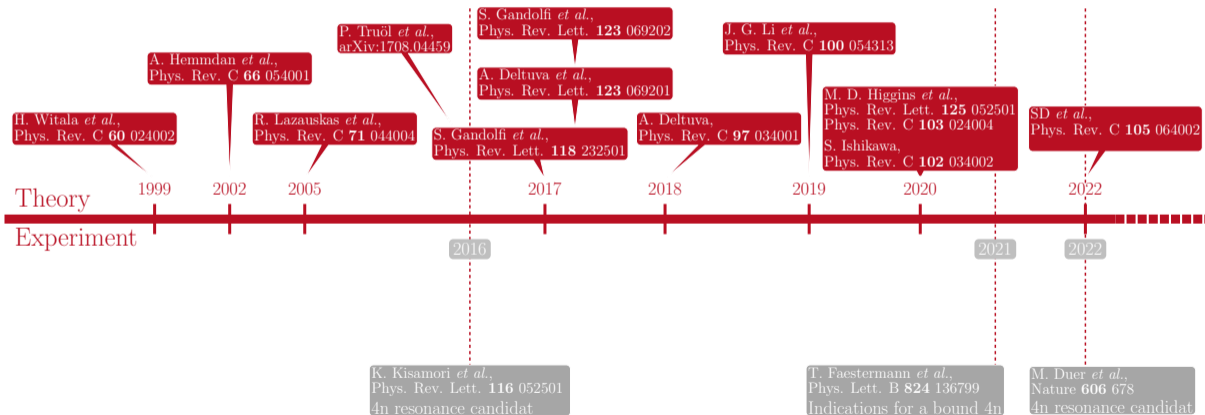
Sebastian Dietz  
SFB Workshop 2022



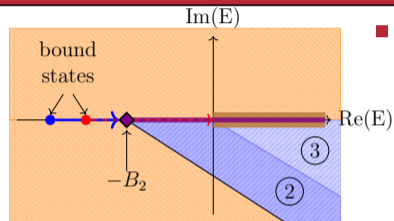
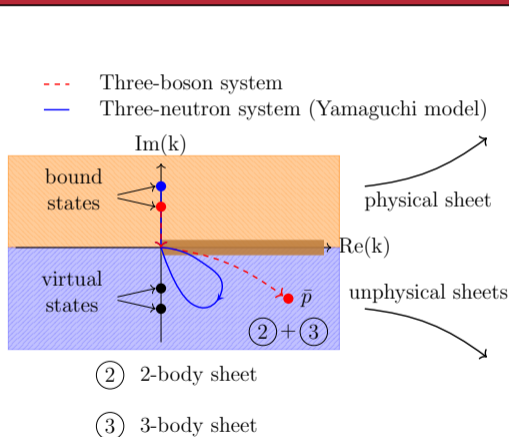
# Three neutrons: The history I



# Three neutrons: The history II



# Sheet structure

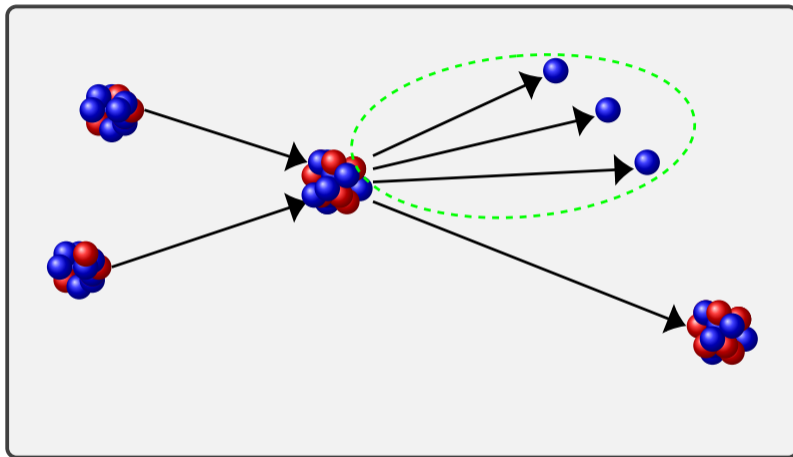


■ SD, H.-W. Hammer, S. König, A. Schwenk, PRC **105** 064002 (2002)

- No evidence for 3n resonance using analytical continuation & finite-volume approach

■ Today:

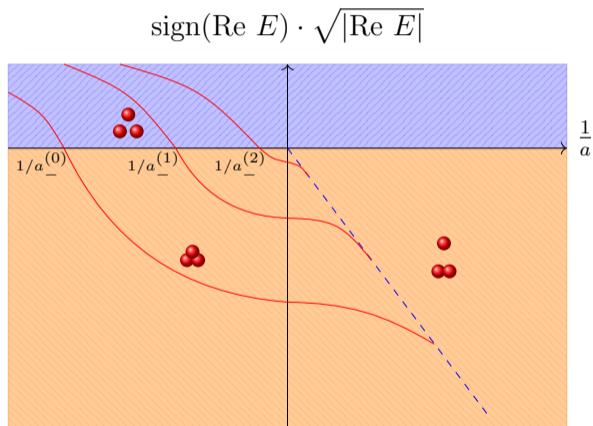
- Search along positive real momentum/ energy axis
- Finite volume



# Three bosons

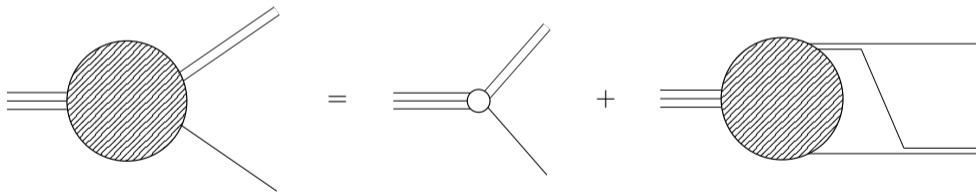
## The Efimov effect

- Use 3b system as benchmark
- Presents resonances for virtual 2b system; similar to 3n



## The three-boson system - Faddeev equation

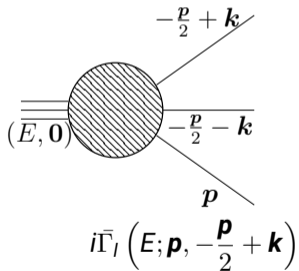
- Starting point: Faddeev equation for amplitude  $\Gamma_l$
- Set three-body force  $D_0$  to zero and vary cutoff  $\Lambda$
- Arbitrary coupling  $g_3$ ; has to be fitted to (experimental) data



$$i\Gamma_l(E; p) = ig_{3,l}p^l + \int_0^\infty dq q^2 Z_{2,l}(E; q, p) \tau(E; q) i\Gamma_l(E; q)$$

# The 3b system - Symmetrization

- Interested in 3b final state; not 2b + b





# The 3b system - Symmetrization

■ Interested in 3b final state; not 2b + b

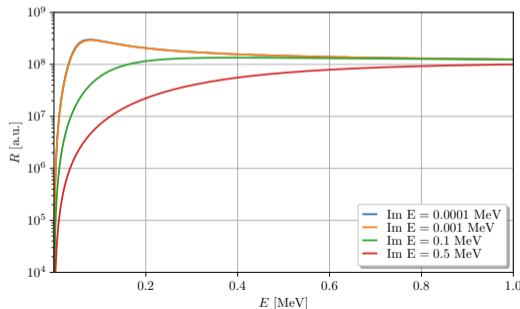
■ Symmetrization due to bosonic nature

$$\begin{aligned}
 & \text{Diagram 1: } (E, \mathbf{0}) \text{ entering a shaded circle, } p \text{ exiting, } -\frac{p}{2} + \mathbf{k} \text{ and } -\frac{p}{2} - \mathbf{k} \text{ exiting.} \\
 & i\bar{\Gamma}_l \left( E; \mathbf{p}, -\frac{\mathbf{p}}{2} + \mathbf{k} \right) \\
 & = (1 + P_{13}P_{23} + P_{23}P_{13}) \text{ Diagram 2} \\
 & \text{Diagram 2: } (E, \mathbf{0}) \text{ entering a shaded circle, } p \text{ exiting, } -\frac{p}{2} + \mathbf{k} \text{ and } -\frac{p}{2} - \mathbf{k} \text{ exiting, with a small white circle at the vertex.}
 \end{aligned}$$

# The 3b system - Production amplitude

- Experimentally measurable: Production amplitude  $R(E)$

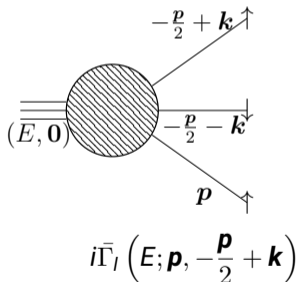
$$R(E) = \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3p}{(2\pi)^3} \left| \bar{\Gamma} \left( E, \mathbf{p}, -\frac{\mathbf{p}}{2} + \mathbf{k} \right) \right|^2 2\pi\delta \left[ E - E_{\mathbf{p}} - E_{-\frac{\mathbf{p}}{2} + \mathbf{k}} - E_{-\frac{\mathbf{p}}{2} - \mathbf{k}} \right]$$



→ See influence of resonance!

# The 3n system - Antisymmetrization

- Antisymmetrization due to fermionic nature



# The 3n system - Antisymmetrization

- Antisymmetrization due to fermionic nature

Diagram illustrating the antisymmetrization of a vertex in a 3n system due to the fermionic nature of the particles. The vertex is represented by a shaded circle.

Left side: Vertex with incoming lines labeled  $(E, \mathbf{0})$  and outgoing lines labeled  $-\frac{p}{2} + \mathbf{k}$  (up),  $-\frac{p}{2} - \mathbf{k}$  (down), and  $p$  (down).

Right side: Two diagrams representing the antisymmetrized vertex, separated by a minus sign. The first diagram has outgoing lines  $-\frac{p}{2} + \mathbf{k}$  (up),  $p$  (down), and  $-\frac{p}{2} - \mathbf{k}$  (down). The second diagram has outgoing lines  $p$  (up),  $-\frac{p}{2} - \mathbf{k}$  (down), and  $-\frac{p}{2} + \mathbf{k}$  (down).

Equation: 
$$i\bar{\Gamma}_l \left( E; \mathbf{p}, -\frac{\mathbf{p}}{2} + \mathbf{k} \right)$$

# The 3n system - Conformal scaling

- Nonrelativistic conformal symmetry: H.-W. Hammer and D. T. Son, Proc. Natl. Acad. Sci. **118**, e2108716118 (2021)  
→ Spectrum is determined (up to overall normalization)  
by scaling dimension  $\Delta$  for energies

$$\frac{1}{ma^2} \approx 0.1 \text{ MeV} \ll E \ll \frac{1}{mr_0^2} \approx 5 \text{ MeV}$$

$N$	$S$	$L$	$\Delta$
3	0.5	0	4.66622
3	0.5	1	4.27272
3	0.5	2	5.60498

Y. Nishida and D. T. Son, Phys. Rev. D **76**, 086004 (2007)  
H. W. Griebhammer, Nucl. Phys. A **760** (2005)



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- Cross section/ amplitude scale as

$$\frac{d\sigma}{dE} \sim E^{\Delta - \frac{5}{2}}, \quad |\Gamma_l(E, \mathbf{p})|^2 \sim E^{\Delta - \frac{7}{2}}, \quad R \sim E^{\Delta - \frac{5}{2}}$$

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- Noninteracting particles

$$\frac{d\sigma}{dE} \sim E^{\frac{3N-5}{2} + \#\nabla}, \quad |\Gamma_l(E, \mathbf{p})|^2 \sim E^{\frac{3N-7}{2} + \#\nabla}, \quad R \sim E^{\frac{3N-5}{2} + \#\nabla}$$

$N$	$S$	$L$	$\Delta$
3	0.5	0	4.66622
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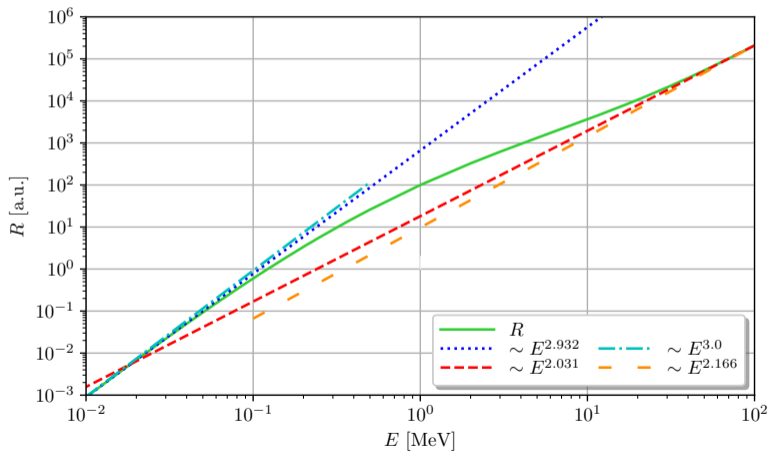
Y. Nishida and D. T. Son, Phys. Rev. D **76**, 086004 (2007)  
 H. W. Griesshammer, Nucl. Phys. A **760** (2005)



# The 3n system - Results

■ S-wave

$$R(E) \sim |\bar{\Gamma}_0|^2$$

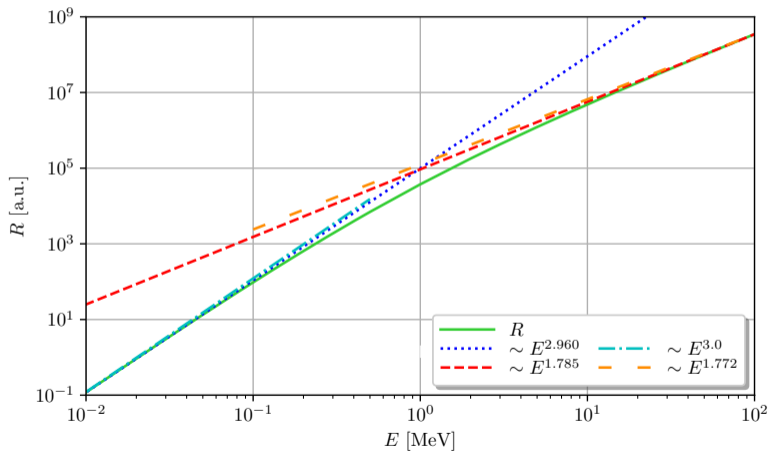




# The 3n system - Results

■ P-wave

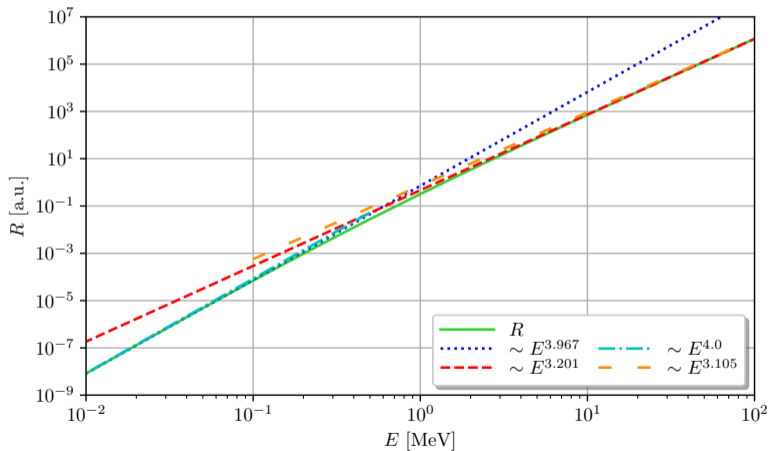
$$R(E) \sim |\bar{\Gamma}_1|^2$$



# The 3n system - Results

■ D-wave

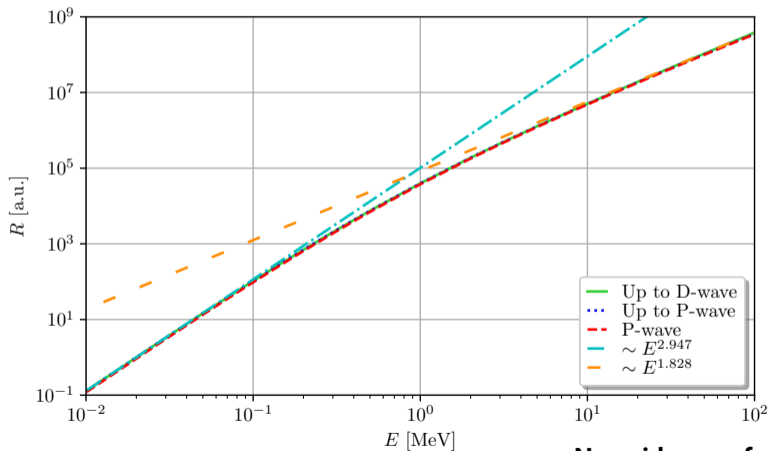
$$R(E) \sim |\bar{\Gamma}_2|^2$$



# The 3n system - Results

■ Up to D-wave

$$R(E) \sim \left| \sum_{l=0}^2 \bar{\Gamma}_l \right|^2$$

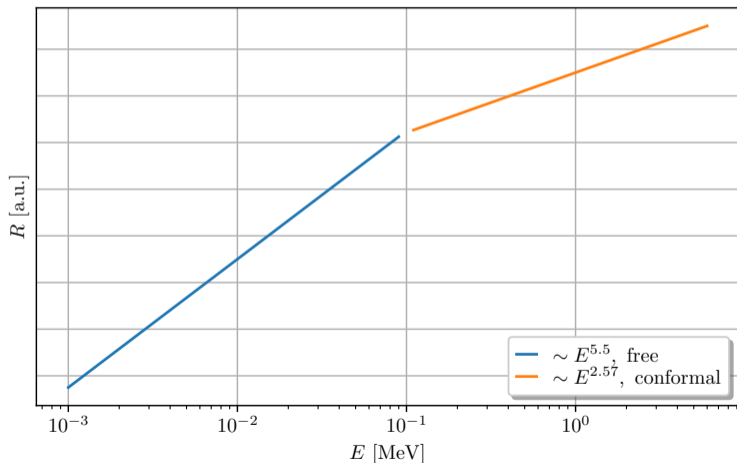


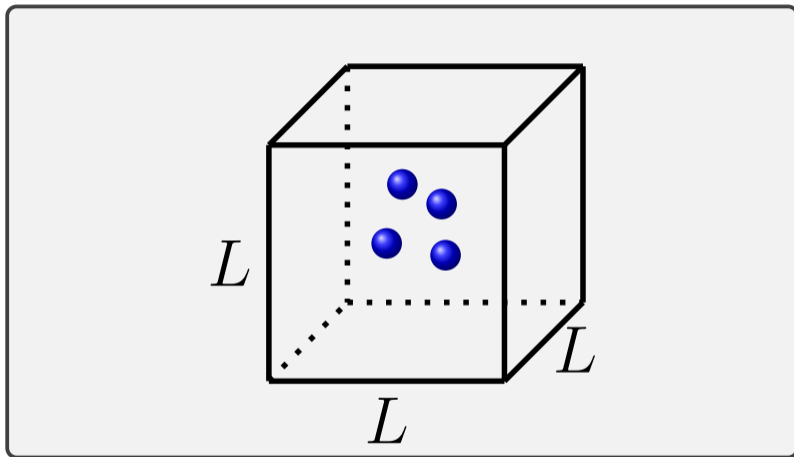
→ **No evidence of resonance!**

# The four-neutron system

Predictions by nonrelativistic conformal field theory (NR-CFT)

- 4n: Predictions for point production amplitude by nonrelativistic conformal field theory
- No peak structure  
→ **No evidence of resonance**

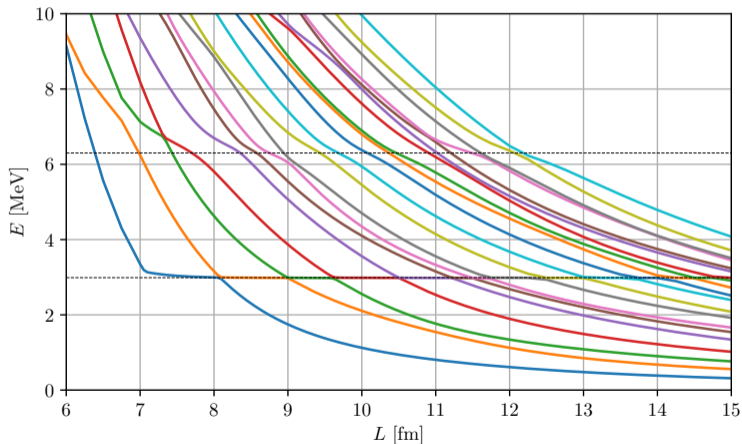




# Finite volume

## Avoided Level Crossing

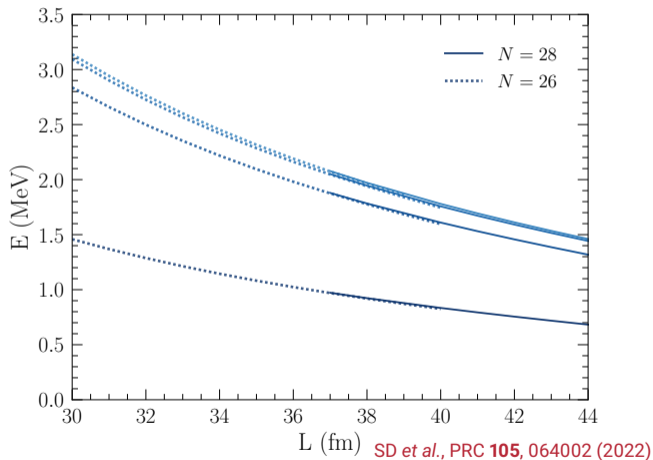
- Resonance in finite volume:  
Avoided level crossing
- S-wave Gaussian toy potential  
with resonances at:
  - $E = (2.9826 - 0.0007i)$  MeV
  - $E = (6.3287 - 0.3252i)$  MeV



# Finite volume

## Three-neutron system

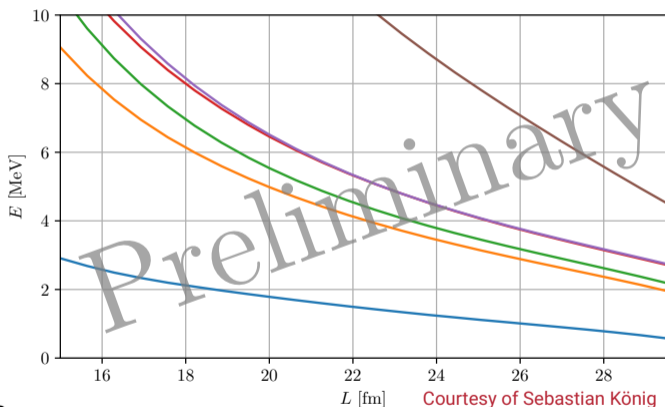
- LO pionless EFT potential for  $a_{nn} = -18.9$  fm
- nn interaction in  $^1S_0$ -channel
- Third neutron in relative P-wave
- Apply discrete variable representation (DVR)
- N: Mesh points for DVR basis
- Super-Gaussian regulator  $\sim \exp(-k^4/\Lambda^4)$
  
- **No sign of three-neutron resonance**



# Finite volume

## Four-neutron system

- LO pionless EFT potential for  $a_{nn} = -18.9$  fm
- nn interaction in  $^1S_0$ -channel
- $S = 0$
- $A_1^+ \rightarrow L = 0, 4$
- $N = 10$
- Super-Gaussian regulator  $\sim \exp(-k^4/\Lambda^4)$
- DVR + Finite volume eigenvector continuation (FVEC)  
Yapa, König, PRC 106, 014309 (2022)
- Numerical convergence needs to be analyzed in more detail



■ **No sign of four-neutron resonance**



# Summary & Outlook

## ■ Summary:

### ■ Point production

- Derived point production amplitude  $R$  for 3b/3n system
- 3b: Showed influence of resonance on  $R$
- Presented NR-CFT & predictions for spectrum
- 3n: Compared to predictions by NR-CFT & showed that no resonance exists
- 4n: Resonance unlikely by NR-CFT

## ■ Outlook:

### ■ Point production

- Calculate  $R$  using Faddeev-Yakubovsky equations  $\rightarrow$  4n

### ■ Finite volume

- Discussed resonances in FV
- 3n: Presented spectrum calculated using DVR & showed that no resonance exists
- 4n: Presented spectrum calculated using DVR + FVEC & showed that no structure of resonance is present

### ■ Finite volume

- Calculate spectrum for Gaussian regulator  $\sim \exp(-k^2/\Lambda^2)$
- Check convergence for higher  $N$

Thank you for your attention!

