

# Ground and dipole-excited states of the halo nucleus ${}^8\text{He}$

FRANCESCA BONAITI, JGU MAINZ (PROJECT B04)

SFB 1245 ANNUAL WORKSHOP @ DARMSTADT

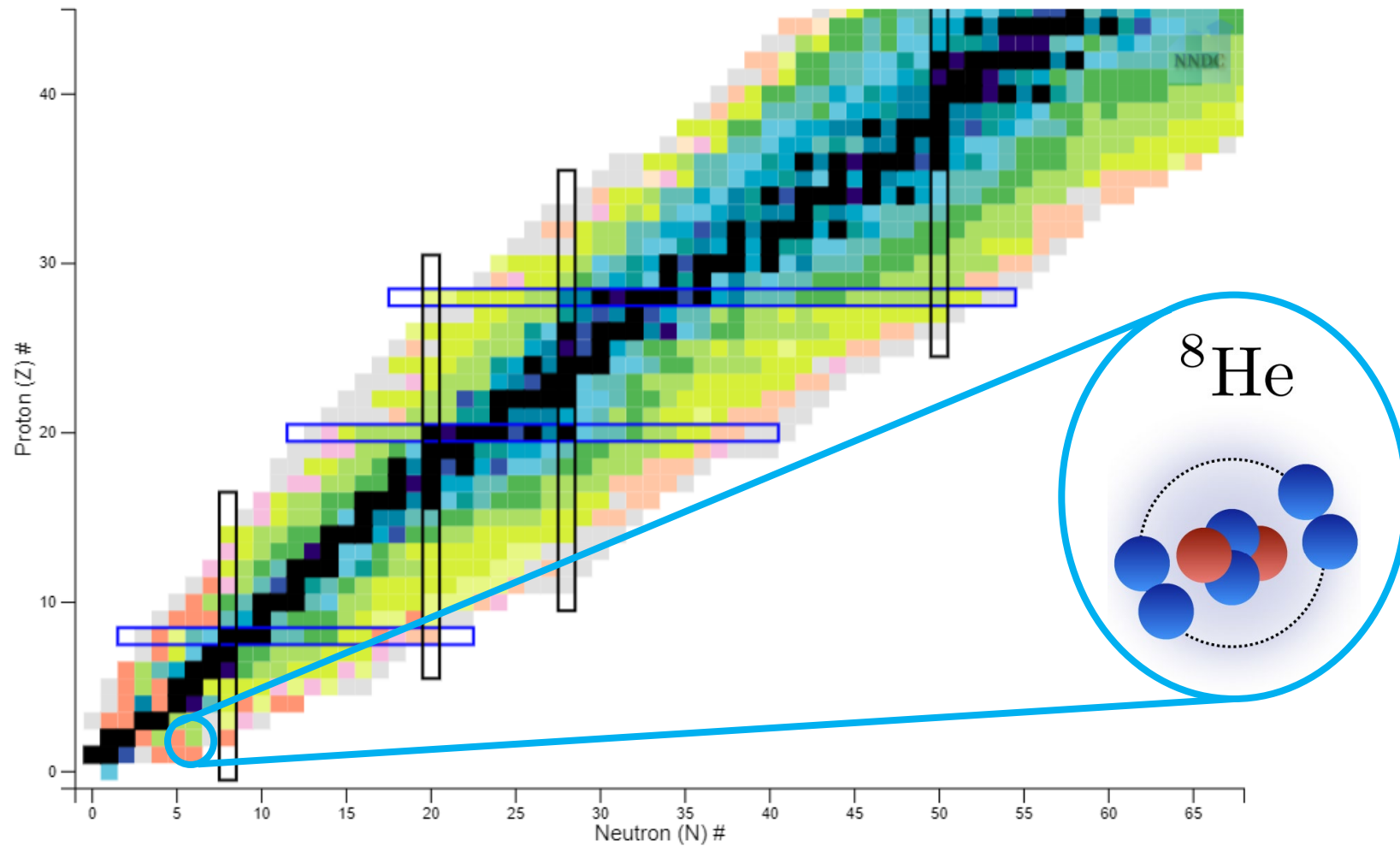
OCTOBER 5, 2022

In collaboration with:

Sonia Bacca

Gaute Hagen (ORNL)

# Motivation



# Motivation

Article | [Open Access](#) | [Published: 27 January 2021](#)

## Measuring the $\alpha$ -particle charge radius with muonic helium-4 ions

[Julian J. Krauth](#) , [Karsten Schuhr](#)



Physics Letters B  
Volume 822, 10 November 2021, 136710

[Nature](#) **589**, 527–531 (2021) | [Cite this article](#)

Proton inelastic scattering reveals deformation in  $^8\text{He}$

M. Holl <sup>a, b</sup>, R. Kanungo <sup>a, b</sup> , Z.H. Sun <sup>c, d</sup>, G. Hagen <sup>c, d</sup>, J.A. Lay <sup>e, f</sup>, A.M. Moro <sup>e, f</sup>, P. Navrátil <sup>b</sup>

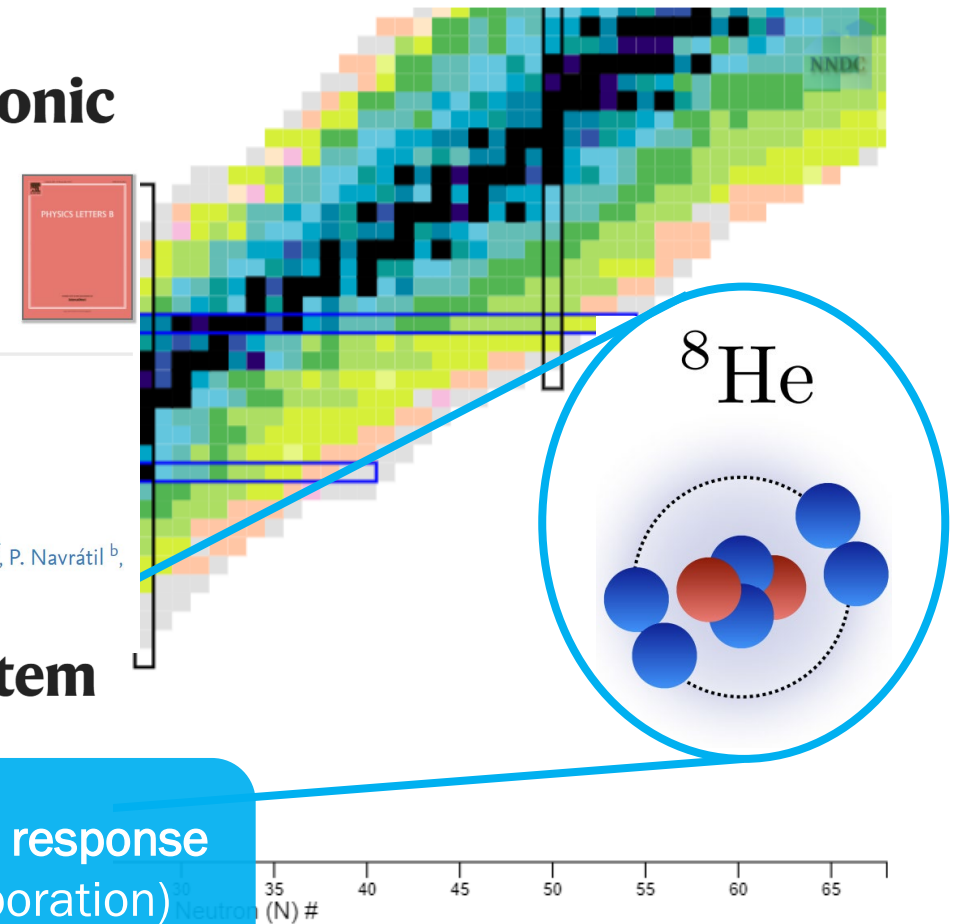
Article | [Open Access](#) | [Published: 22 June 2022](#)

## Observation of a correlated free four-neutron system

[M. Duer](#) , [T. Aumann](#), ... [M. V. Zhukov](#) [+ Show authors](#)

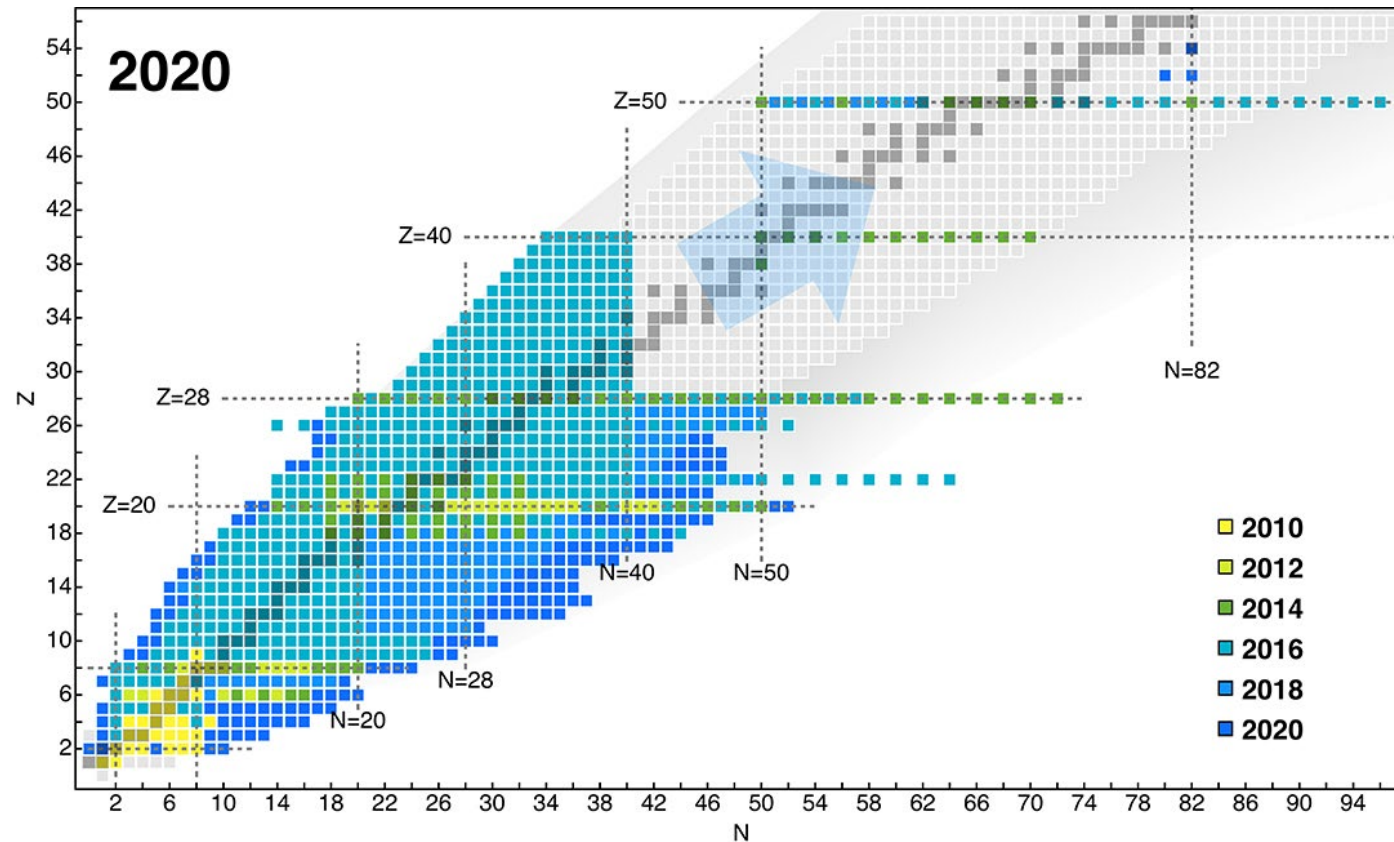
[Nature](#) **606**, 678–682 (2022) | [Cite this article](#)

Low-energy dipole response  
(SAMURAI collaboration)  
→ SFB project A06 (T. Aumann's group)



# Ab initio methods

H. Hegert, Front.in Phys. 8 379 (2020)

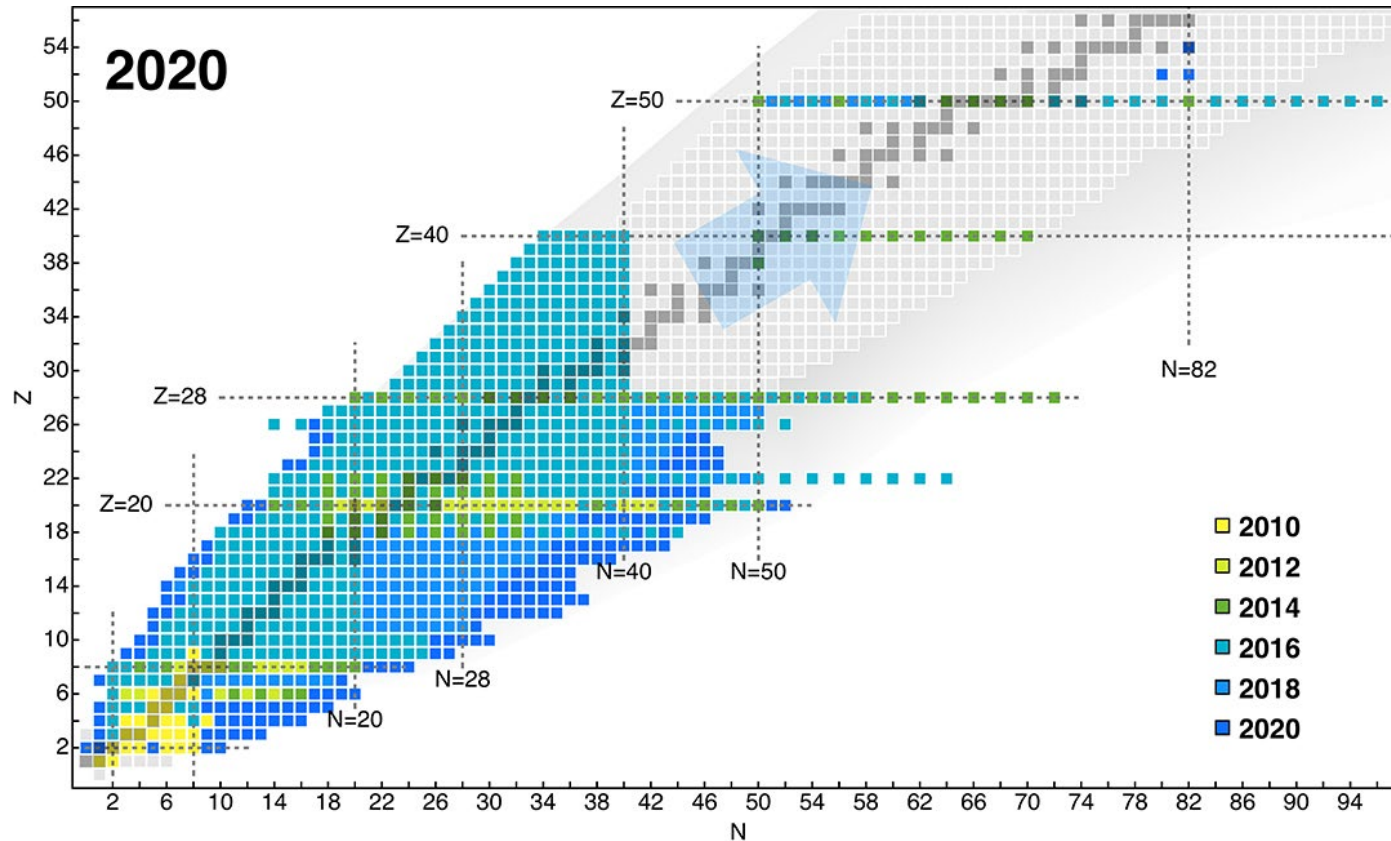


# Ab initio methods



2021:  $^{208}\text{Pb}$   
B. Hu et al,  
arXiv:2112.01125

H. Hegert, Front.in Phys. 8 379 (2020)

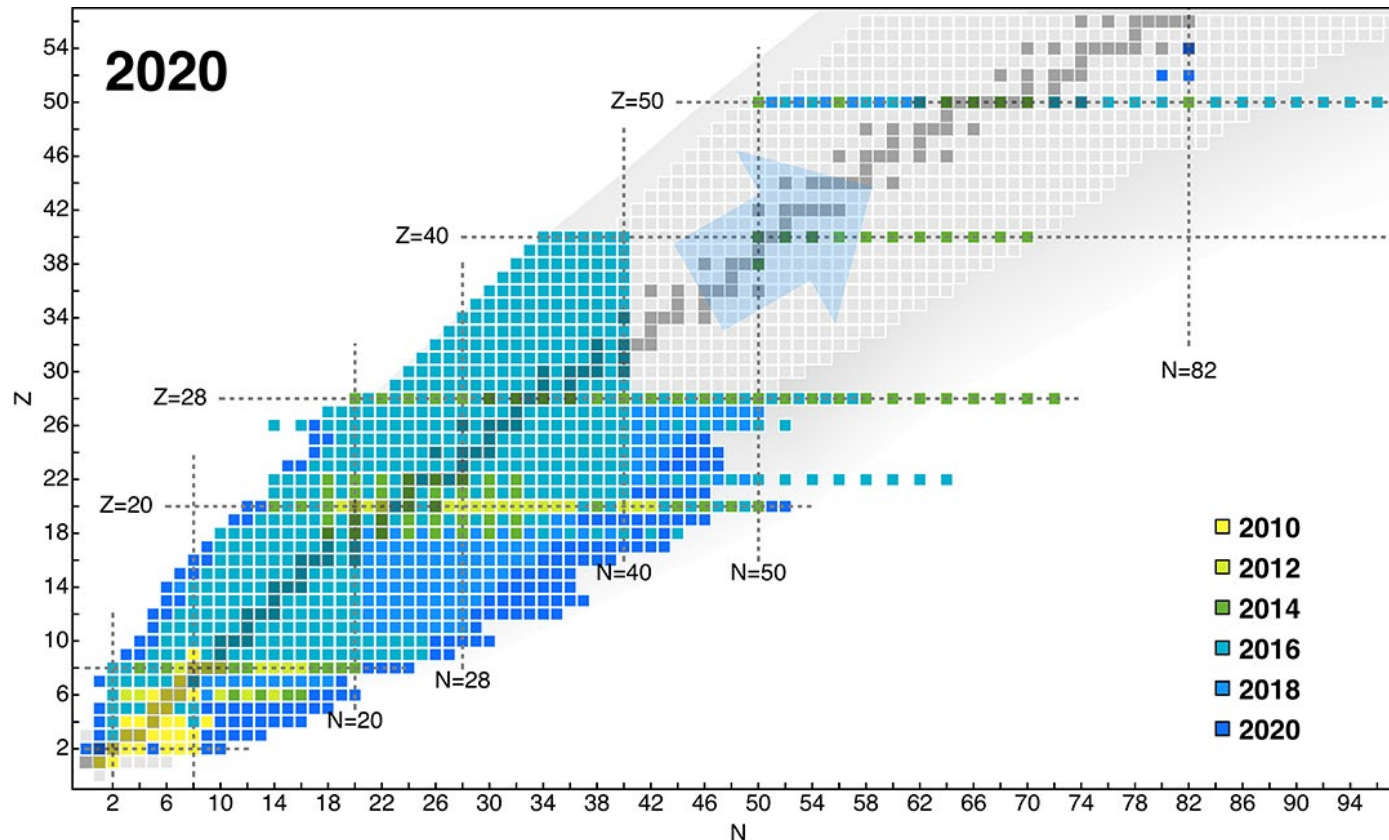




# Ab initio methods

2021:  $^{208}\text{Pb}$   
B. Hu et al,  
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H. Hegert, Front.in Phys. 8 379 (2020)



☐ Main degrees of freedom of the problem: **protons and neutrons.**

☐ Solve

$$H |\psi\rangle = E |\psi\rangle$$

$$H = T + V_{NN} + V_{3N}$$

with **controlled approximations.**

☐ Two ingredients: a **nuclear interaction model** and a **many-body solver.**

# Nuclear interaction models

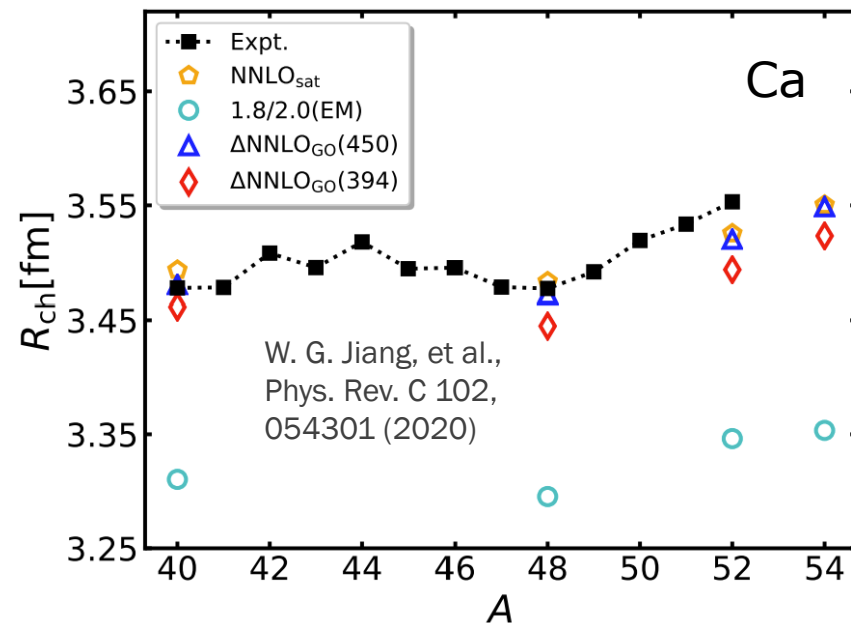
We work with **Chiral Effective Field Theory interactions**:

- ❑ **Low-energy approximation of QCD**, with  $\pi$ ,  $n$ , ( $\Delta$ ) as degrees of freedom.
- ❑ Separation of scales allows for **low-momentum expansion**, many-body forces arise naturally in the theory.
- ❑ Short-range physics enclosed in **low-energy constants**, fitted to experiment.

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**NNLO<sub>sat</sub>** [A. Ekstrom et al, 2015]

- ❑ Fitted on radii and BEs of light and medium-mass nuclei, including carbon and oxygen isotopes.

**ΔNNLO<sub>Go</sub>** [W. G. Jiang et al, 2020]

- ❑ Includes  $\Delta$  explicitly.
- ❑ Fitted on radii and BEs of light nuclei and nuclear matter saturation.
- ❑ Two cutoffs: 394 and 450 MeV.



# Coupled-cluster theory

- Starting point: **Hartree-Fock** reference state on the HO basis  $|\phi\rangle$
- Correlations are included via **exponential ansatz**:

$$|\psi\rangle = e^T |\phi\rangle$$

with

$$T = \sum t_i^a a_a^\dagger a_i + \sum t_{ij}^{ab} a_a^\dagger a_b^\dagger a_j a_i + \sum t_{ijk}^{abc} a_a^\dagger a_b^\dagger a_c^\dagger a_k a_j a_i + \dots$$

→ coefficients  
obtained via  
**coupled-cluster  
equations**

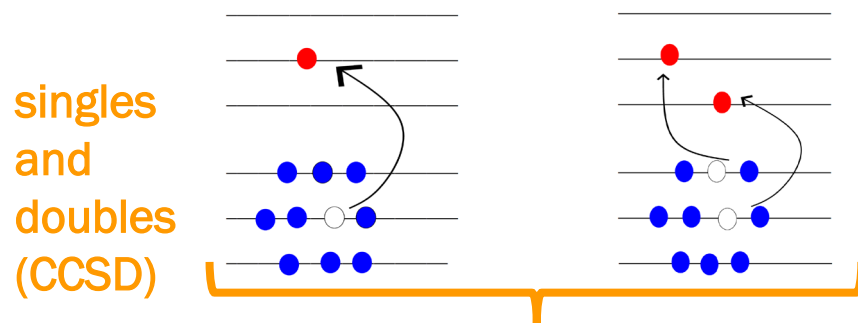
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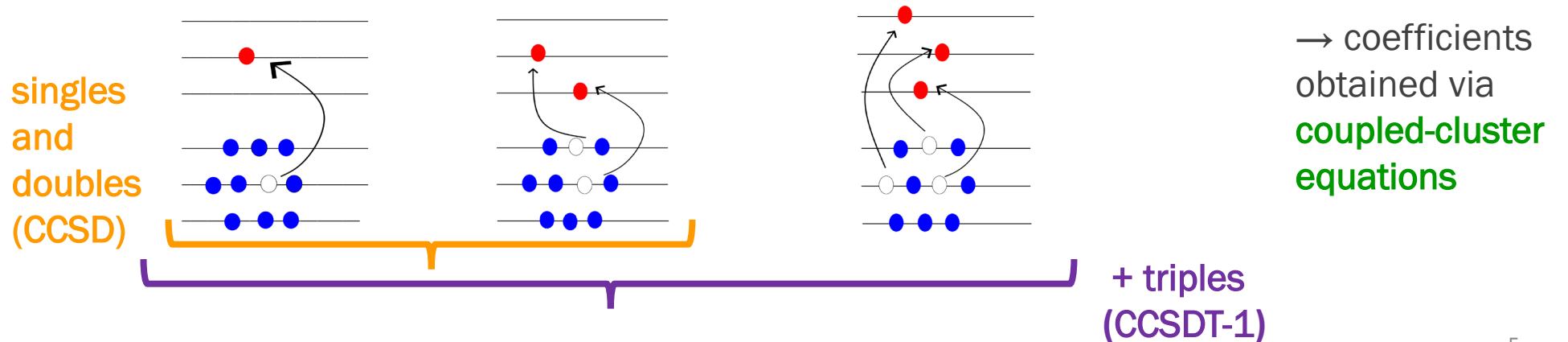
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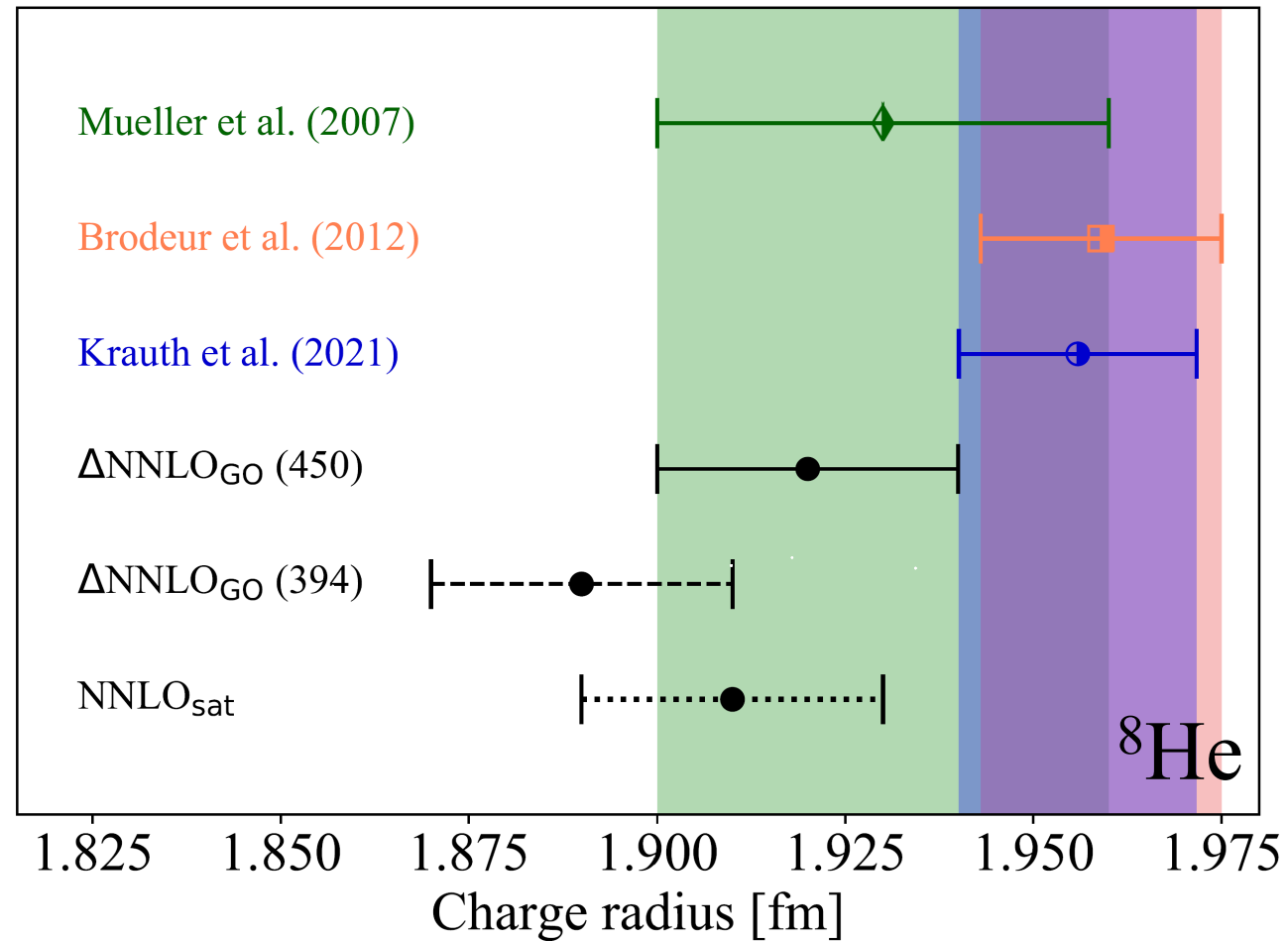
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# Charge radius



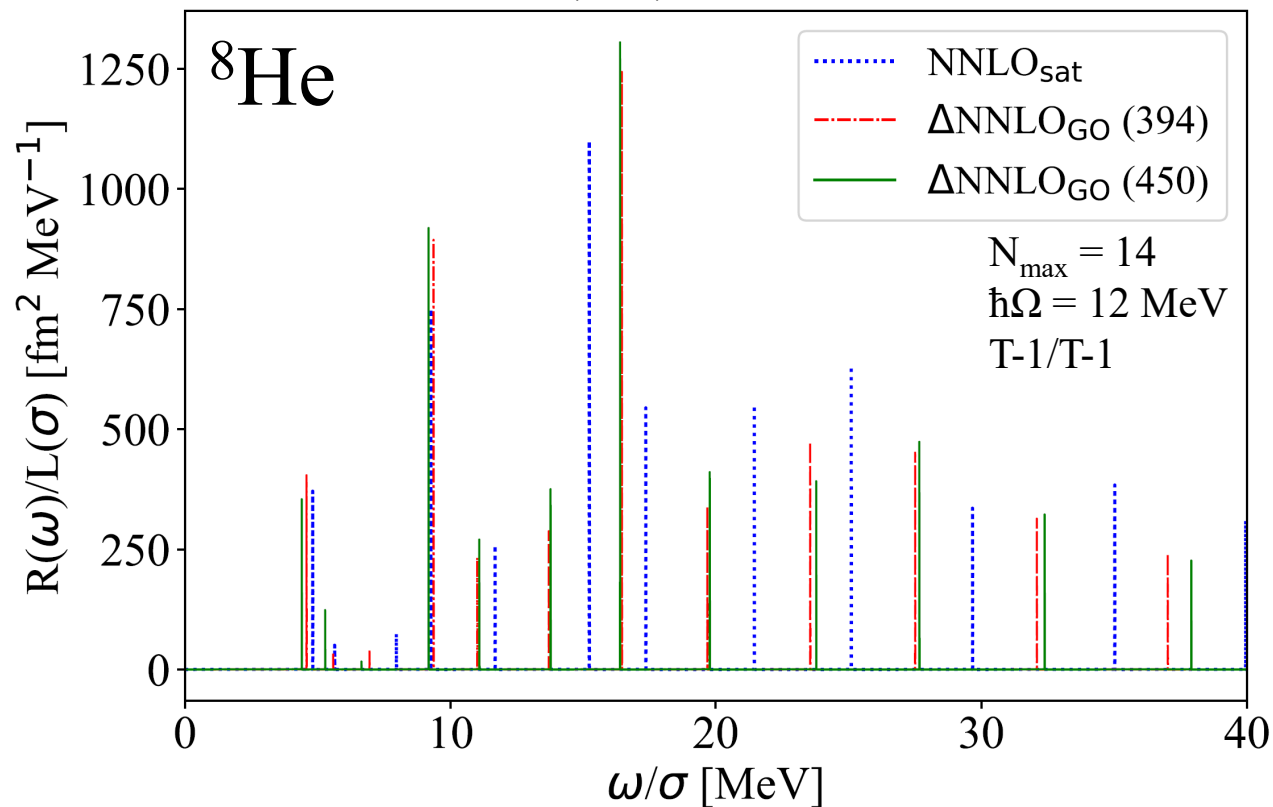
# From ground to dipole-excited states

- Dipole excitations are described by the **nuclear response function**.

$$R(\omega) = \sum_f \langle f | \hat{\Theta} | 0 \rangle |^2 \delta(\omega - E_f + E_0)$$

continuum problem  
addressed via **Lorentz  
Integral Transform** method

FB et al., PRC 105, 034313 (2022)



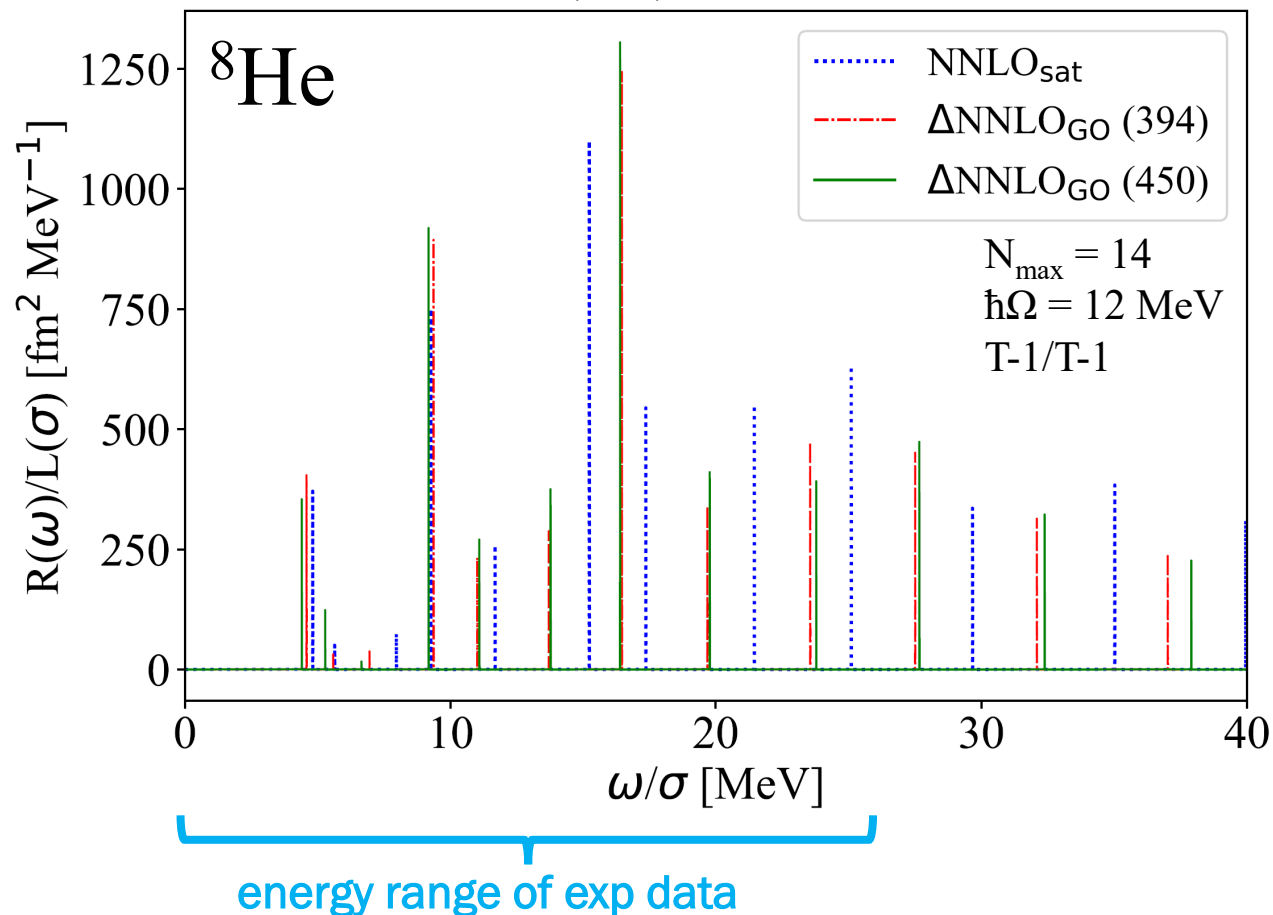
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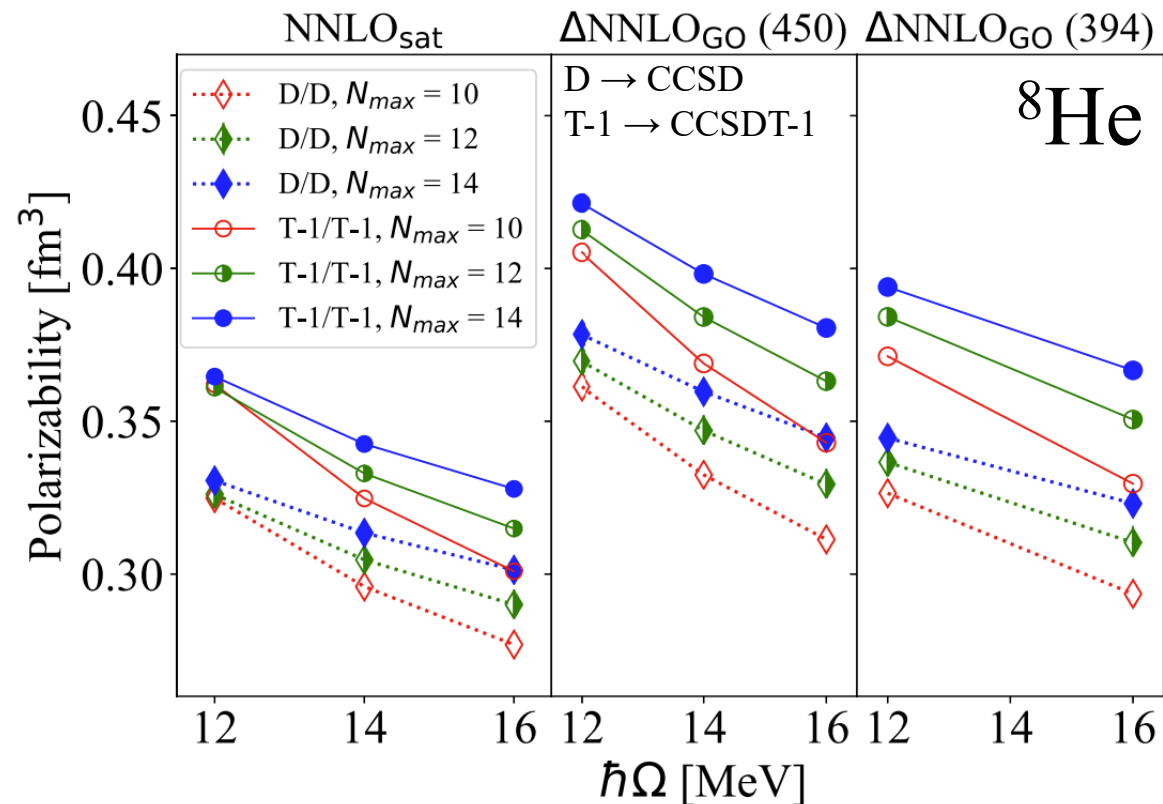
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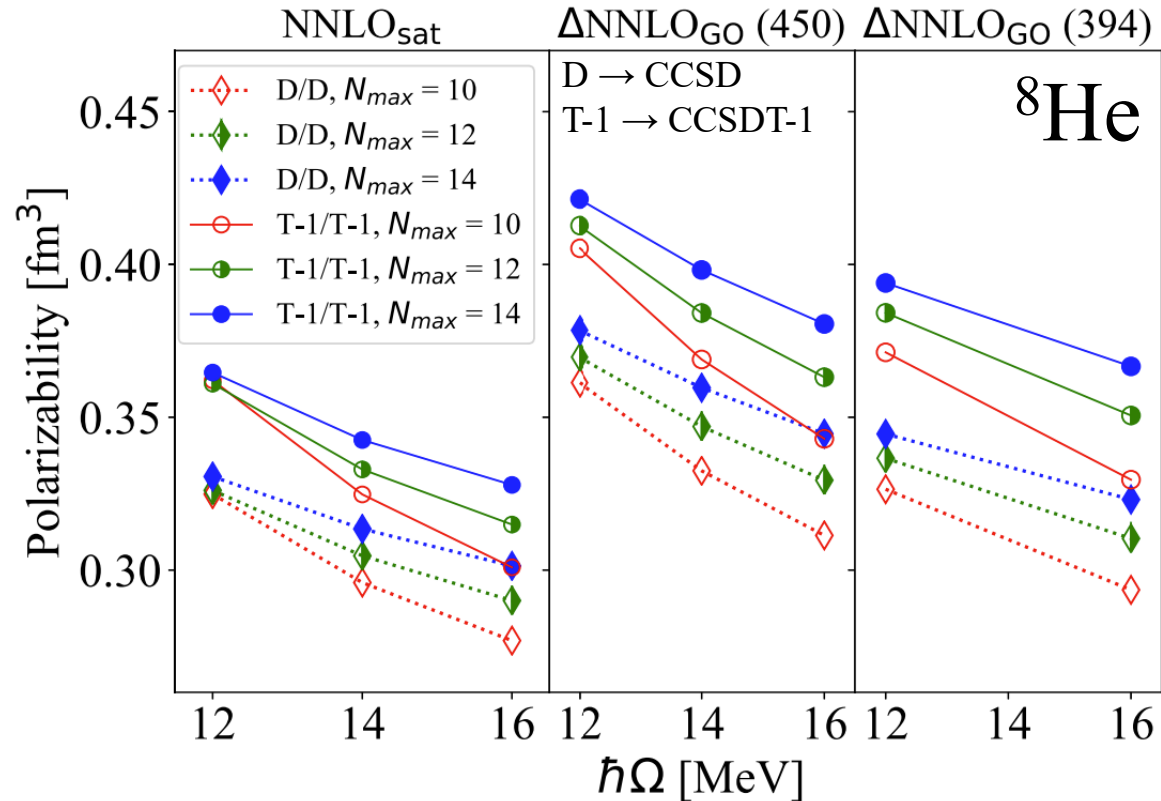
# Dipole polarizability



$$\alpha_D = 2\alpha\hbar c \int_0^\infty d\omega \omega^{-1} R(\omega)$$

FB et al., PRC 105, 034313 (2022)

# Dipole polarizability



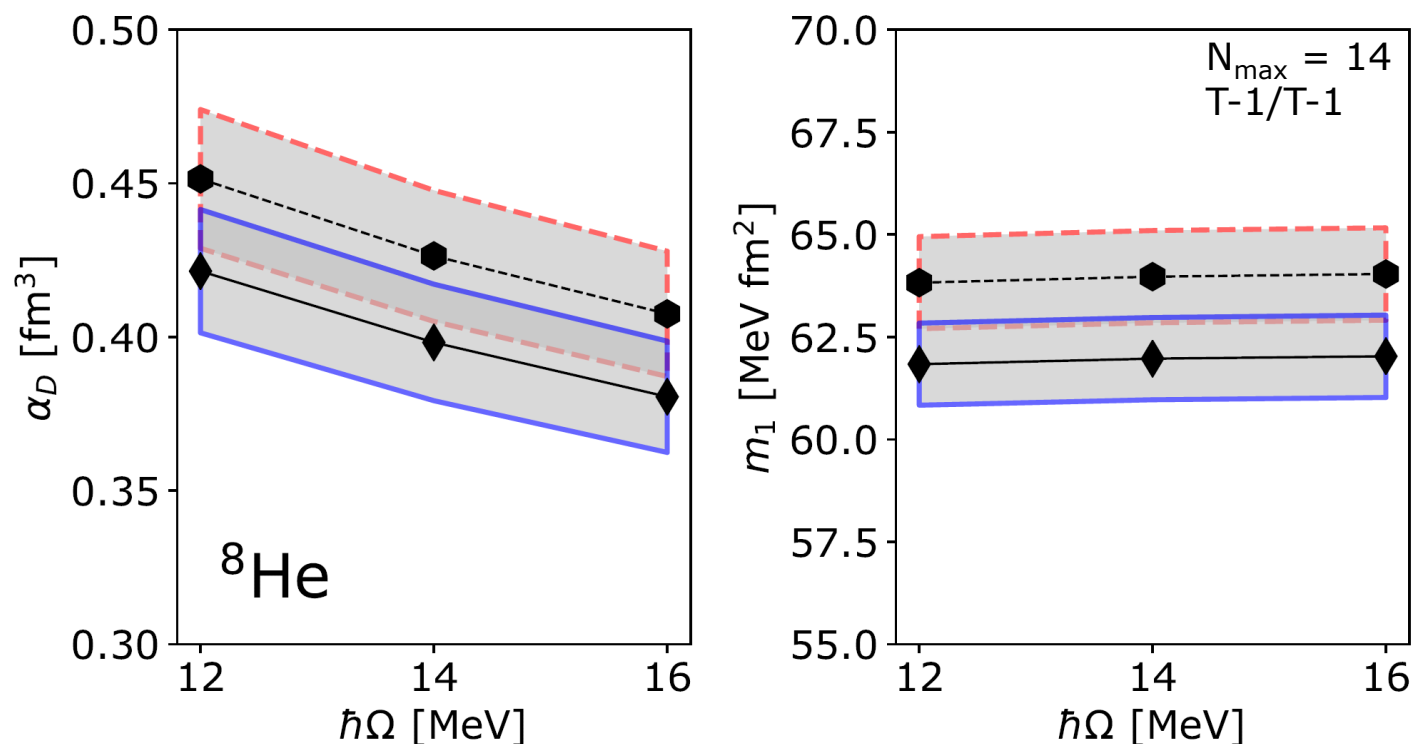
$$\alpha_D = 2\alpha\hbar c \int_0^\infty d\omega \omega^{-1} R(\omega)$$

Interaction	$\alpha_D$ (fm <sup>3</sup> )
NNLO <sub>sat</sub>	0.37(3)
$\Delta$ NNLO <sub>GO</sub> (450)	0.42(3)
$\Delta$ NNLO <sub>GO</sub> (394)	0.39(2)

□ 5 times larger than  $\alpha_D(^4\text{He}) = 0.074(9)$  fm<sup>3</sup> [Arkatoev et al, 1975, 1980, Pachucki et al, 2006].

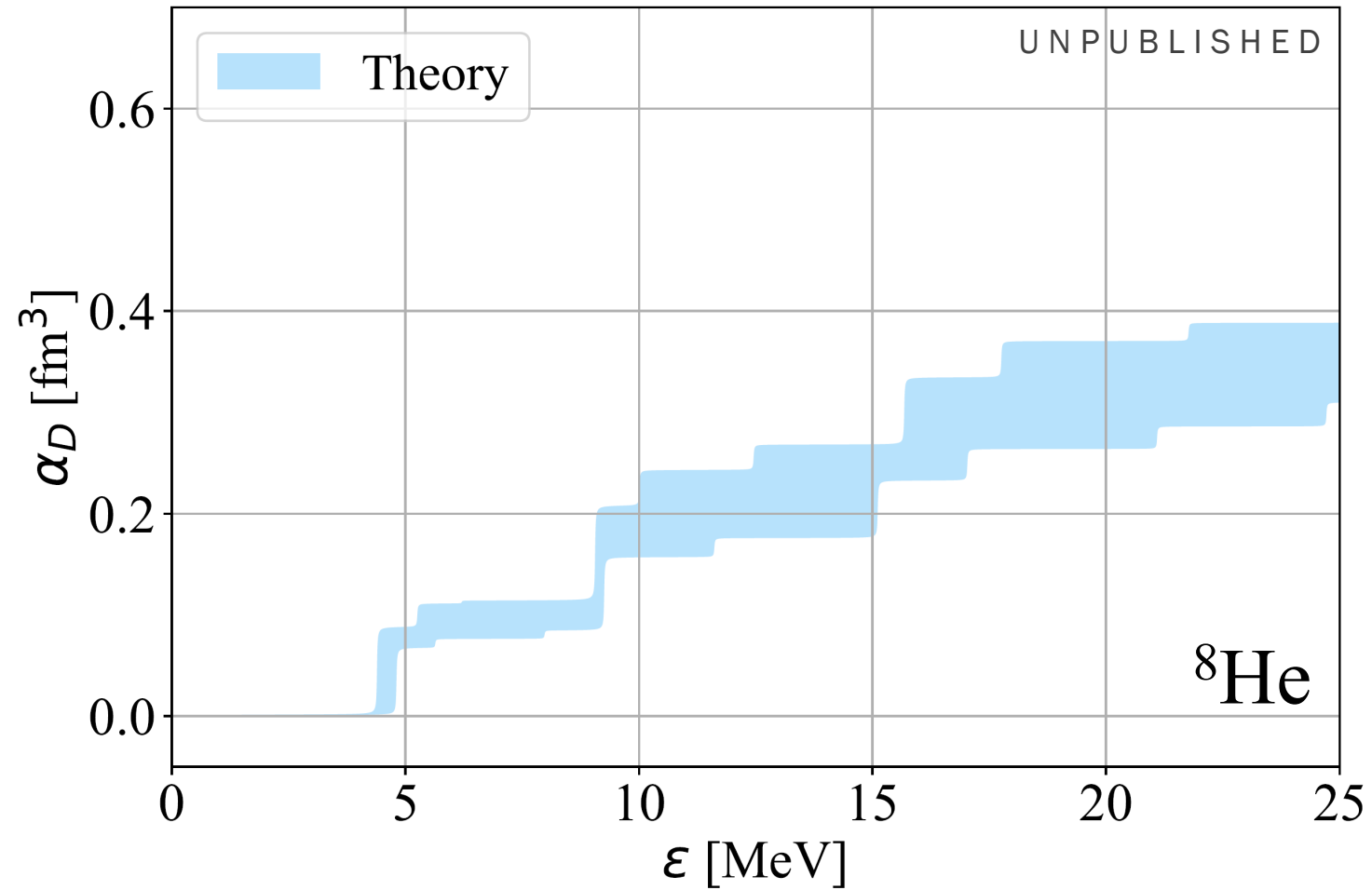


# Comparison between chiral orders

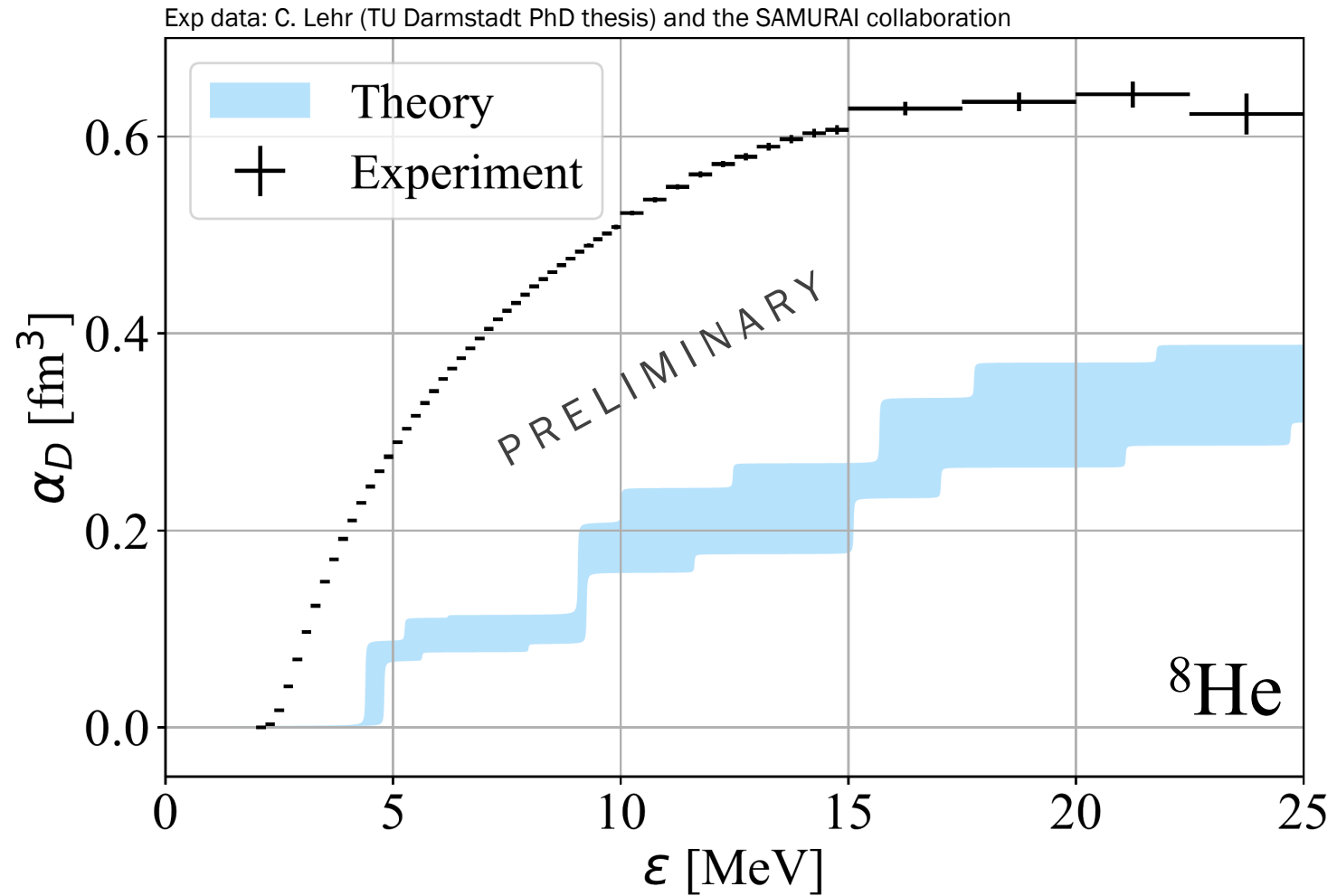


B. Acharya, S. Bacca, FB et al,  
arXiv:2210.04632

# Running sums



# Running sums



more weight to  
**low-energy**  
states

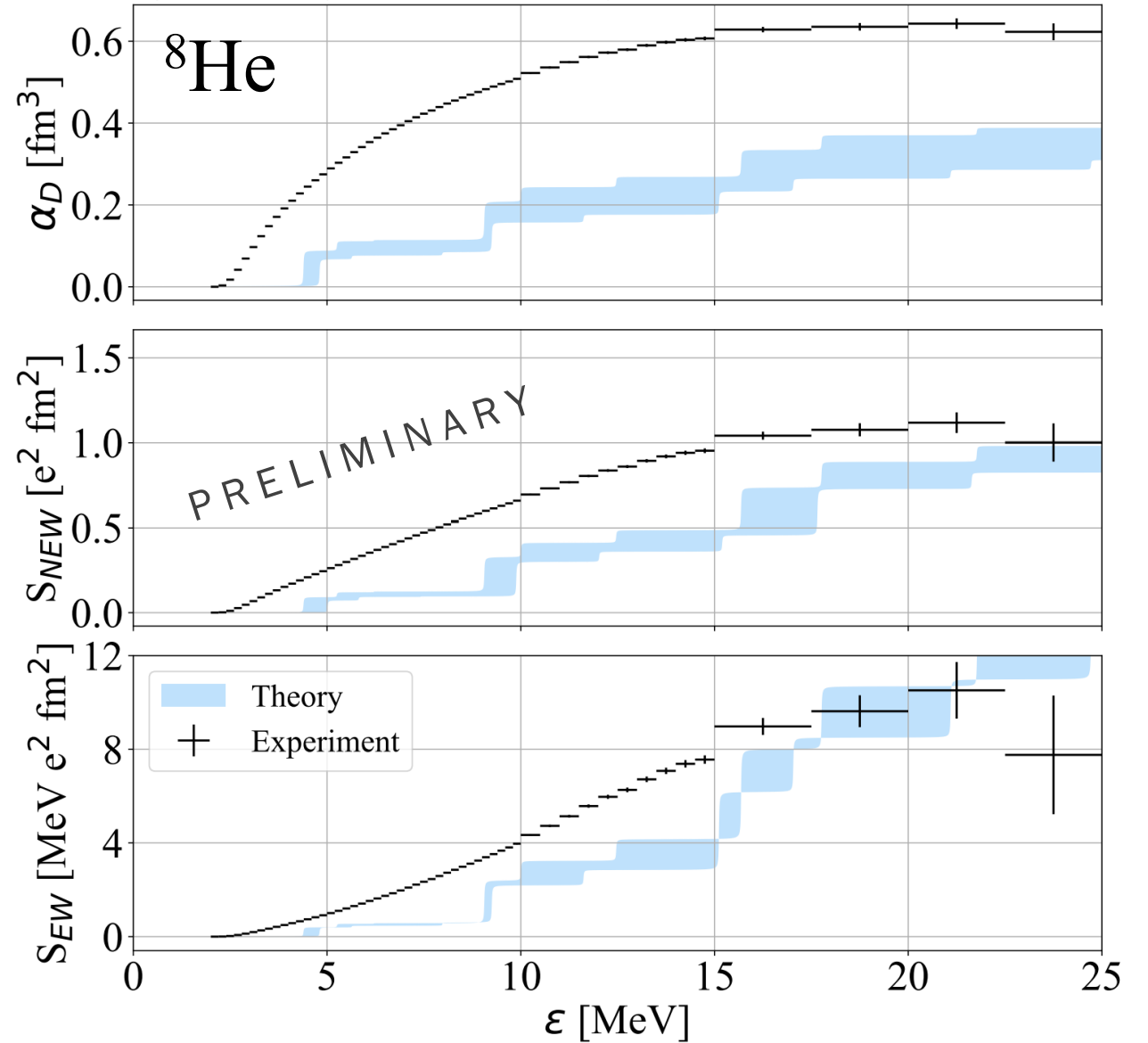
$$\alpha_D \propto \int_0^\epsilon d\omega \omega^{-1} R(\omega)$$

$$S_{NEW} \propto \int_0^\epsilon d\omega \omega^0 R(\omega)$$

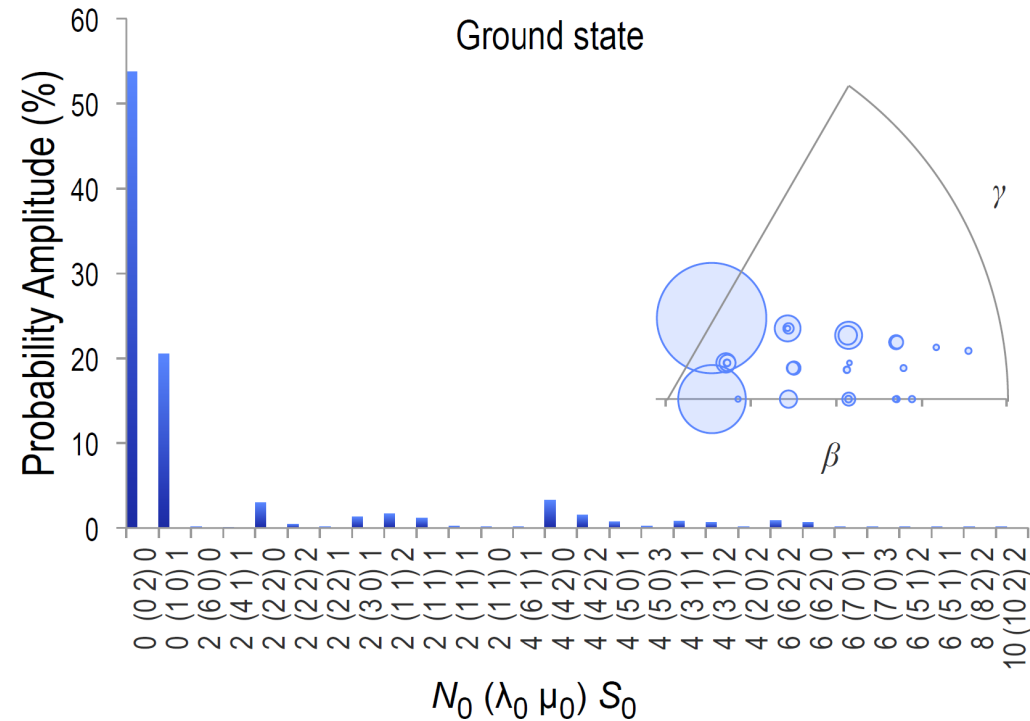
$$S_{EW} \propto \int_0^\epsilon d\omega \omega R(\omega)$$

more weight  
to **high-energy**  
states

Exp data: C. Lehr (TU Darmstadt PhD thesis) and the SAMURAI collaboration



# Deformation in $^8\text{He}$ ?

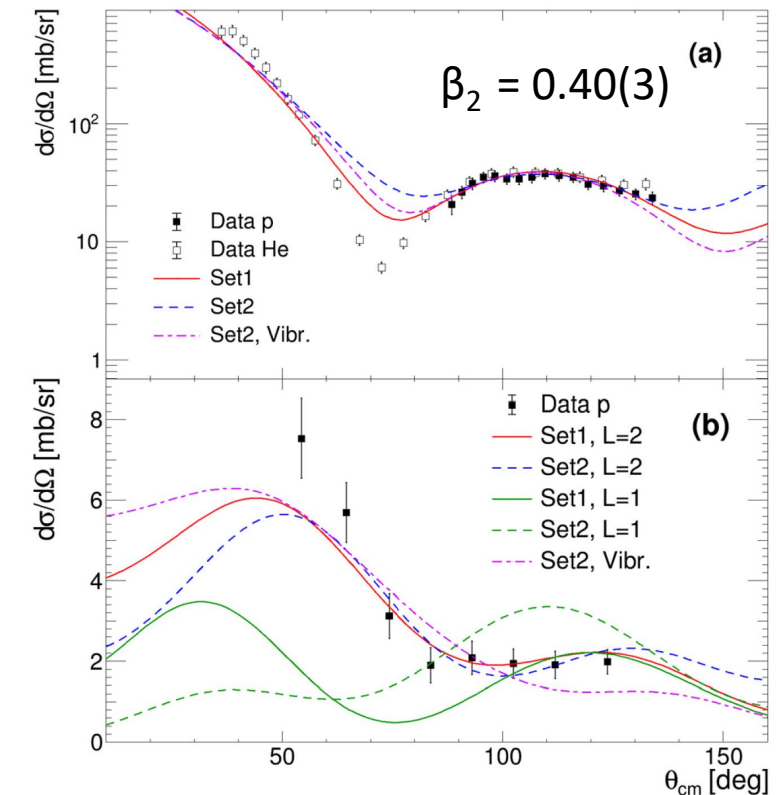


## SA-NCSM ground state calculations

Launey et al., EPJ Special Topics 229, 2429-2441 (2020)

## First 2+ excited state

M. Holl et al, Phys. Lett. B 822 136710 (2021)



# Coupled cluster theory for deformed nuclei

- ❑ Coupled-cluster computations starting from **axially-symmetric reference states** now possible [S. J. Novario et al, 2020].
- ❑ Need of **symmetry restoration** [G. Hagen et al, 2022].
- ❑ To do that, we calculate the expectation value:

$$E_J = \frac{\langle \tilde{\Psi} | P_J H | \Psi \rangle}{\langle \tilde{\Psi} | P_J | \Psi \rangle}$$

- ❑ In this way we project a state with  $J_z = 0$  on total angular momentum  $J$ .



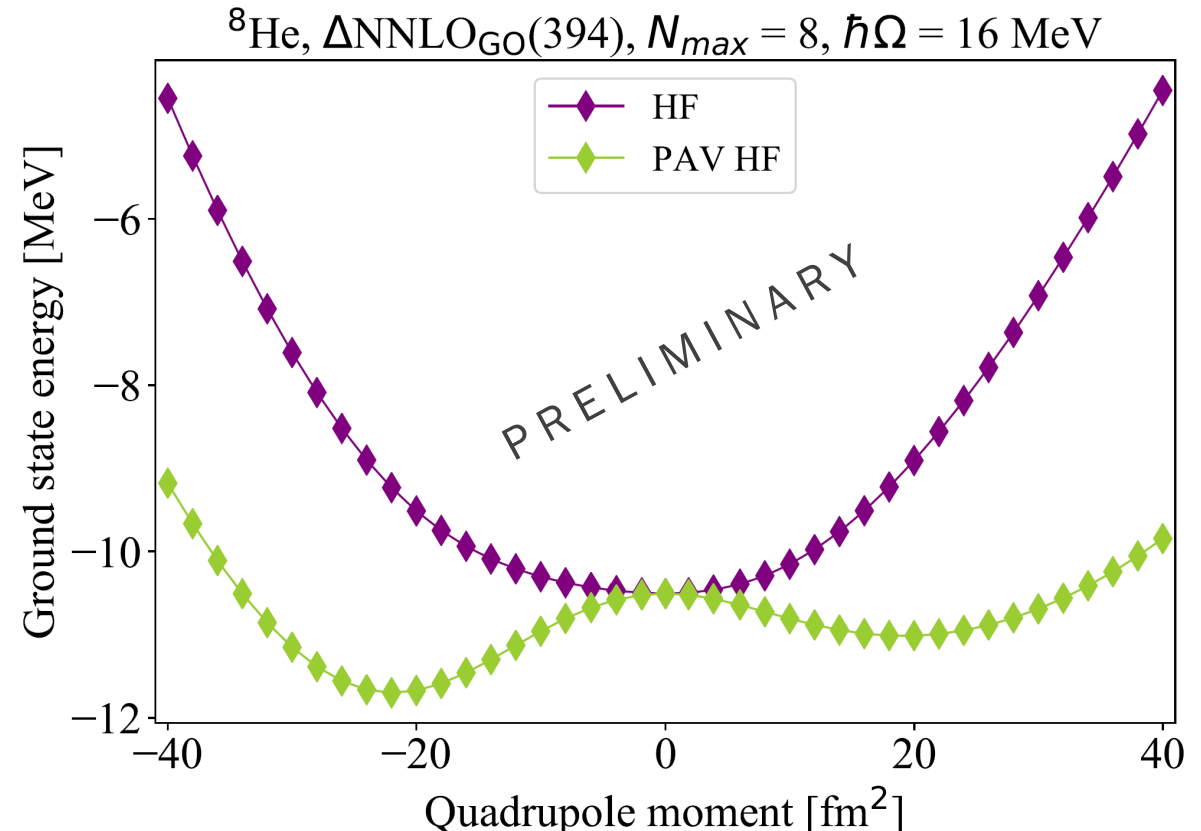
Guggenheim Museum in New York City, photo by me

# Deformed reference states for ${}^8\text{He}$

We start from **Hartree-Fock calculations** where:

- we assume  $J_z$  conservation,
- we minimize the energy under the constraint of a **fixed expectation value for the quadrupole moment**.

We then perform an **angular momentum projection** after variation (PAV) of the  $E_{\text{gs}}$  vs  $Q$  curve.

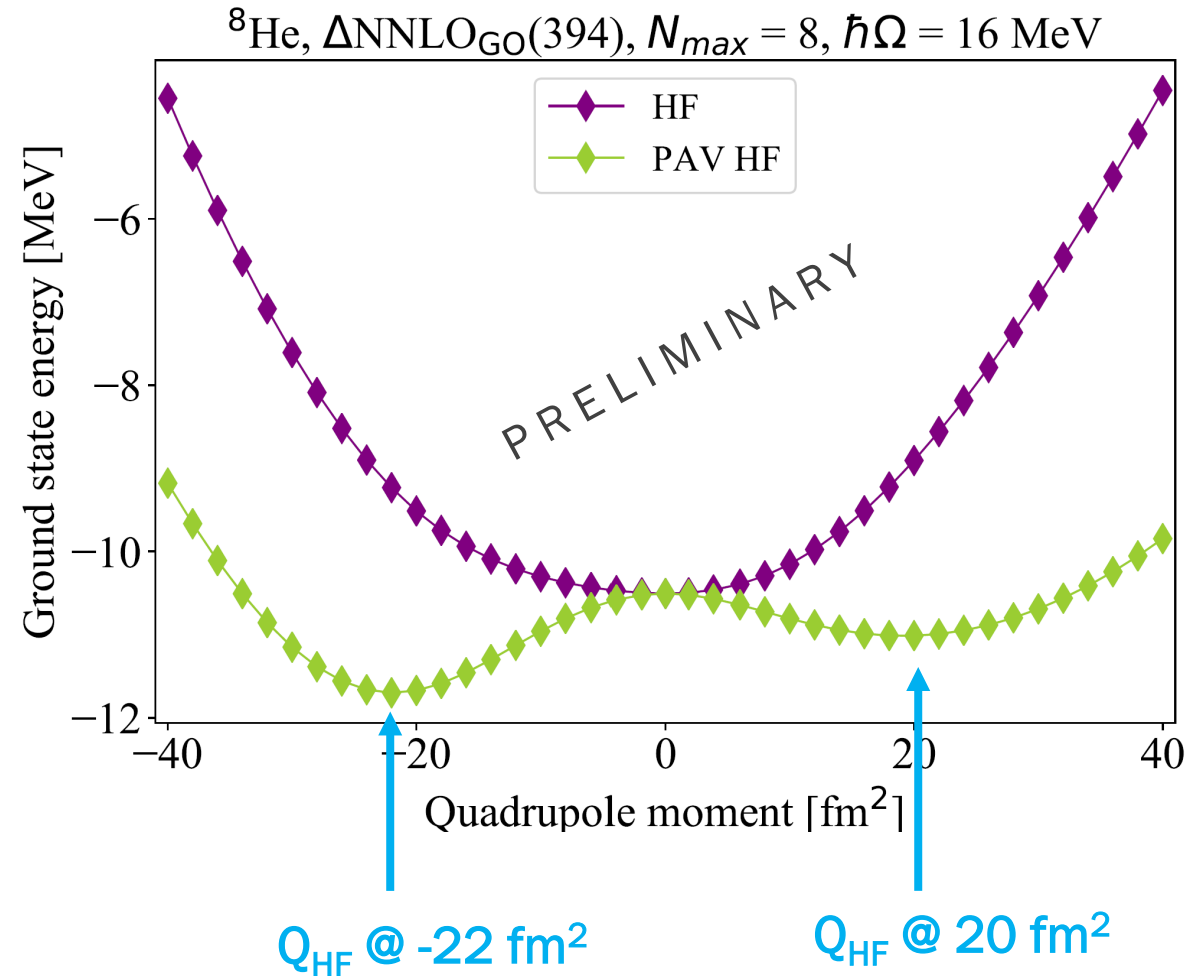


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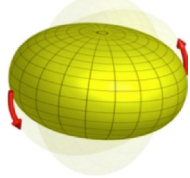
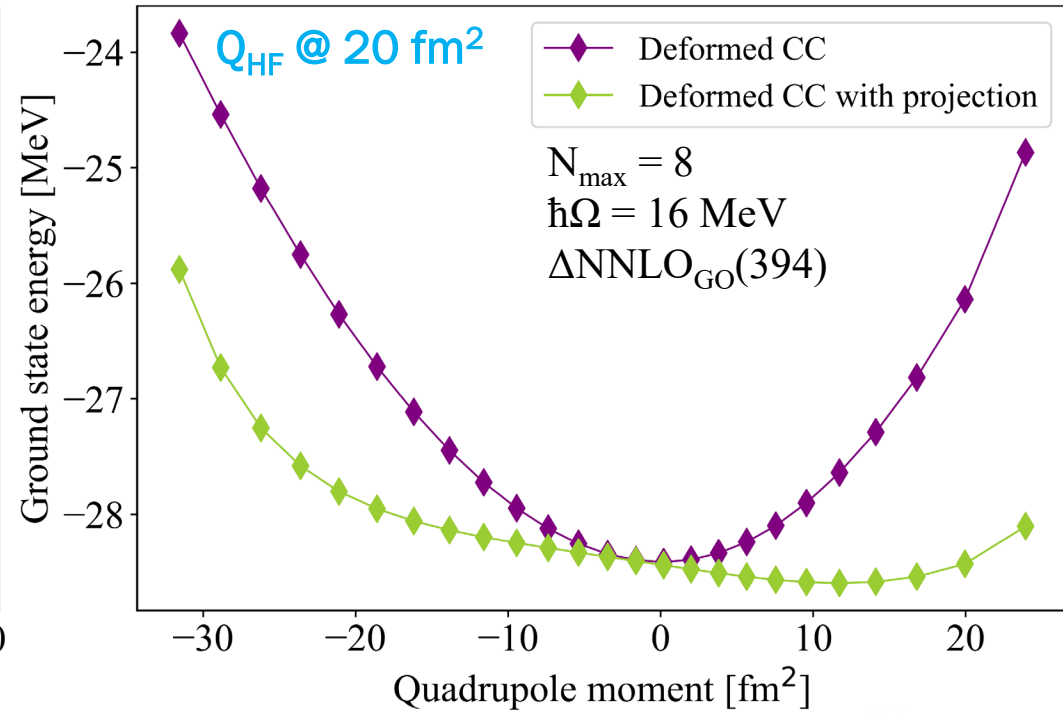
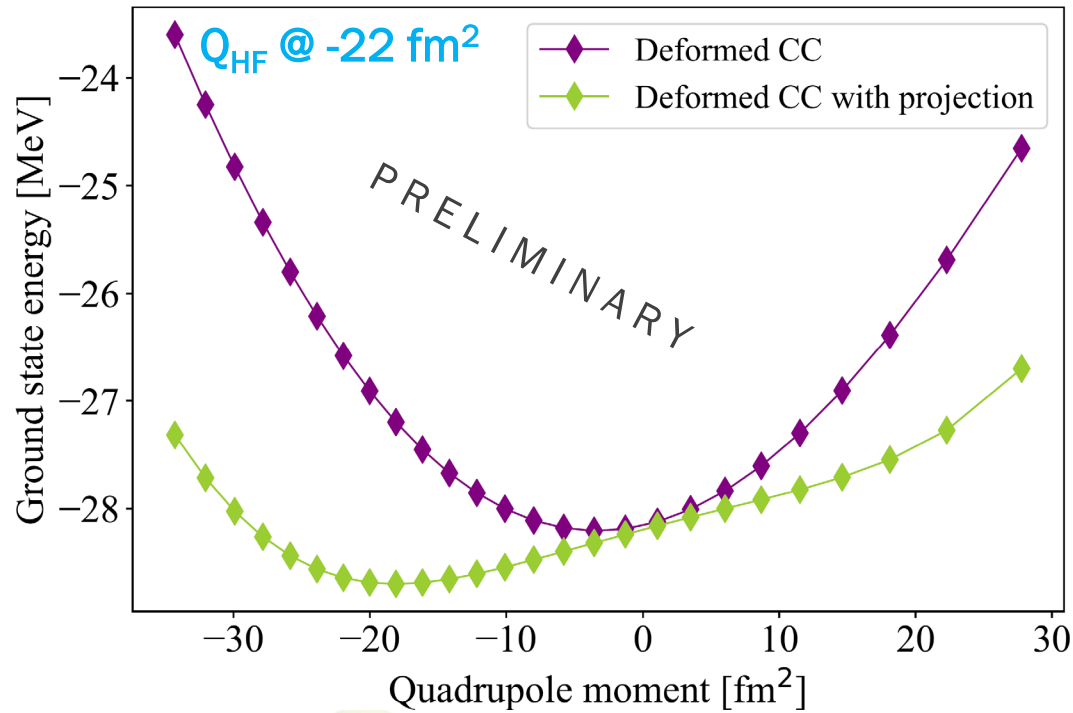
- only axial symmetry is assumed,
- we minimize the energy under the constraint of a **fixed expectation value for the quadrupole moment**.

We then perform an **angular momentum projection** after variation (PAV) of the  $E_{\text{gs}}$  vs  $Q$  curve.

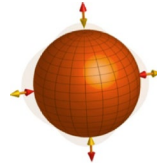




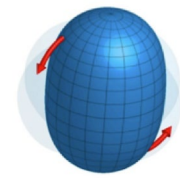
# Deformed CC results for $^8\text{He}$



$Q < 0$   
oblate



$Q = 0$   
spherical



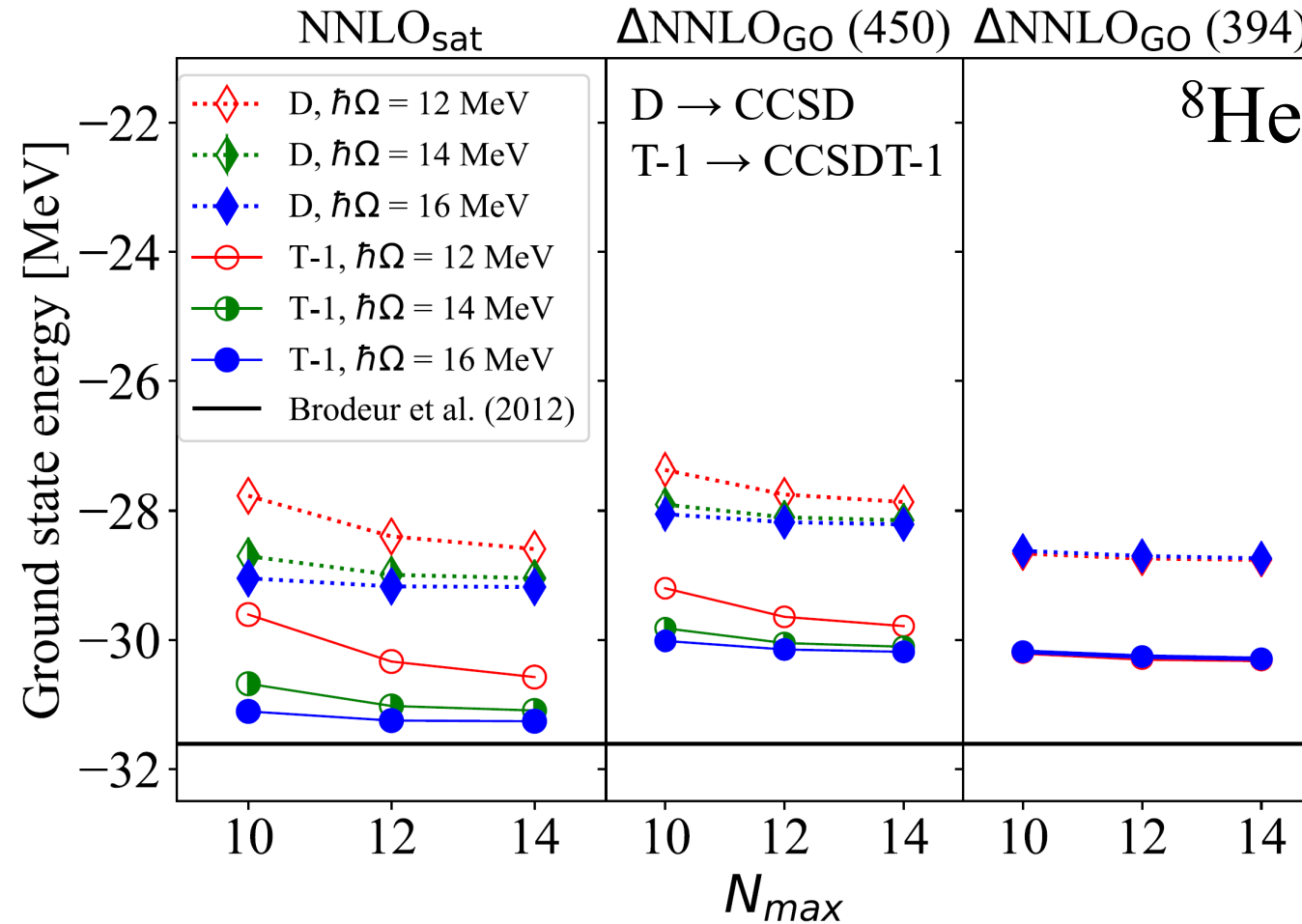
$Q > 0$   
prolate

# Conclusions and outlook

- ❑ Good agreement between theory and experiment for ground-state properties of  $^8\text{He}$ .
- ❑ Our theory may miss some correlations in the description of the **low-lying dipole states**.
- ❑ Neglecting the **nuclear deformation effects** could be a possible reason.
- ❑ Symmetry-restored CC calculations point to an **interplay of prolate and oblate shapes** in the ground state of  $^8\text{He}$ .
- ❑ Our plan is to extend our  $\alpha_D$  calculations to the deformed coupled cluster framework.

BACKUP

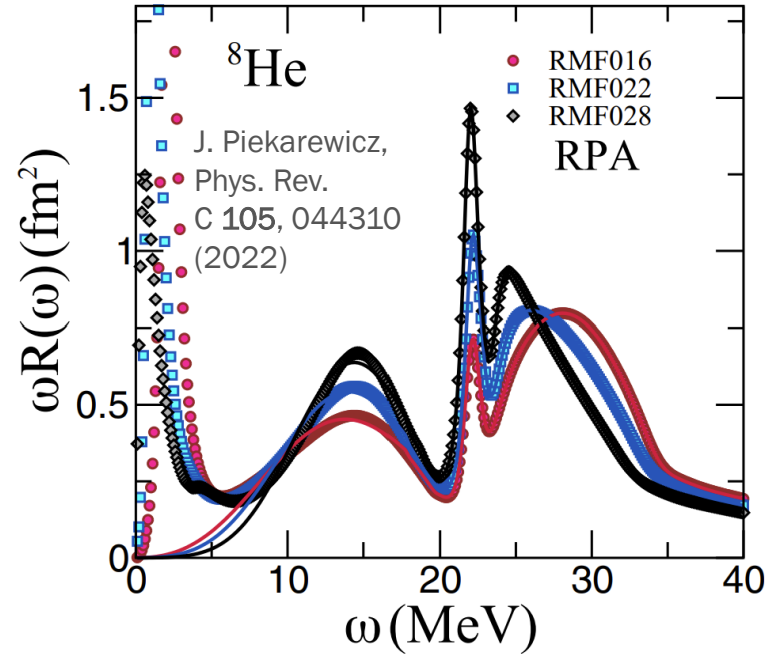
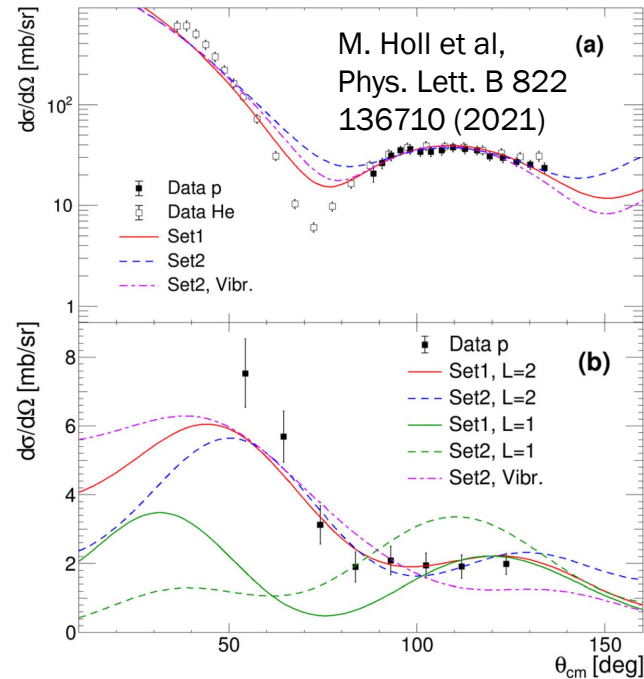
# Ground state energy



# Low-lying dipole states in $^8\text{He}$

## Proton inelastic scattering on $^8\text{He}$

No sign of dipole resonances below 6 MeV, compatible with NCSMC calculations with NN-N4LO.

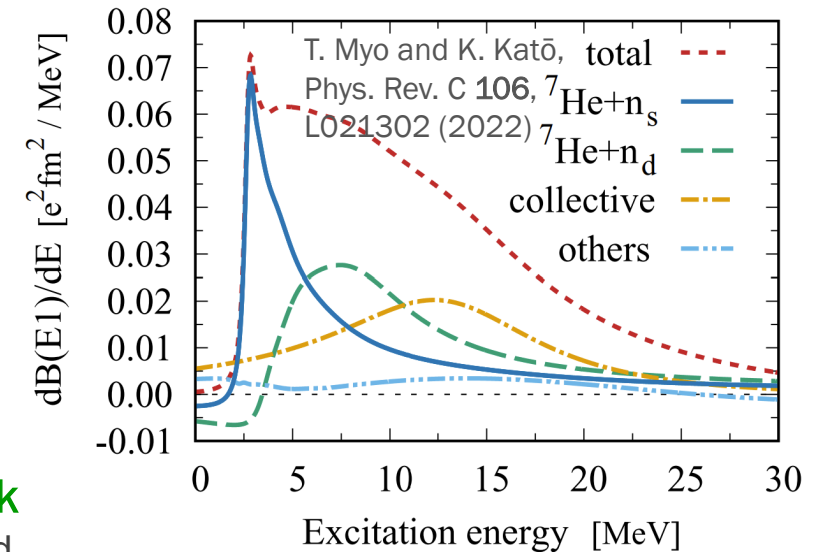


## Dipole response in DFT-RPA framework

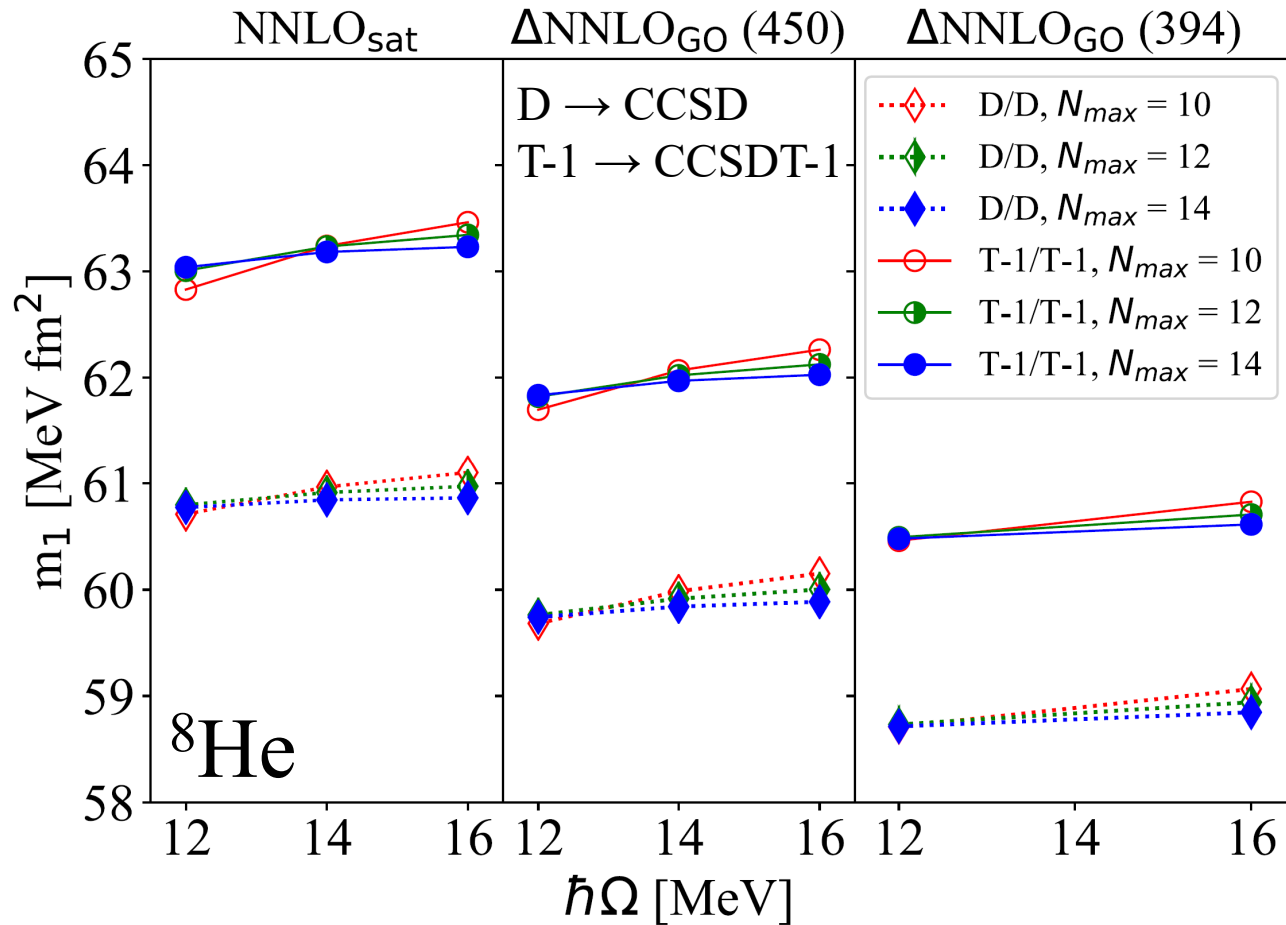
Presence of a soft dipole mode disfavoured (attributed to spurious CM contamination).

## Cluster orbital shell model calculation

Predicted strength at low-energy, associated with  $^7\text{He} + n$  channel.



# Energy-weighted dipole sum rule



$$m_1 = \int d\omega \omega R(\omega)$$

The **photoabsorption cross section** is:

$$\sigma_\gamma(\omega) = 4\pi^2 \alpha \omega R(\omega)$$

so we get:

$$m_1 \propto \int d\omega \sigma_\gamma(\omega)$$

Interaction	$m_1$ (MeV fm <sup>2</sup> )
NNLO <sub>sat</sub>	63(1)
$\Delta\text{NNLO}_{\text{GO}}(450)$	62(1)
$\Delta\text{NNLO}_{\text{GO}}(394)$	60.5(9)

# Cluster sum rule

- How much of the **dipole strength of  $^8\text{He}$**  is related to the **relative motion between core and halo?**
- Let us describe  $^8\text{He}$  in terms of an  **$^4\text{He}$  core** and a **four-neutron halo**.
- Comparing the  $m_1$  sum rule for  $^8\text{He}$  to

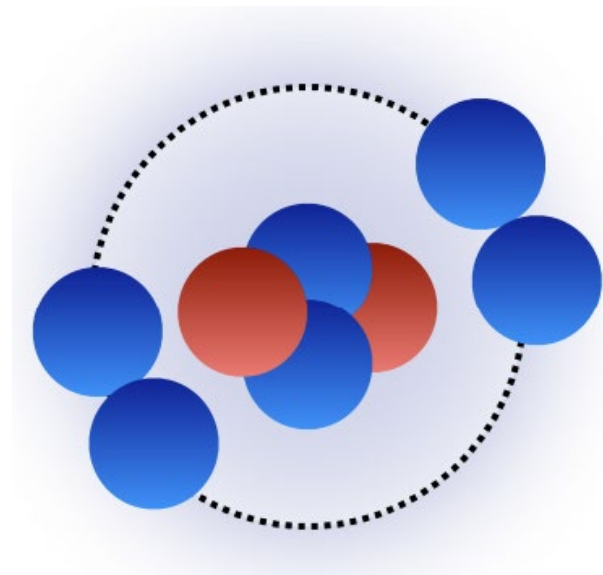
$$\int_{\omega_{th}}^{\infty} d\omega [\sigma_{\gamma}(\omega) - \sigma_{\gamma}^{cl_1}(\omega) - \sigma_{\gamma}^{cl_2}(\omega)] =$$

$$5.974 \frac{(Z_1 A_2 - Z_2 A_1)^2}{A A_1 A_2} \text{MeV fm}^2 (1 + \kappa).$$

we probe a **new dipole degree of freedom**, connected to the relative motion of two “clusters” inside the nucleus (**cluster sum rule**).

- Using  $\text{NNLO}_{\text{sat}}$ , and the  $m_1$  sum rule value for  $^4\text{He}$  ( $\approx 41 \text{ MeV fm}^2$ ), we get:

$$\frac{S_{\text{cluster}}}{S_{^8\text{He}}} \approx 30\%$$



# From ground to dipole-excited states

- Dipole excitations in nuclei can be studied calculating the **nuclear response function**.

$$R(\omega) = \sum_f |\langle f | \hat{\Theta} | 0 \rangle|^2 \delta(\omega - E_f + E_0)$$



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- To avoid the **continuum problem** we calculate a **Lorentz Integral Transform** of the response.

$$L(\sigma, \Gamma) = \frac{\Gamma}{\pi} \int d\omega \frac{R(\omega)}{(\omega - \sigma)^2 + \Gamma^2}$$

- In this way we obtain a **bound-state-like problem**, that we can handle in CC theory.

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- Then via **inversion** we get to the response.

