Recent developments in the study of dense matter

Status report on project B05: Nuclear matter equation of state for astrophysical applications

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with

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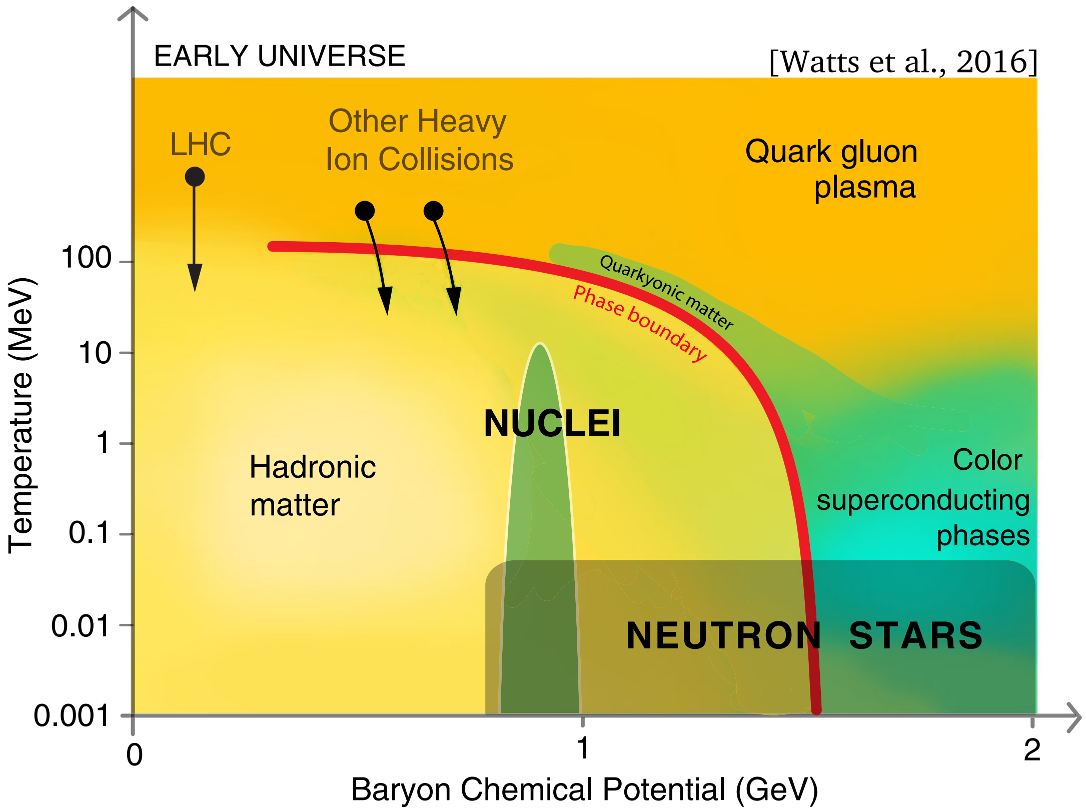
Christian Drischler, Corbinian Wellenhofer and Kai Hebeler

2nd CRC 1245 Workshop
Schloss Waldthausen, 2017

QCD phase diagram:
Neutron stars and the cold dense EoS



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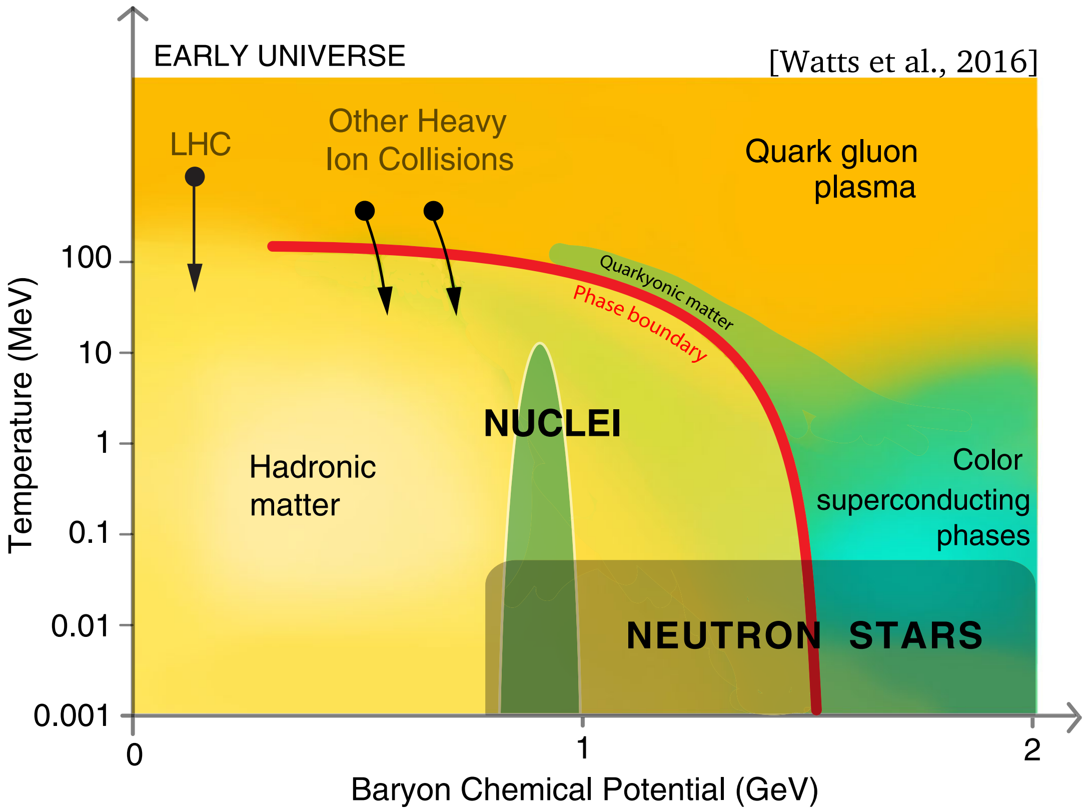

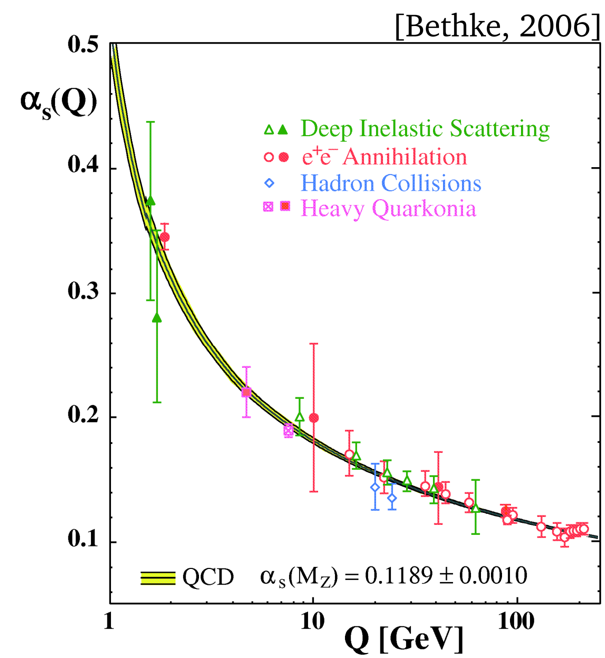

$\lesssim 60\,\rho_{\mathrm{sat}}$ [Kurkela et al., 2014]

Perturbative methods

QCD phase diagram: Neutron stars and the cold dense EoS

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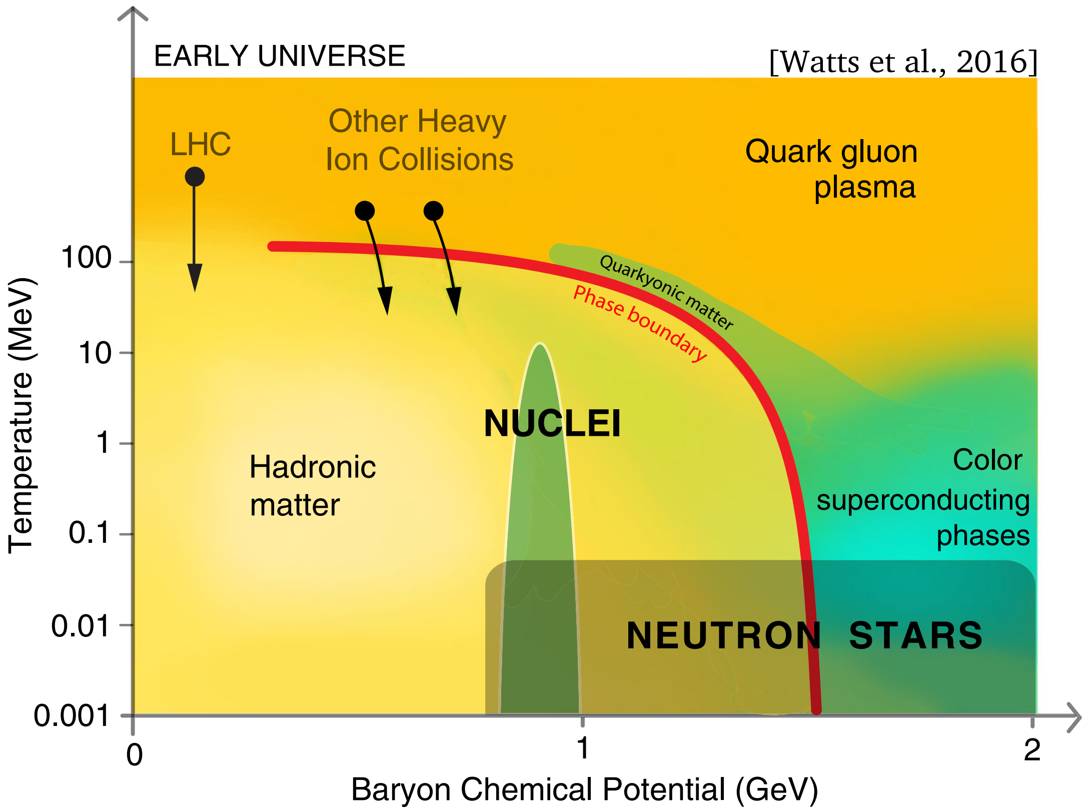

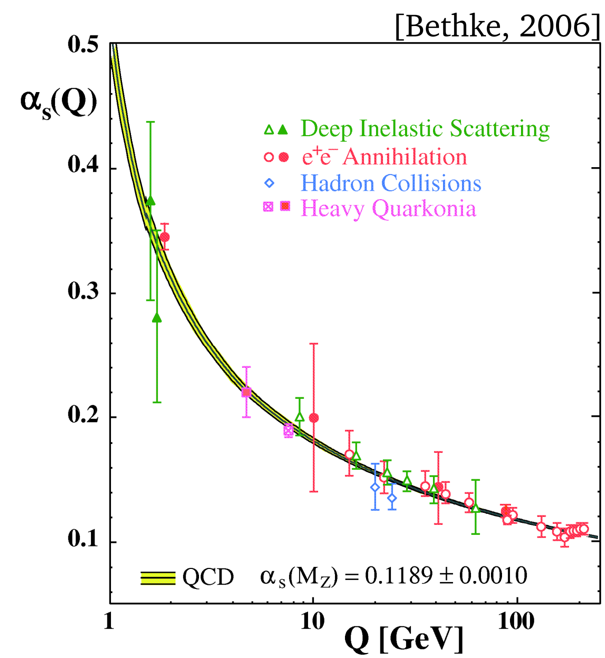

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$\approx$ Perturbative methods

Chiral effective field theory

Functional methods

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Outline

- Chiral effective field theory (at lower densities)
  - Efficient Monte-Carlo framework for MBPT
  - Nuclear thermodynamics from χEFT interactions
- Functional renormalization group (at higher densities)
  - Fierz-complete four-quark interactions in hot and dense QCD (2 flavors)
  - Ground state properties and phases
- Conclusions and outlook

Christian Drischler

Corbinian Wellenhofer

Martin Pospiech

Marc Leonhardt

[Drischler, Hebeler, Schwenk, in preparation]

[Wellenhofer, Holt, Kaiser, Weise; ’14, ’15, ‘16]

[Braun, ML, Pospiech, arXiv:1705.00074]

[Braun, ML, Pospiech, in preparation]

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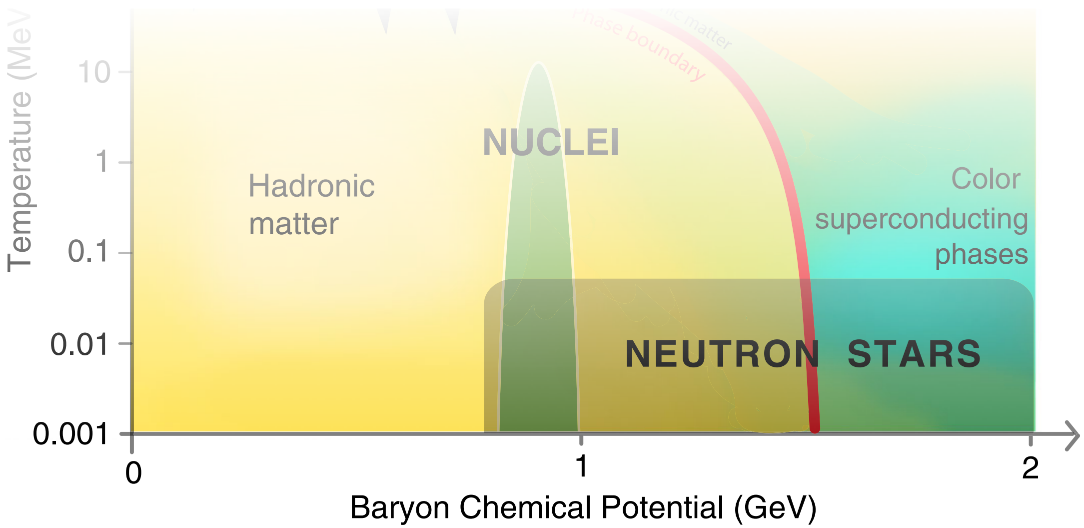
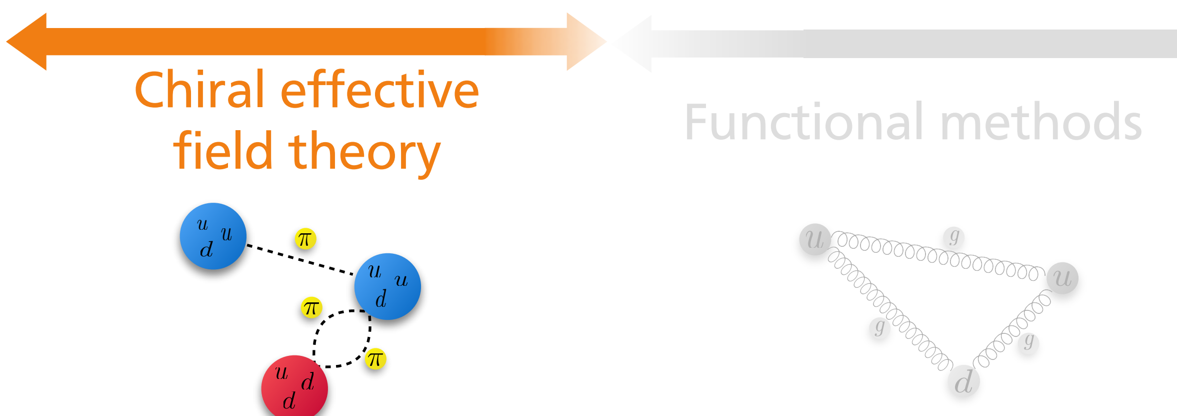
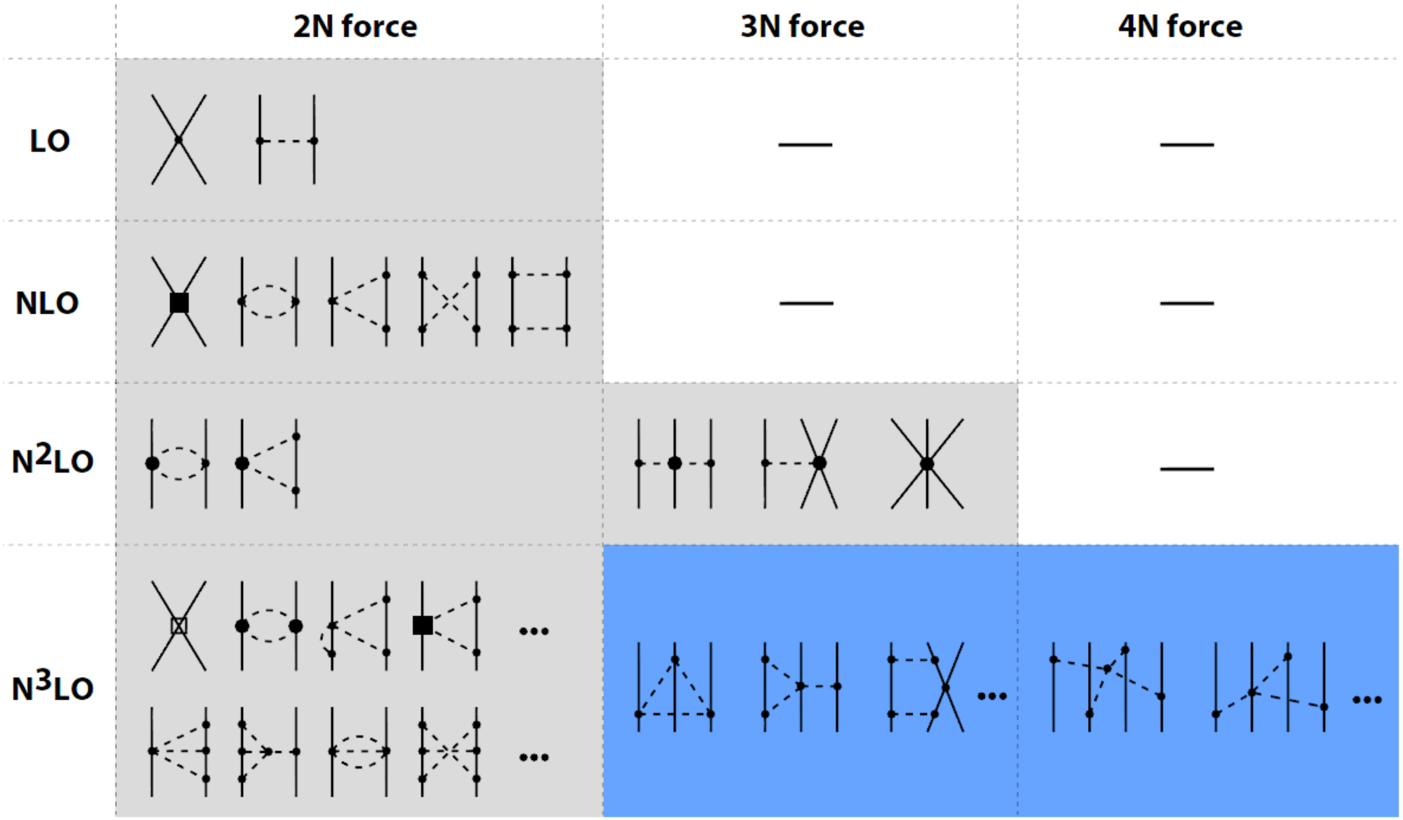




Nuclear matter EOS for astrophysical applications
Chiral effective field theory



... and ongoing work at N⁴LO, N⁵LO, ...

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Kaiser, Machleidt, Meißner, ...

Nuclear matter EOS for astrophysical applications

Efficient Monte-Carlo framework for MBPT

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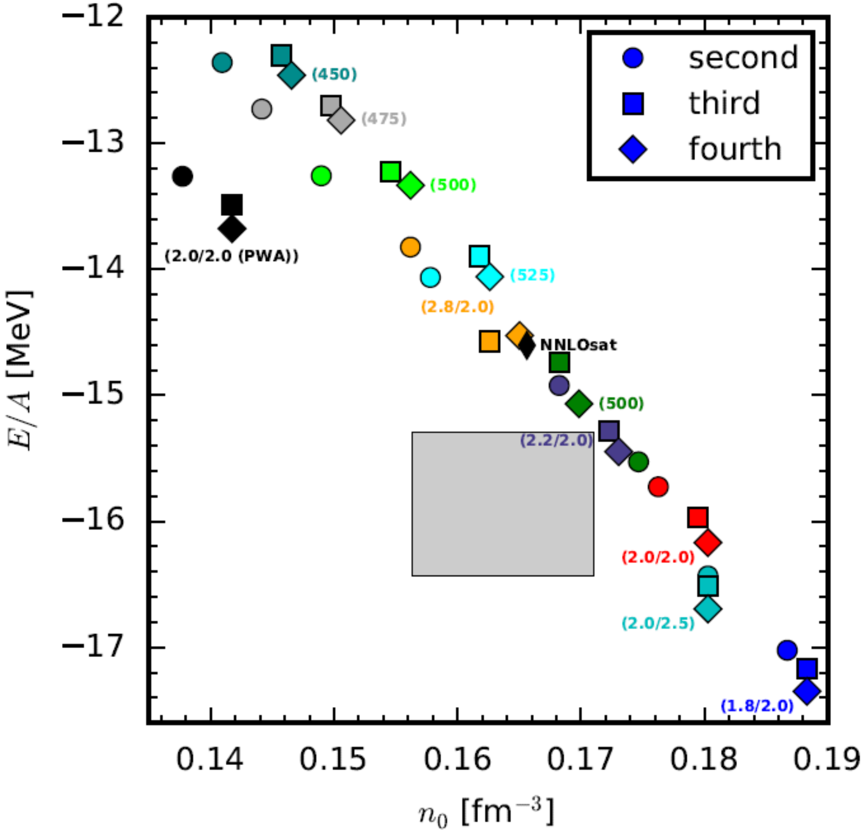

- based on **analytical expressions**
  - NN, 3N, 4N forces @ N³LO (no PW's)
  - MBPT for up to 4th order (*automatic* code generation)

[Drischler, Hebeler, Schwenk, in preparation]

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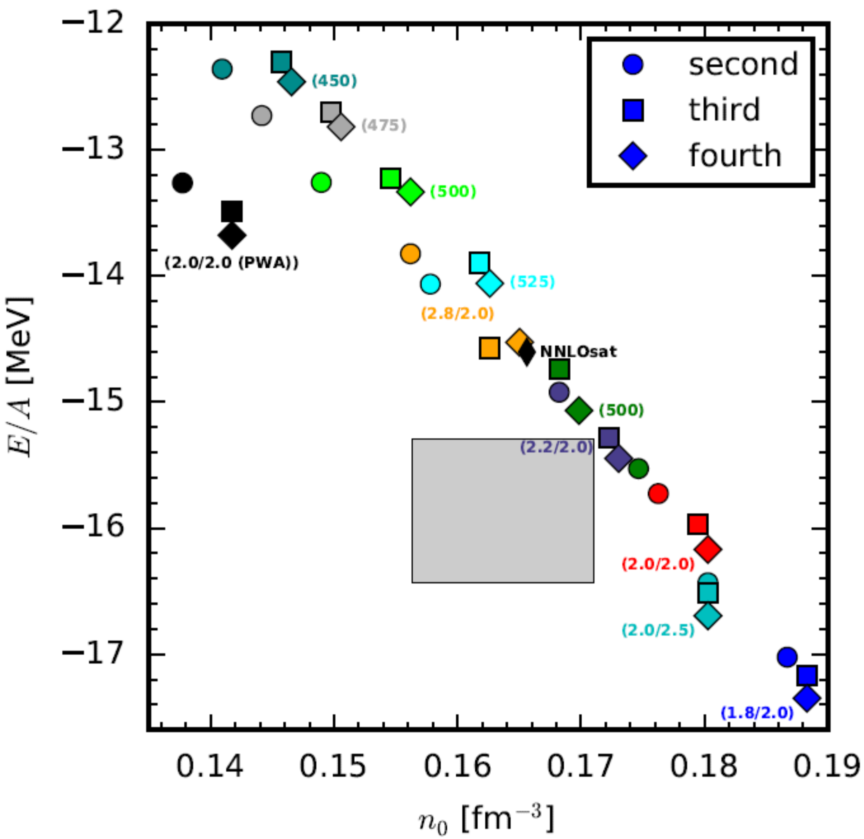
Nuclear matter EOS for astrophysical applications

Efficient Monte-Carlo framework for MBPT



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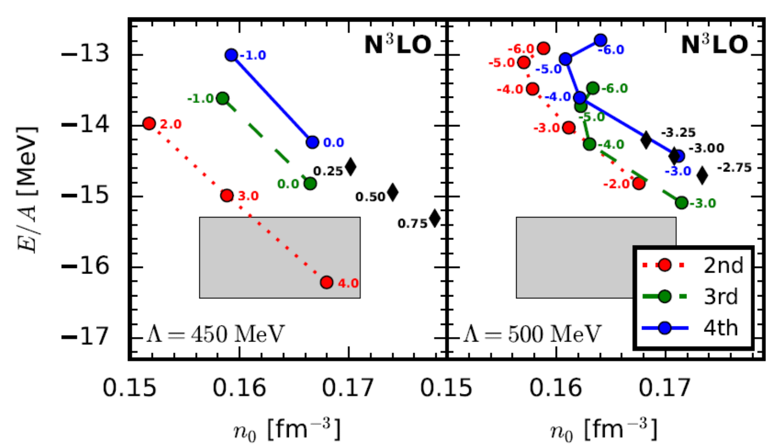
- **aim:** guiding fits of next-generation interactions in terms of **saturation**
- fit 3N LECs c_D/c_E @ N³LO to ³H and study saturation



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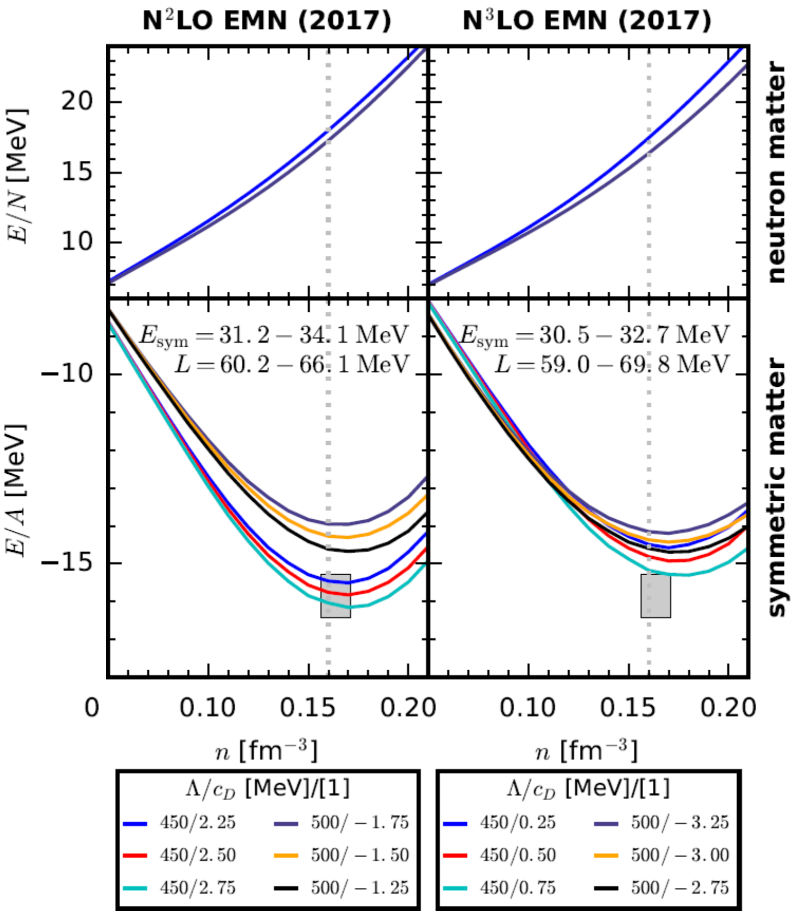

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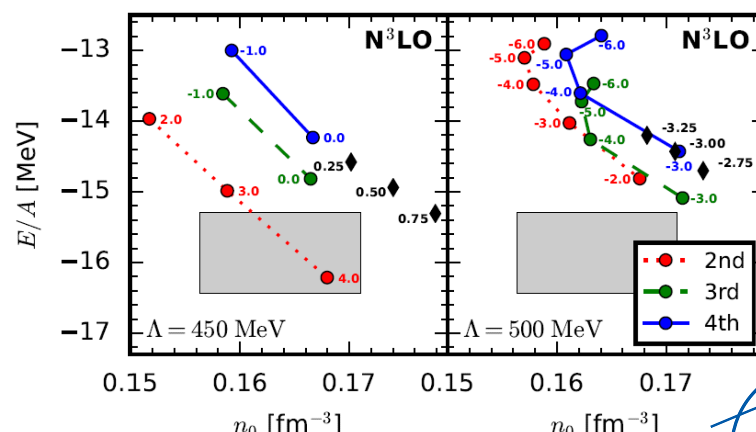

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Nuclear thermodynamics
from chiral effective field theory interactions

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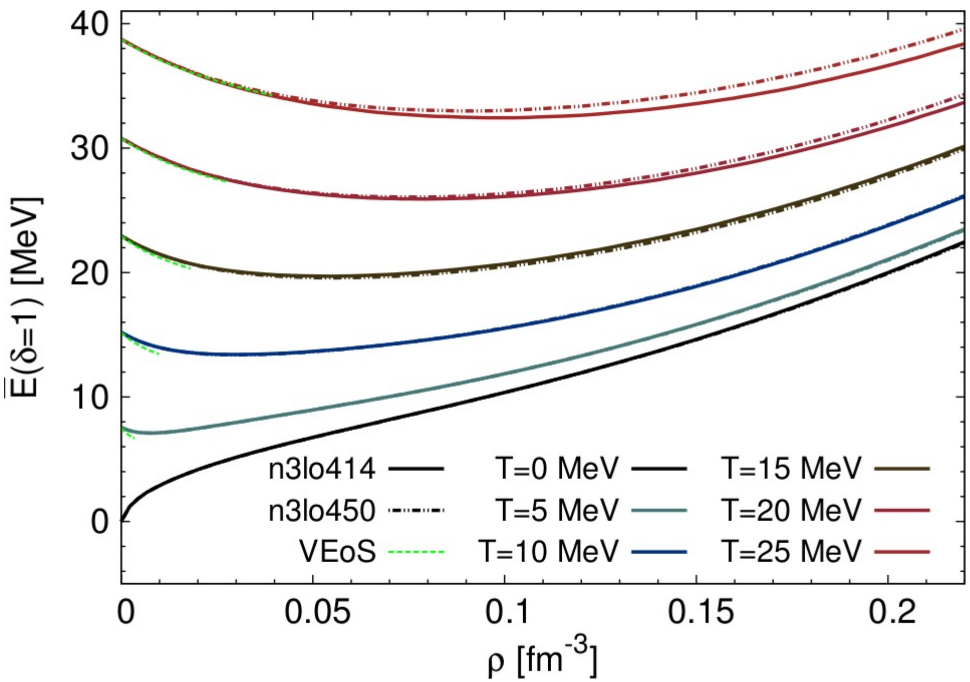

Figure: Internal energy of pure neutron matter (δ=1; VEoS: virial expansion)

[Wellenhofer, Holt, Kaiser, Weise, PRC **89**, 064009 (2014)]
[Wellenhofer, Holt, Kaiser, PRC **92**, 015801 (2015)]
[Wellenhofer, Holt, Kaiser, PRC **93**, 055802 (2016)]
[Schwenk, Horowitz, Phys. Lett. **B638**, 153-159 (2006)]

- **Compute thermodynamic properties of nuclear matter**
  - Needed for neutron star and supernova simulations
  - Large parameter space: temperature **T**, nucleon density **ρ**, isospin asymmetry **δ=1-2Y** (where Y is the proton fraction)

- **Good benchmark results**

  e.g.: good agreement with virial expansion at low densities (see Figure)

- **Future work:**

  single-particle properties, improved calculations (better uncertainty estimates), ...

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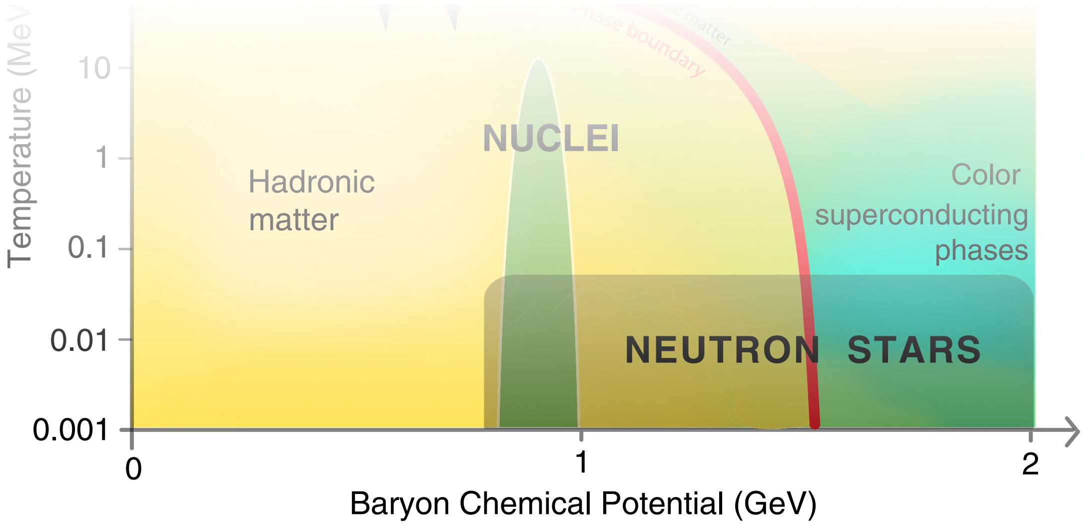
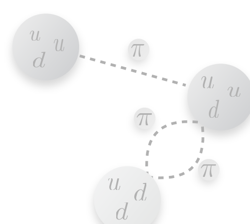
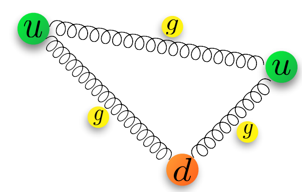


Chiral effective field theory



Functional methods



Functional renormalization group (FRG)

From high to low energies in QCD

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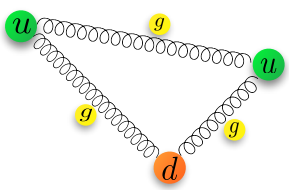
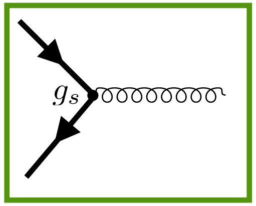
RG steps





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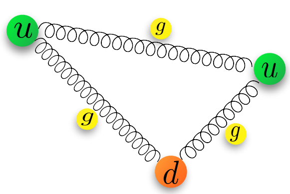
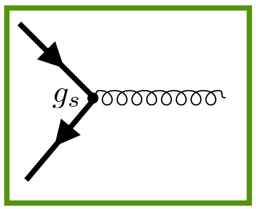
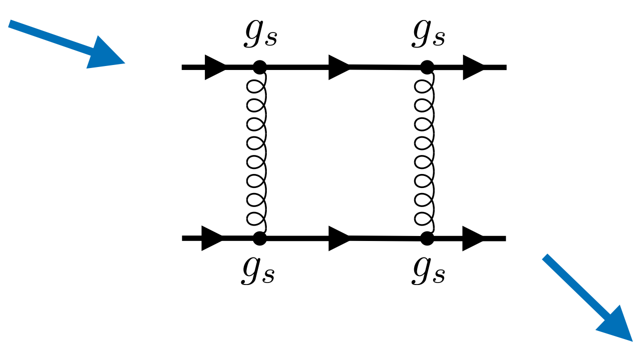
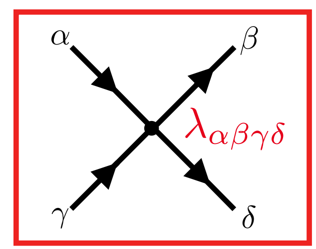






Four-quark interactions



1. Important effective interactions
2. Encode information on the ground state and spontaneous symmetry breaking
3. Full treatment possible and crucial

Four-quark interactions and symmetry breaking

Aspects of the low-energy sector in QCD

$$S[\bar{\psi},\psi] = \int_x \left\{ \bar{\psi} \mathrm{i} \partial\!\!\!/ \psi \; + \frac{1}{2} \bar{\lambda}_{(\sigma-\pi)} \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5 \vec{\tau} \psi)^2 \right] \right\}$$

Four-quark interactions and symmetry breaking

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Partial bosonization

$$\sigma \sim \bar{\psi}\psi$$
$$\vec{\pi} \sim \bar{\psi}\gamma_5\vec{\tau}\psi$$

Symmetry of the ground state

$$U_B \sim \frac{1}{\bar{\lambda}_{(\sigma-\pi)}}(\sigma^2 + \pi^2) + \dots$$

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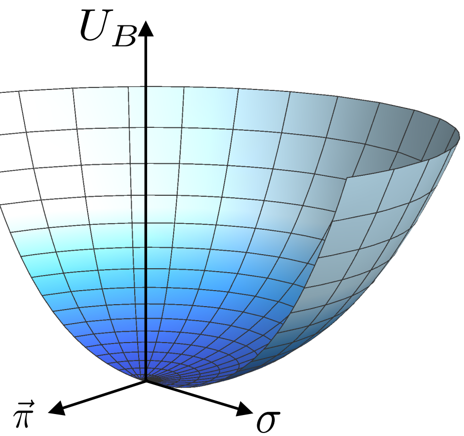
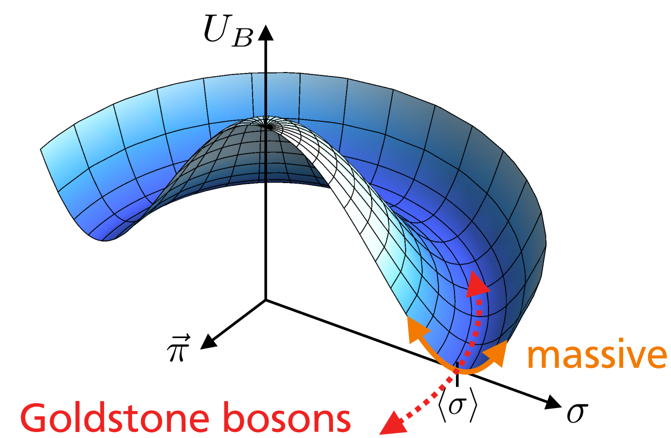


$$\left. \frac{1}{\bar{\lambda}_{(\sigma-\pi)}} \right|_{k_0} \longrightarrow 0$$



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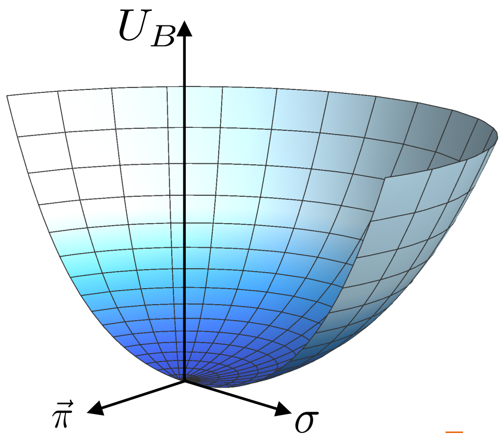

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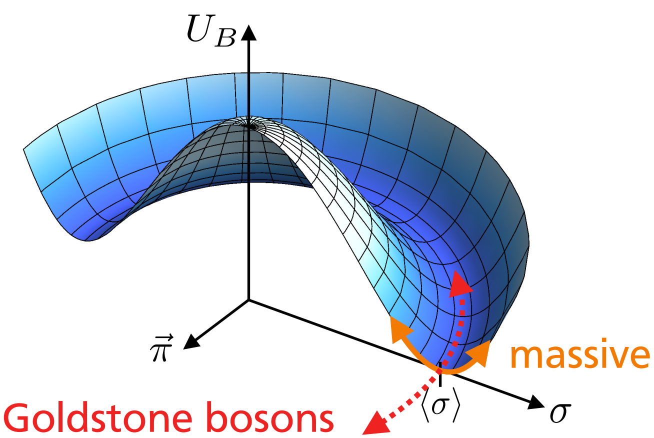
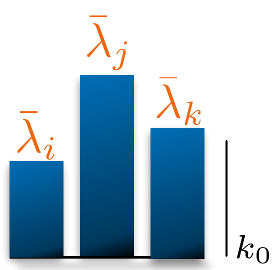


Assessing relative interaction strengths



Formation of specific condensates

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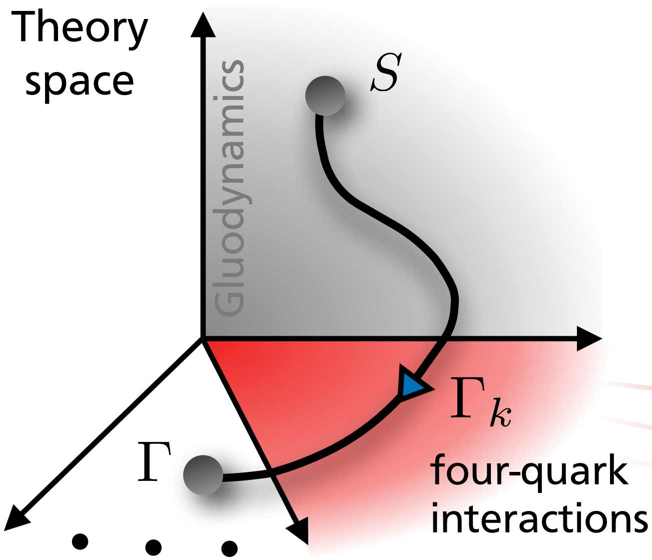
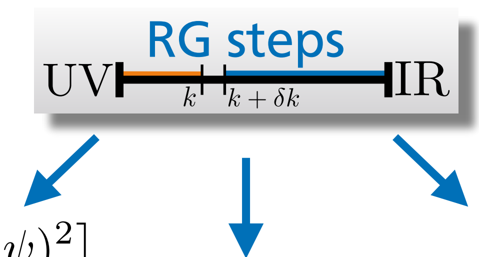
Four-quark interactions and symmetries

Fierz-complete basis of interactions

Studies so far: Incomplete subsets, typically 1-3 four-quark interaction channels.



Ambiguities! Neglected contributions!



$$\left[(\bar{\psi}\gamma_\mu\psi)^2\right]$$

$$\left[(\bar{\psi}\gamma_\mu\gamma_5 T^a\psi)^2\right]$$

⋮

Everything that is not forbidden is allowed.

Four-quark interactions and symmetries

Fierz-complete basis of interactions

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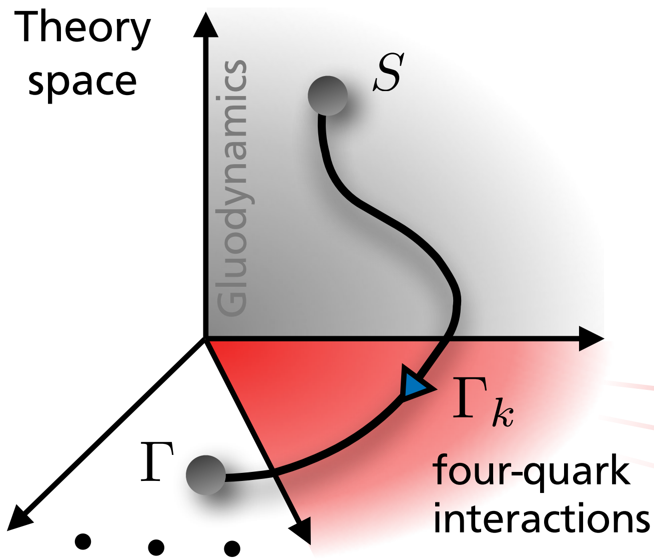

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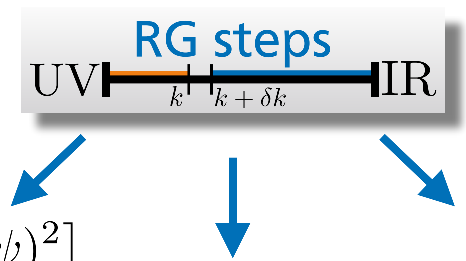

$[(\bar{\psi}\gamma_\mu\psi)^2]$

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⋮

Everything that is not forbidden is allowed.

Only constrained by symmetries!

T (time reversal) ✓ P (parity) ✓ C ✗ (charge conj.)

Lorentz group: $\mathrm{SO}(1,3) \longrightarrow \mathrm{SO}(3)$

Flavor space: $\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2) \otimes \mathrm{U}_V(1)$

Color space: $\mathrm{SU}(N_c)$

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Four-quark interactions and symmetries

Fierz-complete basis of interactions

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$$\Gamma_k[\bar{\psi}, \psi] = \int_x \left\{ \bar{\psi}(\mathrm{i}Z_{\|}\gamma_0\partial_0 + \mathrm{i}Z_{\perp}\gamma_i\partial_i - \mathrm{i}Z_{\mu}\mu\gamma_0)\psi + \frac{1}{2}\sum_i \bar{\lambda}_i \, \mathcal{L}^i_{(\bar{\psi}\psi)^2} \right\}$$

In total 20 channels meet symmetry constraints → Fierz identities → Fierz-complete basis: 10 channels

$$\mathcal{L}^{(\sigma-\pi)}_{(\bar{\psi}\psi)^2} = (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\vec{\tau}\psi)^2$$

↔ formation of chiral condensate

$$\mathcal{L}^{\mathrm{csc}}_{(\bar{\psi}\psi)^2} \sim (\mathrm{i}\bar{\psi}\gamma_5\tau_A t_c^{A'}\mathcal{C}\bar{\psi}^T)(\mathrm{i}\psi^T\mathcal{C}\gamma_5\tau_A t_c^{A'}\psi) \qquad J^P = 0^+$$

↔ formation of diquark condensate

[Rapp, Schäfer, Shuryak, Velkovsky, 1998]

+ 8 more interaction channels

[Braun, ML, Pospiech '17]

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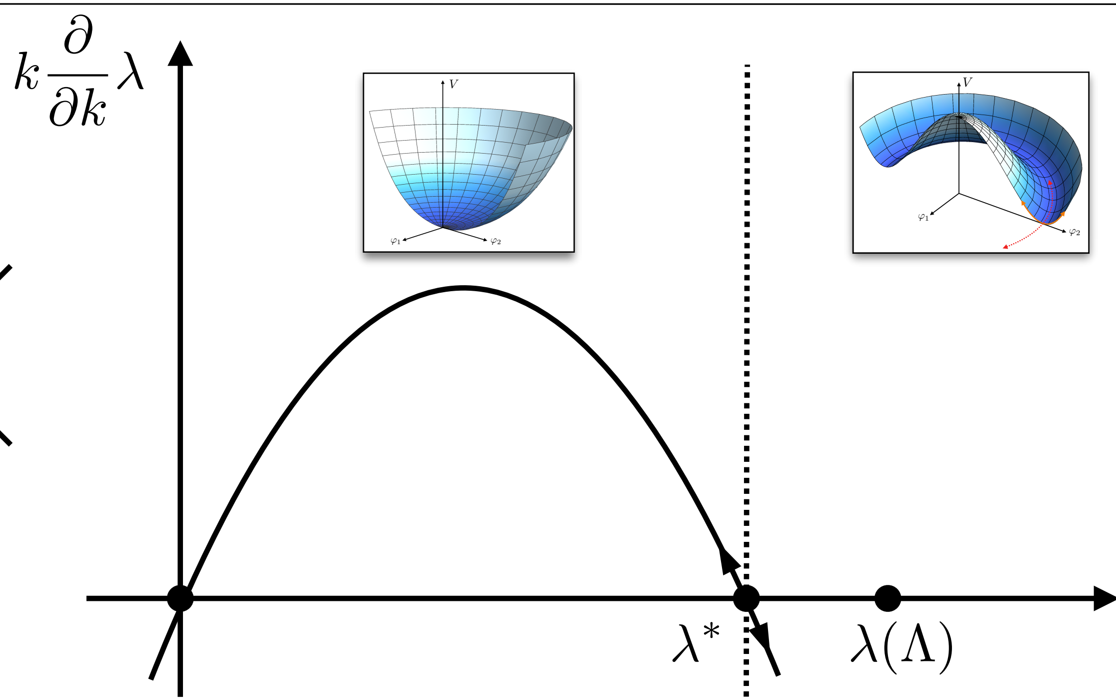
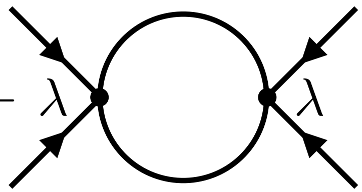
RG flow of four-quark interactions

Qualitative behavior and the effect of external parameters



RG flow equation:

$$k\frac{\partial}{\partial k}\lambda = 2\lambda - \lambda$$  $$\lambda$$

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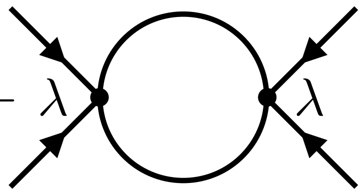
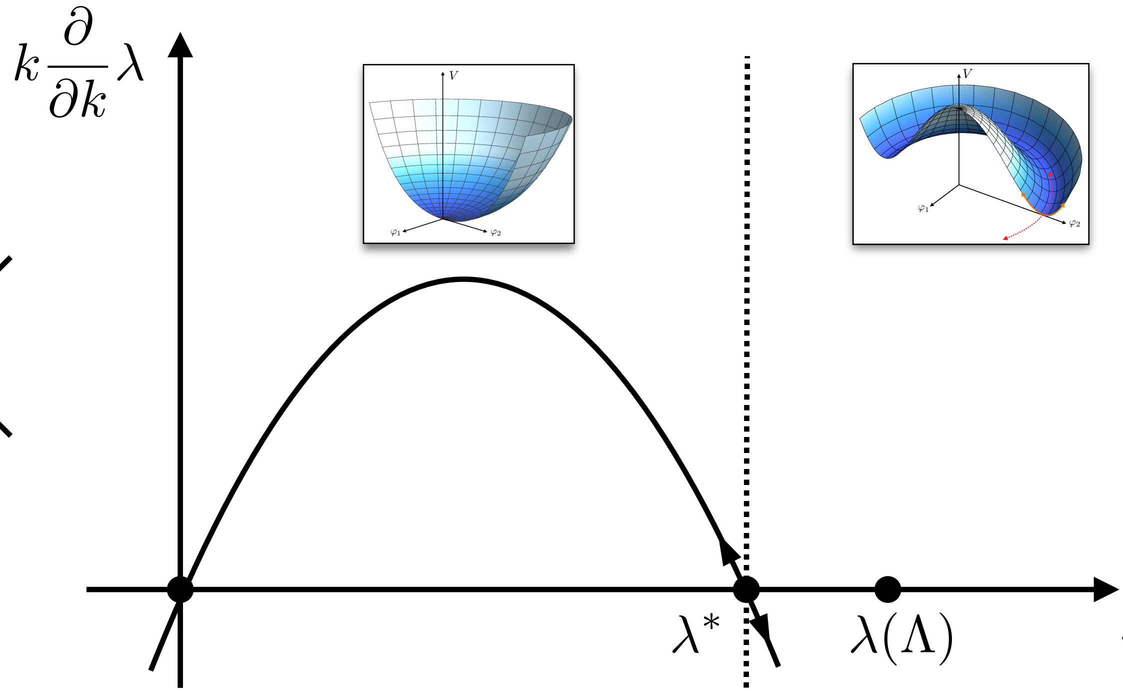
$$k\frac{\partial}{\partial k}\lambda = 2\lambda - \lambda$$  λ



Scale fixing procedure:

- Only $\lambda_{(\sigma-\pi)}(\Lambda)$ assumes finite value as inspired by gluon-induced four-quark flows [Braun, 2011]
- UV value tuned so that specific symmetry-breakdown scale k_0 is obtained which corresponds to a quark mass of 300 MeV in the vacuum limit

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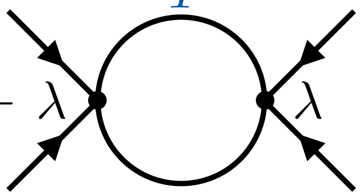
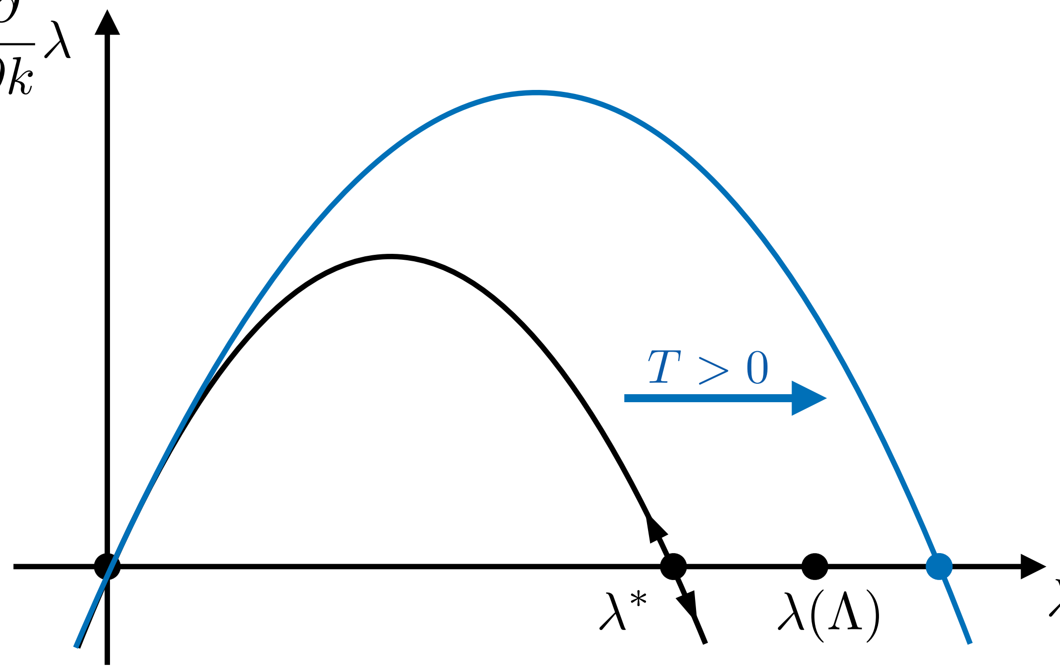
$$k\frac{\partial}{\partial k}\lambda = 2\lambda - \lambda$$  λ



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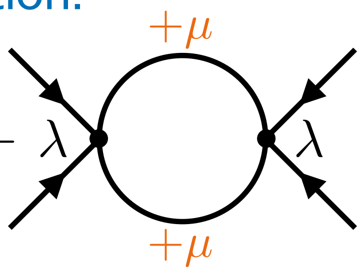

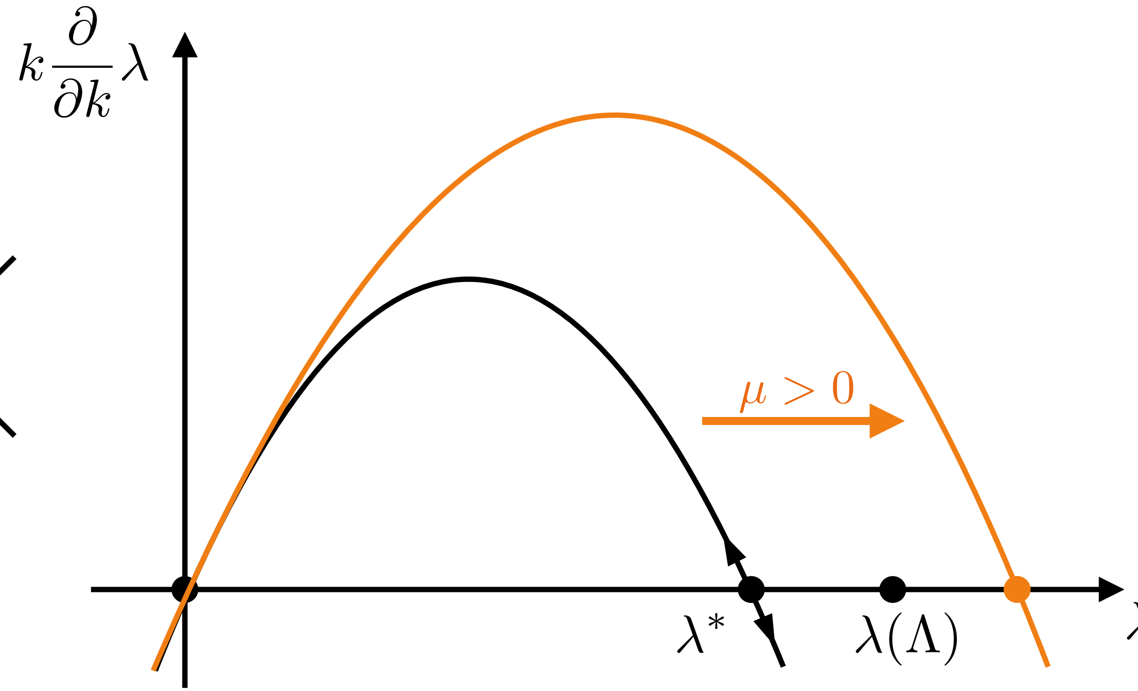

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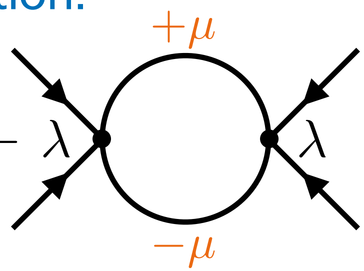
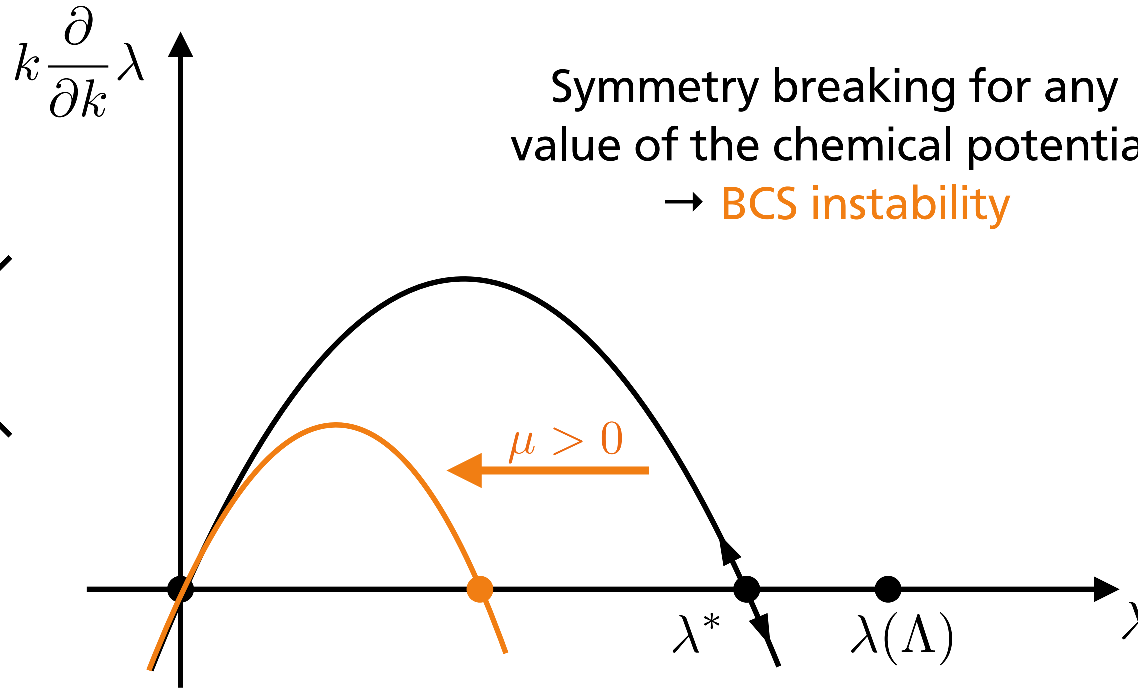


Symmetry breaking for any value of the chemical potential → BCS instability



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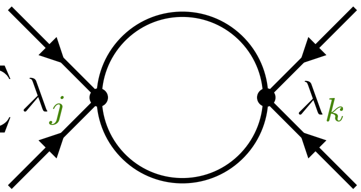
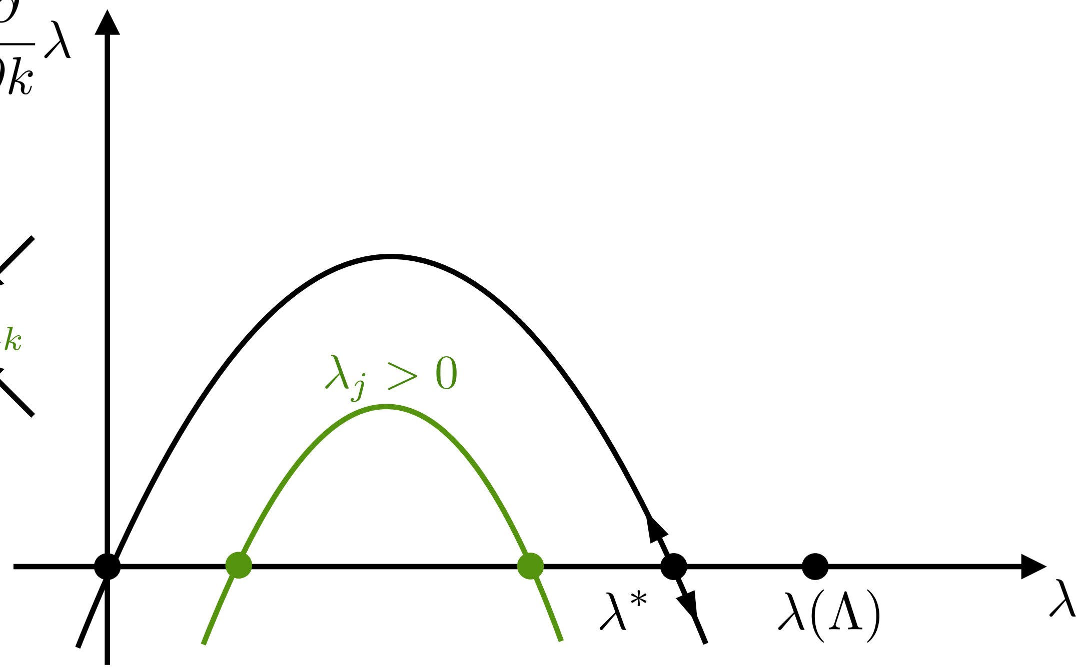




Scale fixing procedure:

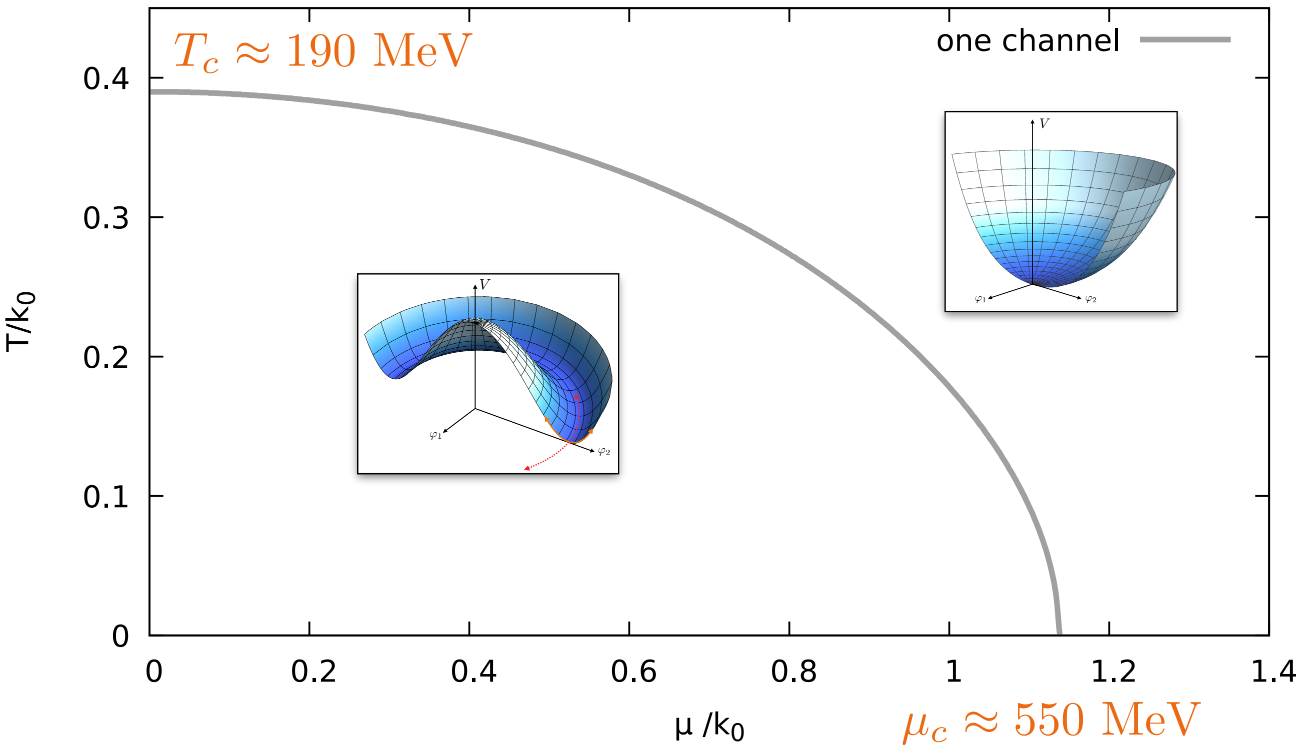
- Only $\lambda_{(\sigma-\pi)}(\Lambda)$ assumes finite value as inspired by gluon-induced four-quark flows [Braun, 2011]
- UV value tuned so that specific symmetry-breakdown scale k_0 is obtained which corresponds to a quark mass of 300 MeV in the vacuum limit

Exploring the phase diagram

Fixed-point structure and patterns of symmetry breaking

Ansatz one-channel approximation

$$\Gamma_k[\bar{\psi}, \psi] = \int_x \left\{ (\text{kinetic term}) + \frac{1}{2} \bar{\lambda}_{(\sigma-\pi)} \left[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5 \vec{\tau} \psi)^2 \right] \right\}$$



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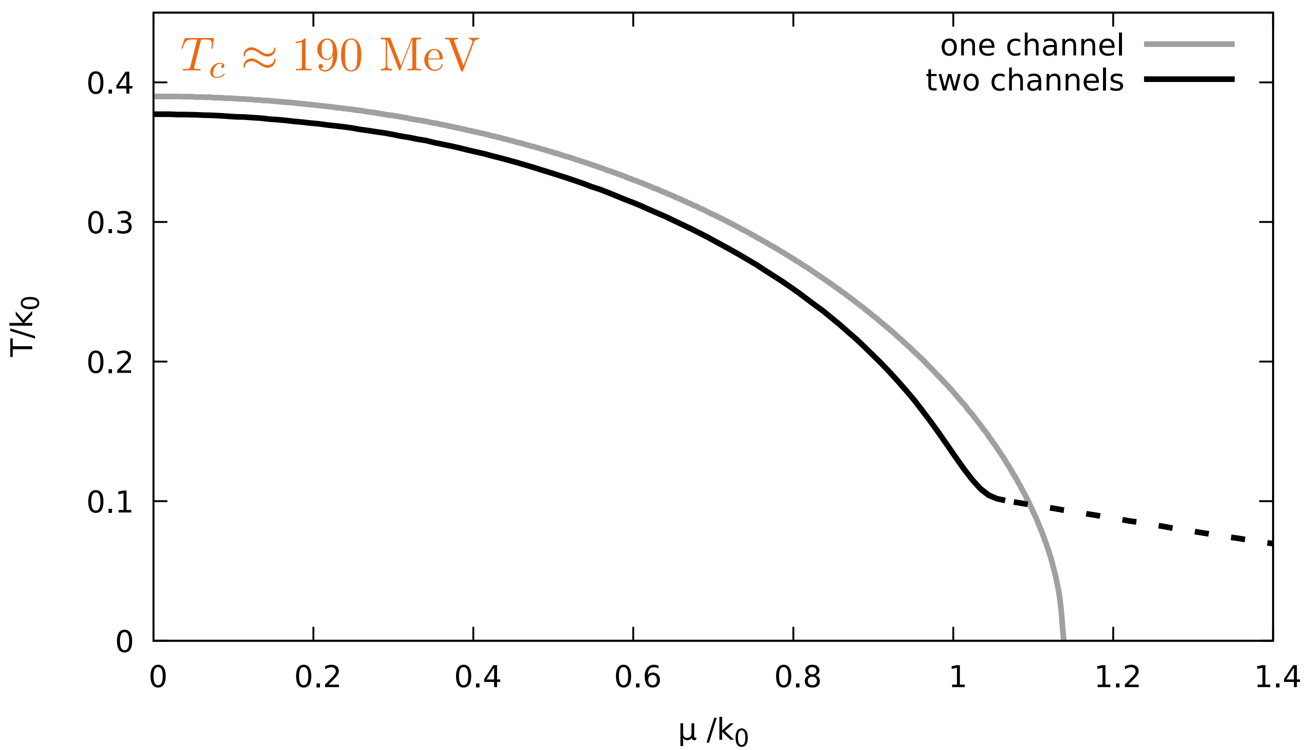

Exploring the phase diagram

Fixed-point structure and patterns of symmetry breaking

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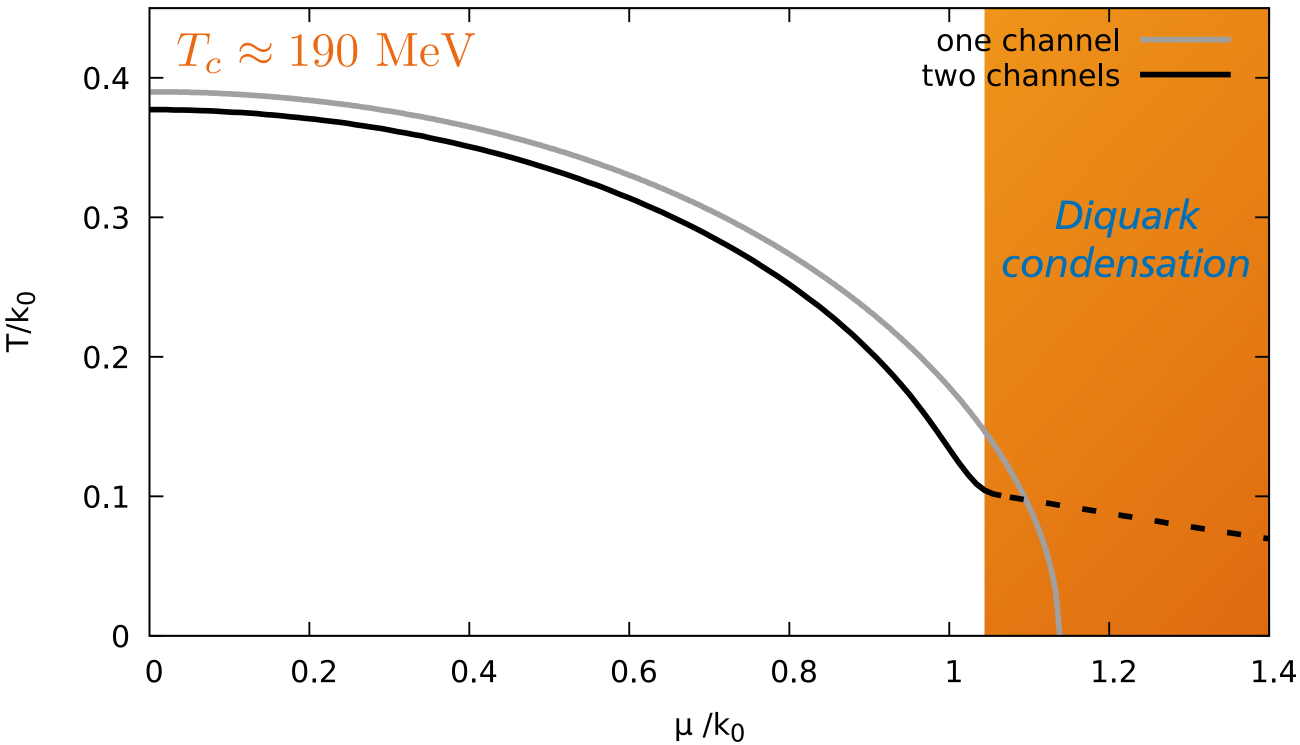

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Exploring the phase diagram

Fixed-point structure and patterns of symmetry breaking

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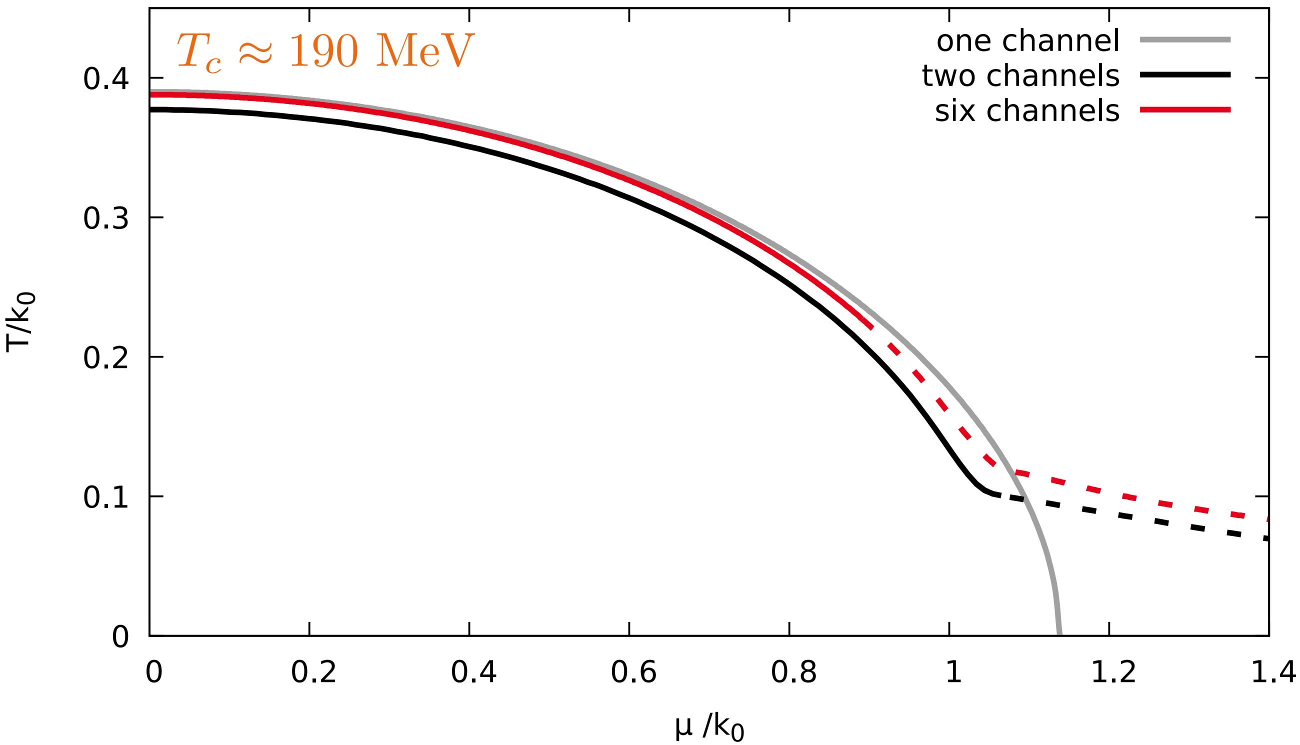
Ansatz Fierz-complete at $T = 0$ and $\mu = 0$

$$\Gamma_k[\bar{\psi}, \psi] = \int_x \left\{ (\text{kinetic term}) + \frac{1}{2} \sum_{i=1}^{6} \bar{\lambda}_i \, \mathcal{L}^i_{(\bar{\psi}\psi)^2} \right\}$$

six channels



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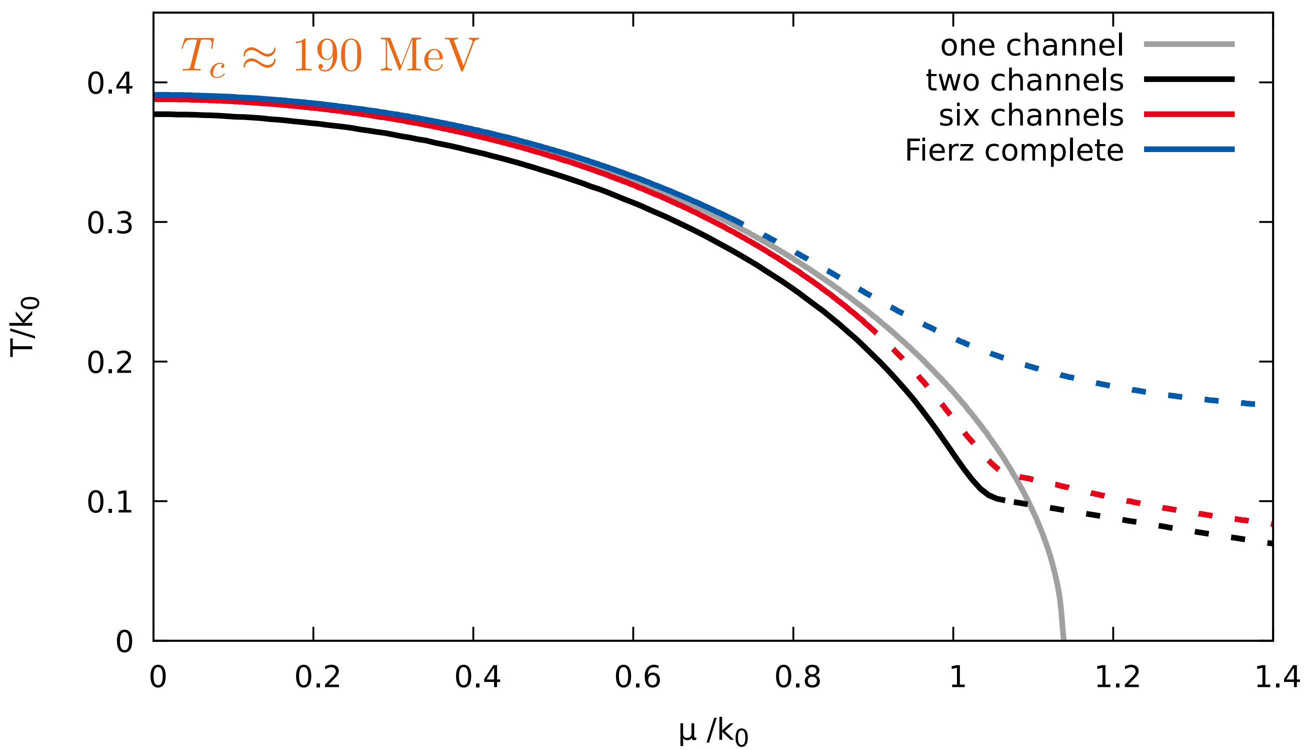
Exploring the phase diagram

Fixed-point structure and patterns of symmetry breaking

Fierz-complete ansatz

$$\Gamma_k[\bar{\psi},\psi] = \int_x \left\{ (\text{kinetic term}) + \frac{1}{2}\sum_i \bar{\lambda}_i \, \mathcal{L}^i_{(\bar{\psi}\psi)^2} \right\}$$

comprises all 10 channels



Exploring the phase diagram

Fixed-point structure and patterns of symmetry breaking

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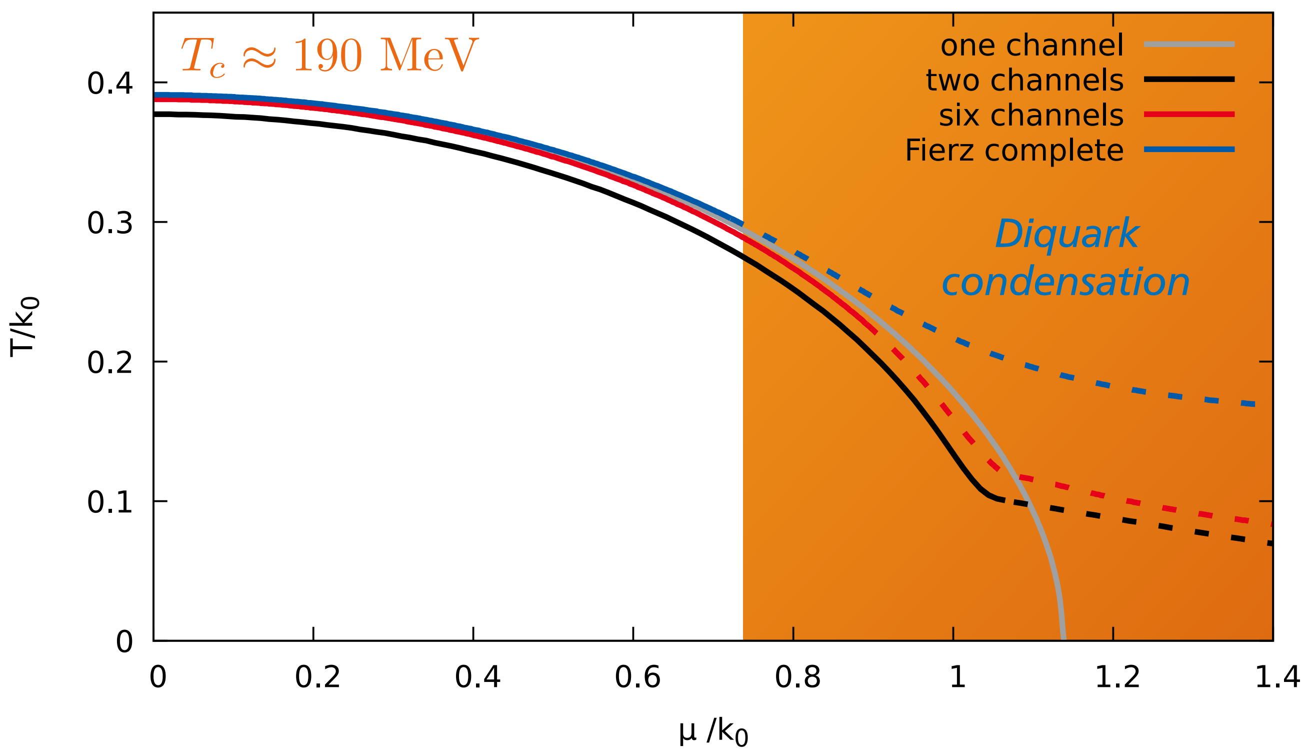

$T_c \sim \Delta_{csc}(T = 0)$

[BCS theory]

Implications for EoS!

Conclusions and outlook

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Chiral effective field theory at lower densities

- Efficient Monte-Carlo framework for MBPT (automatic code generation; 4th order)
- Improve fits of LECs by guiding in terms of nuclear saturation
- Nuclear thermodynamics from χEFT interactions: $T,\ \rho,\ \delta$

Outlook Apply saturation guided fitting to next-generation interactions,
extract single-particle properties from nuclear thermodynamics

Functional renormalization group at higher densities

- First Fierz-complete study of effective action
- Importance of Fierz-completeness to probe the regime at high quark chemical potential and low temperature
- Forming of diquark condensate (color superconducting phase)

Outlook Inclusion of dynamic gauge fields (equations worked out)
and first estimate of EoS, work in progress.

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Backup

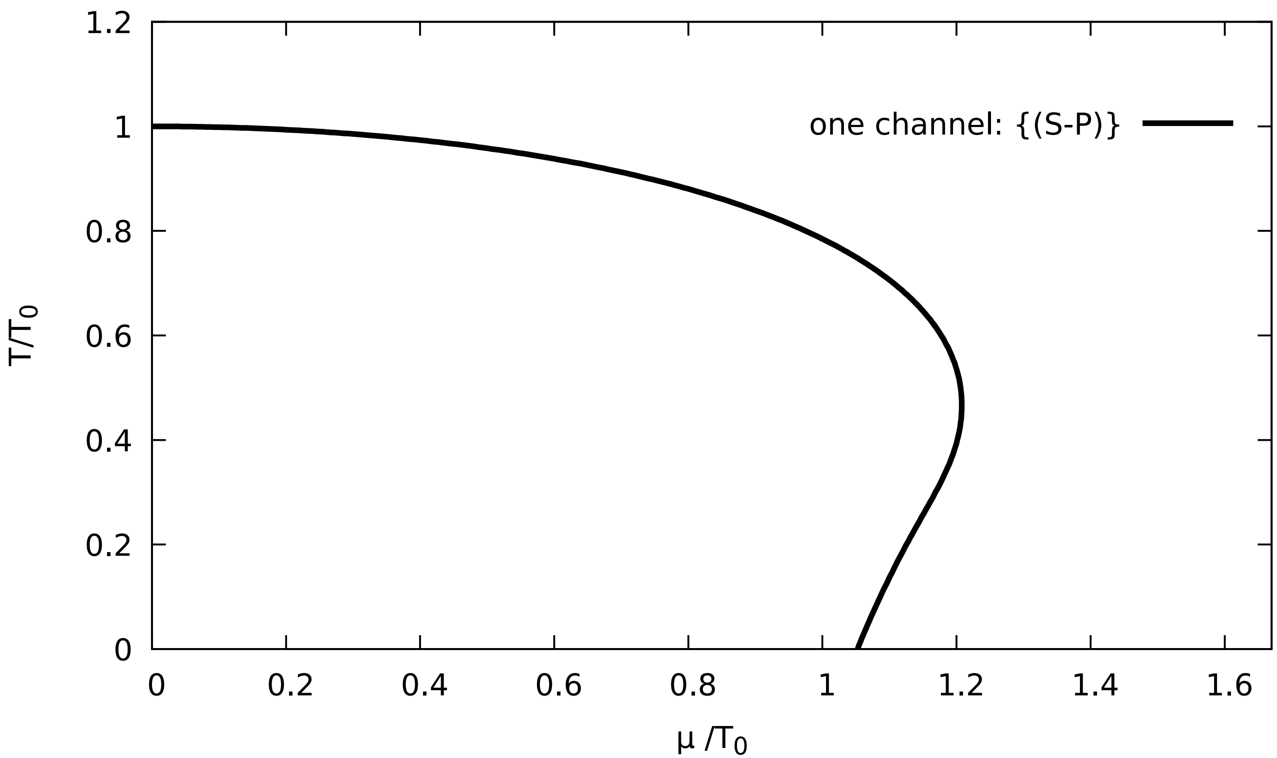
Difermion-type degrees of freedom

$$\begin{Bmatrix}(\mathrm{S}-\mathrm{P})\\(\mathrm{V}_{\parallel})\\(\mathrm{V}_{\perp})\end{Bmatrix} \rightarrow \begin{Bmatrix}(\mathrm{S}-\mathrm{P})\\(\mathrm{S}\mathcal{C}-\mathrm{P}\mathcal{C})\\(\mathrm{A}_{\parallel}\mathcal{C})\end{Bmatrix}$$

$$\mathcal{C} = \mathrm{i}\gamma_2\gamma_0$$

$$(\mathrm{S}\mathcal{C}-\mathrm{P}\mathcal{C}) \equiv (\bar{\psi}\mathcal{C}\bar{\psi}^T)(\psi^T\mathcal{C}\psi) - (\bar{\psi}\gamma_5\mathcal{C}\bar{\psi}^T)(\psi^T\mathcal{C}\gamma_5\psi)$$

$$(\mathrm{A}_{\parallel}\mathcal{C}) \equiv (\bar{\psi}\gamma_0\gamma_5\mathcal{C}\bar{\psi}^T)(\psi^T\mathcal{C}\gamma_0\gamma_5\psi)$$



Difermion-type degrees of freedom

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$$\begin{Bmatrix} (\mathrm{S}-\mathrm{P}) \\ (\mathrm{V}_{\parallel}) \\ (\mathrm{V}_{\perp}) \end{Bmatrix} \rightarrow \begin{Bmatrix} (\mathrm{S}-\mathrm{P}) \\ (\mathrm{S}\mathcal{C}-\mathrm{P}\mathcal{C}) \\ (\mathrm{A}_{\parallel}\mathcal{C}) \end{Bmatrix}$$

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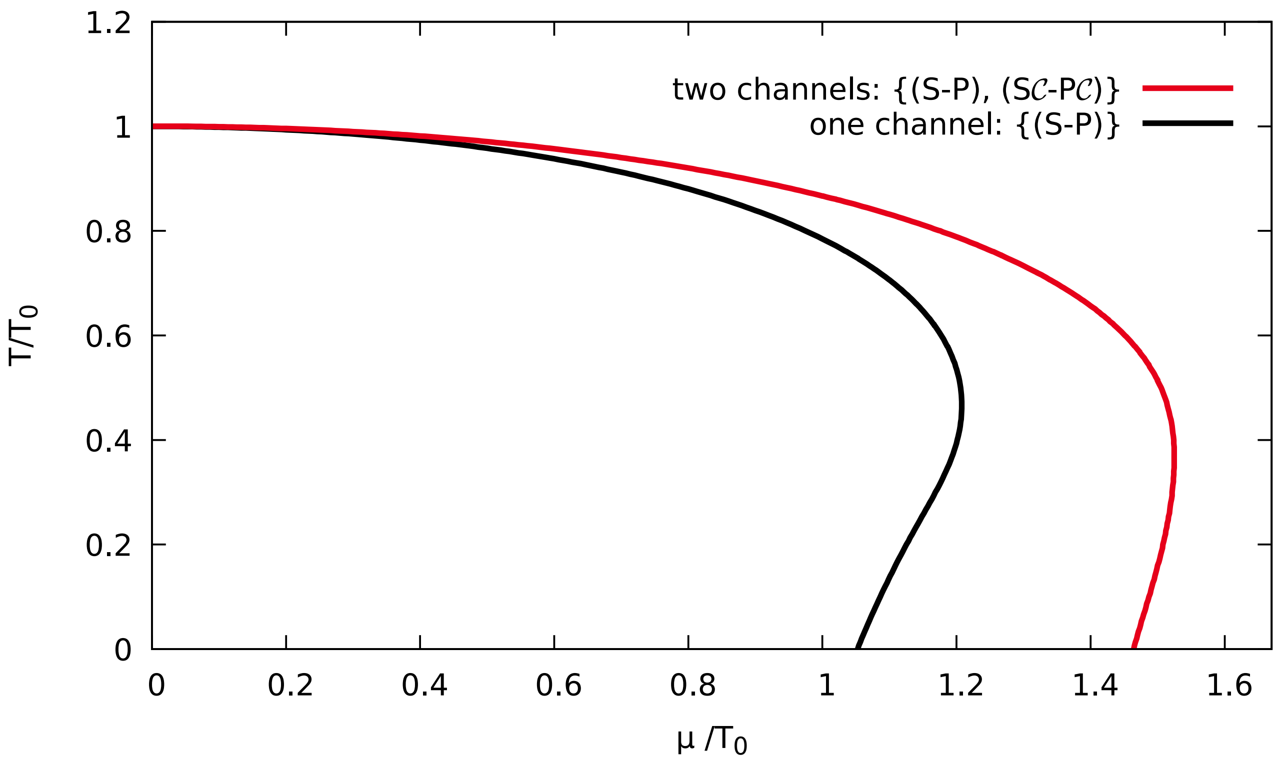

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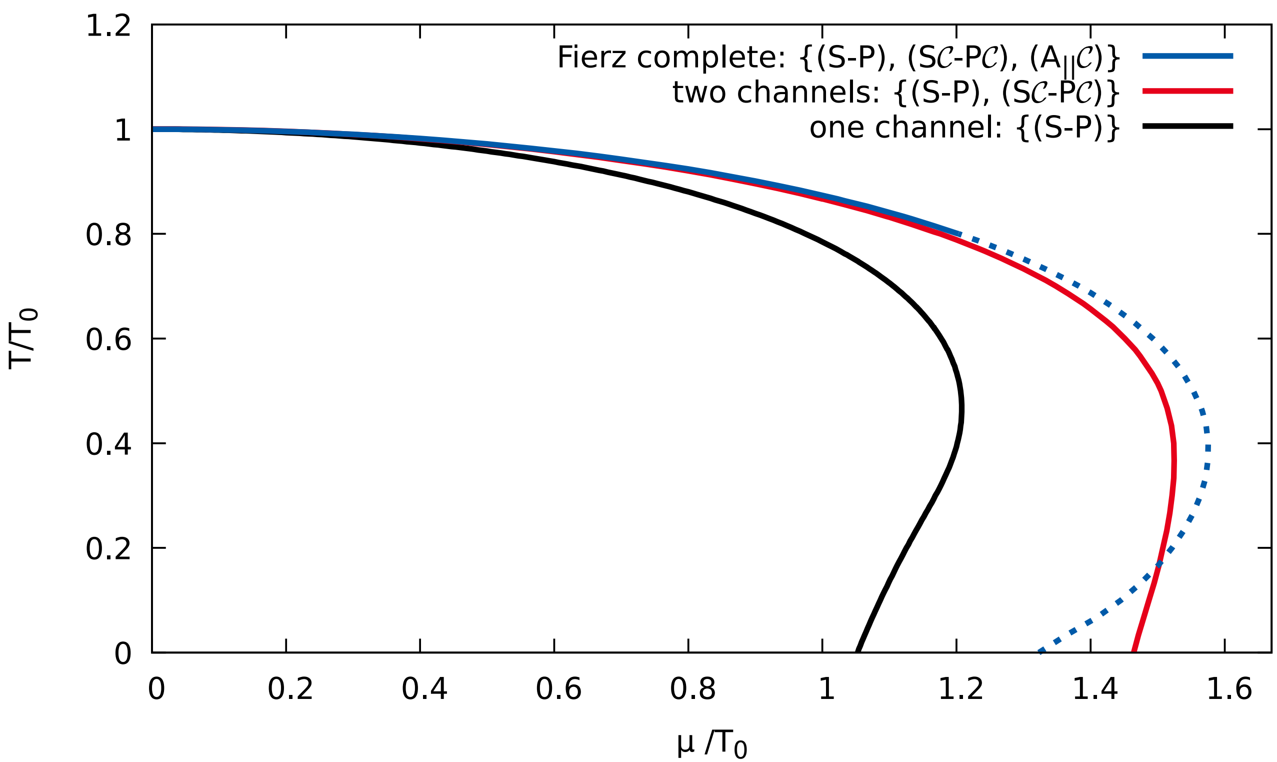

Backup

$$\mathcal{L}^{(\mathrm{S+P})_-^{\mathrm{adj}}}_{(\bar{\psi}\psi)^2} \quad \mathcal{L}^{(\mathrm{V+A})_\parallel}_{(\bar{\psi}\psi)^2} \quad \mathcal{L}^{(\mathrm{V+A})_\perp}_{(\bar{\psi}\psi)^2} \quad \mathcal{L}^{(\mathrm{V-A})_\parallel}_{(\bar{\psi}\psi)^2} \quad \mathcal{L}^{(\mathrm{V-A})_\perp}_{(\bar{\psi}\psi)^2} \quad \mathcal{L}^{(\mathrm{V+A})_\parallel^{\mathrm{adj}}}_{(\bar{\psi}\psi)^2} \quad \mathcal{L}^{(\mathrm{V-A})_\perp^{\mathrm{adj}}}_{(\bar{\psi}\psi)^2}$$

Four-quark interactions and symmetries

Fierz-complete basis of interactions

$$\Gamma_k[\bar\psi,\psi] = \int_x \left\{ \bar\psi(\mathrm{i}Z_{\parallel}\gamma_0\partial_0 + \mathrm{i}Z_{\perp}\gamma_i\partial_i - \mathrm{i}Z_{\mu}\mu\gamma_0)\psi + \frac{1}{2}\sum_i \bar\lambda_i\, \mathcal{L}^i_{(\bar\psi\psi)^2} \right\}$$

In total 20 channels meet symmetry constraints → Fierz identities → Fierz-complete basis: 10 channels

$\mathrm{U}_A(1)$ breaking channels:

$$\mathcal{L}^{(\sigma-\pi)}_{(\bar\psi\psi)^2} = (\bar\psi\psi)^2 - (\bar\psi\gamma_5\vec\tau\psi)^2$$

↔ formation of chiral condensate

$$\mathcal{L}^{\mathrm{csc}}_{(\bar\psi\psi)^2} \sim (\mathrm{i}\bar\psi\gamma_5\tau_A t_c^{A'}\mathcal{C}\bar\psi^T)(\mathrm{i}\psi^T\mathcal{C}\gamma_5\tau_A t_c^{A'}\psi) \qquad J^P = 0^+$$

[Rapp, Schäfer, Shuryak, Velkovsky, 1998]

↔ formation of diquark condensate

$$\mathcal{L}^{\mathrm{det}}_{(\bar\psi\psi)^2} = (\bar\psi\psi)^2 + (\bar\psi\gamma_5\psi)^2 - (\bar\psi\vec\tau\psi)^2 - (\bar\psi\gamma_5\vec\tau\psi)^2$$

$$\mathcal{L}^{(\mathrm{S+P})^{\mathrm{adj}}_-}_{(\bar\psi\psi)^2} = (\bar\psi T^a\psi)^2 - (\bar\psi\gamma_5\vec\tau T^a\psi)^2 + (\bar\psi\gamma_5 T^a)^2 - (\bar\psi\vec\tau T^a\psi)^2$$

+ six $\mathrm{U}_A(1)$ symmetric channels

[Braun, ML, Pospiech '17]

RG flow of four-quark interactions

Qualitative behavior and the effect of external parameters

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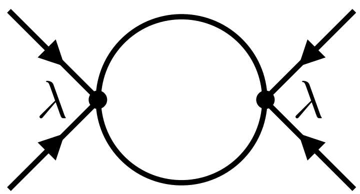
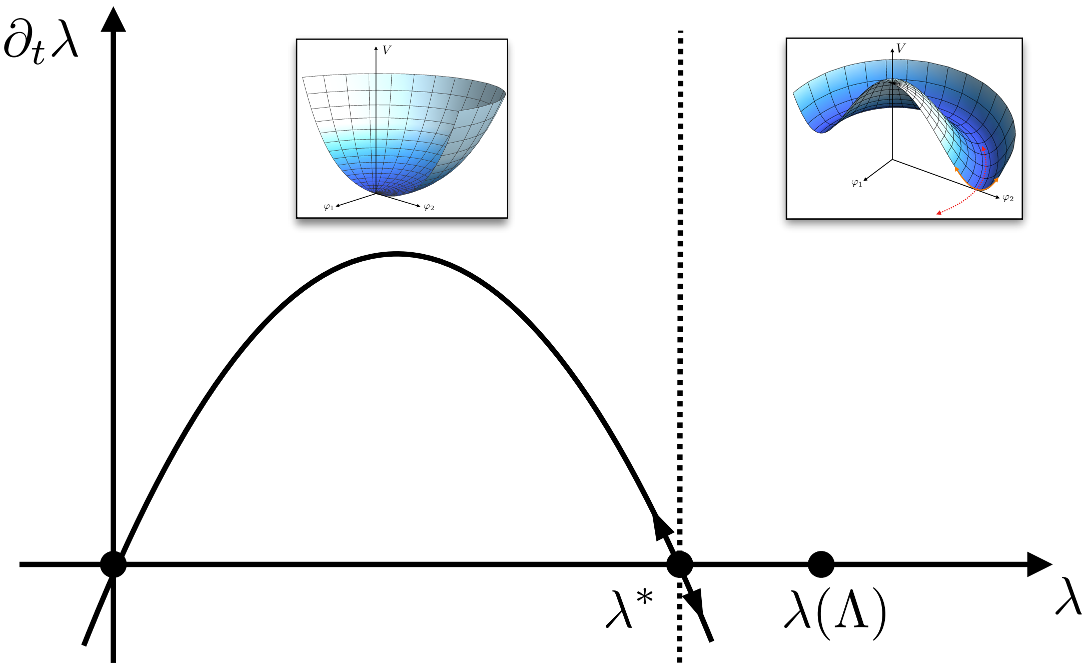
RG flow equation:

$\partial_t \lambda = 2\lambda \; - \; \lambda$  λ



Scale fixing procedure:

- Only $\lambda_{(\sigma-\pi)}(\Lambda)$ assumes finite value as inspired by gluon-induced four-quark flows [Braun, 2011]
- UV value tuned so that specific scale k_0 of symmetry-breakdown is obtained (defined by $1/\lambda(k_0) = 0$, sets the scale for low-energy observables)
- One-channel approximation can be mapped onto mean-field gap-equation to access deep infrared: $m_q(k_0) \approx 300 \text{ MeV}, \quad m_\sigma(k_0) \approx 800 \text{ MeV}$

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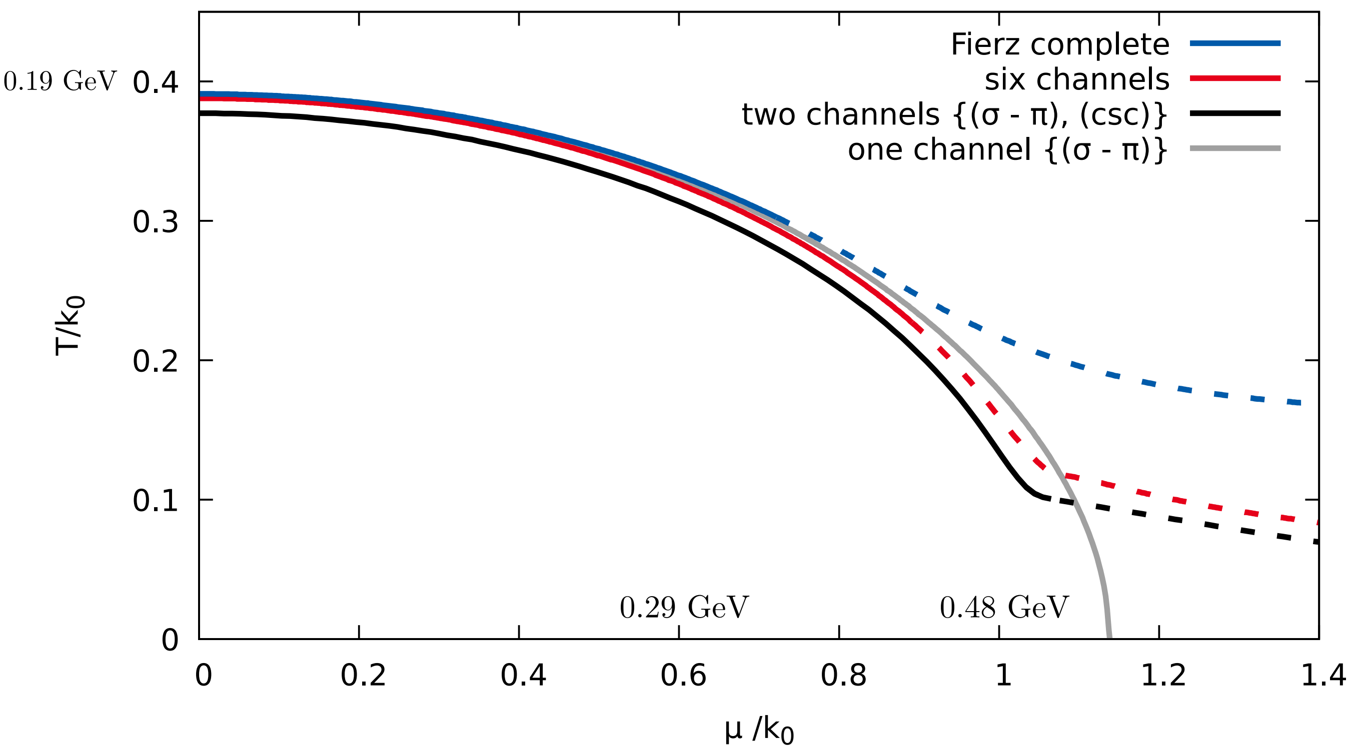
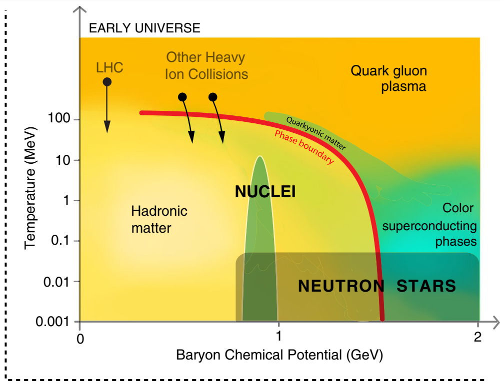
Structure of the phase boundary

$$\lambda_{\mathrm{UV}}^{\sigma\text{-}\pi} \approx 7.317,\ \ \lambda_{\mathrm{UV}}^{(i)} = 0 \text{ for } i \neq \sigma\text{-}\pi \longrightarrow k_0/\Lambda \approx 0.483 \quad \longleftrightarrow \quad m_\psi \approx 0.3 \text{ GeV}$$

[Braun, 2006]
[Mitter, Pawlowski, Strodthoff, 2014]

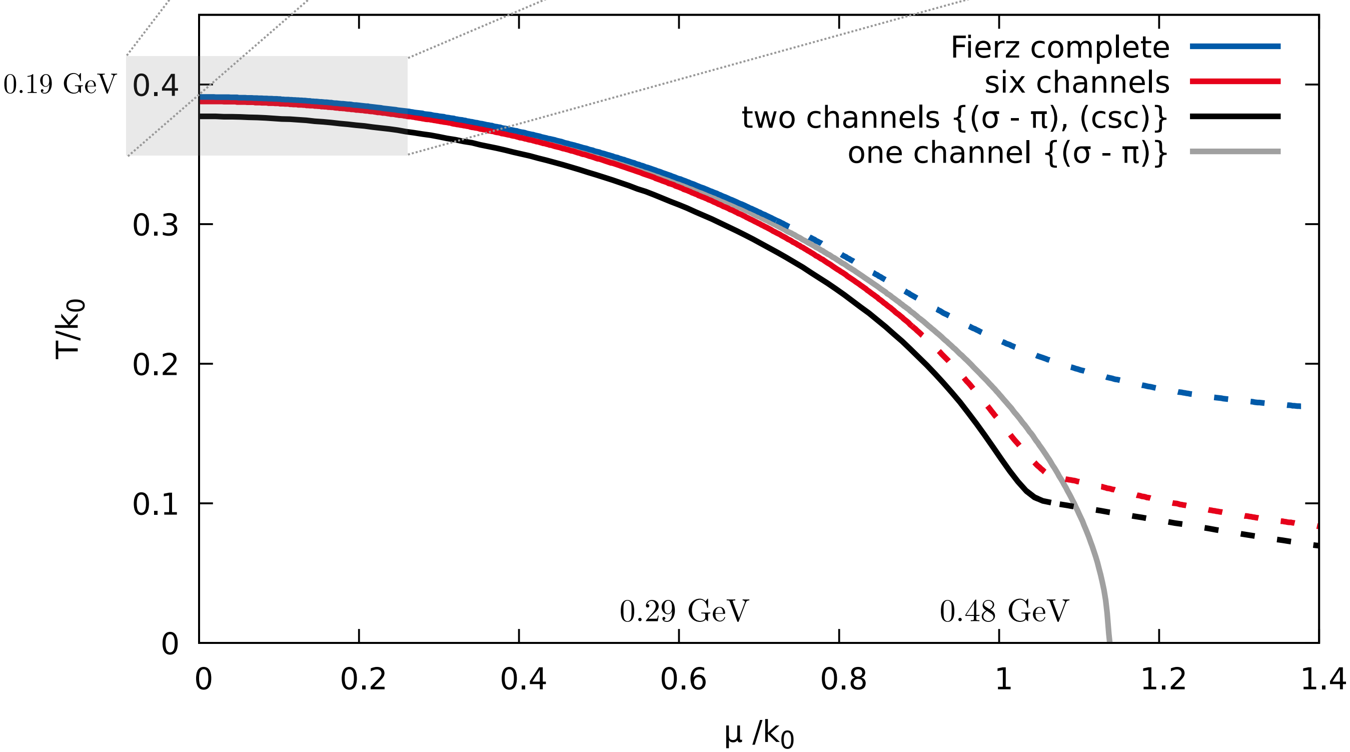




Structure of the phase boundary

Curvature $\kappa := -T_\chi(0) \frac{\mathrm{d}T_\chi(\mu^2)}{\mathrm{d}\mu^2}\Big|_{\mu=0}$

$\kappa \approx 0.06$



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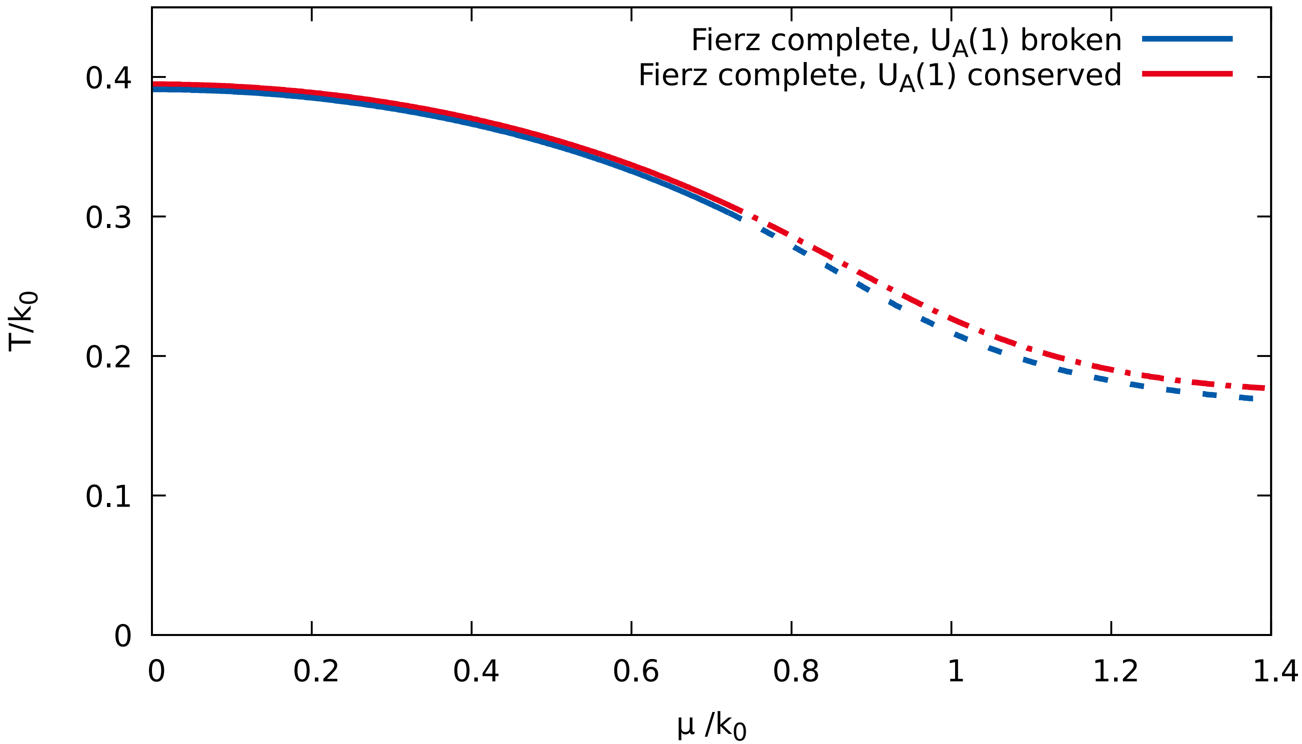
$$\lambda_{\mathrm{UV}}^{\sigma\text{-}\pi} \approx 7.317, \quad \lambda_{\mathrm{UV}}^{(i)} = 0 \text{ for } i \neq \sigma\text{-}\pi \quad \longrightarrow \quad k_{\mathrm{cr}}/\Lambda \approx 0.483 \quad \longleftrightarrow \quad m_{\psi}/\Lambda \approx 0.3$$



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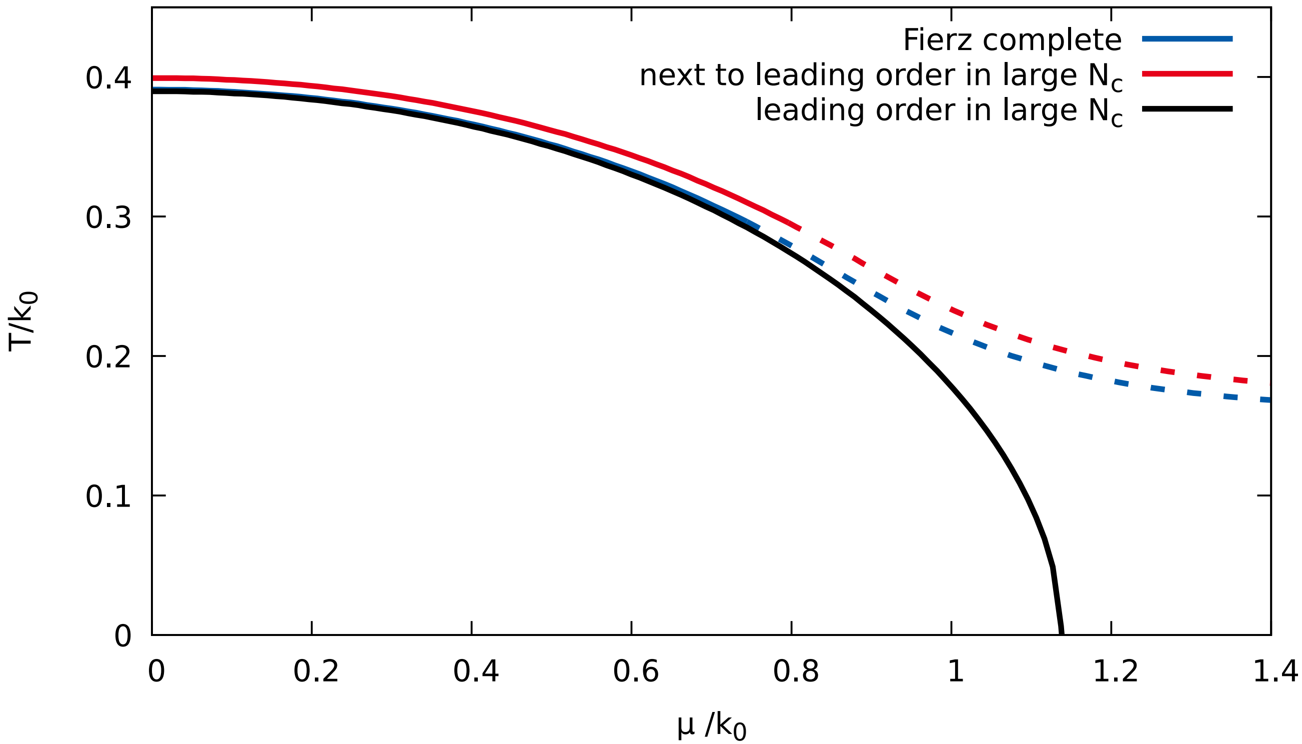

Exploring the phase diagram

Fixed-point structure and patterns of symmetry breaking

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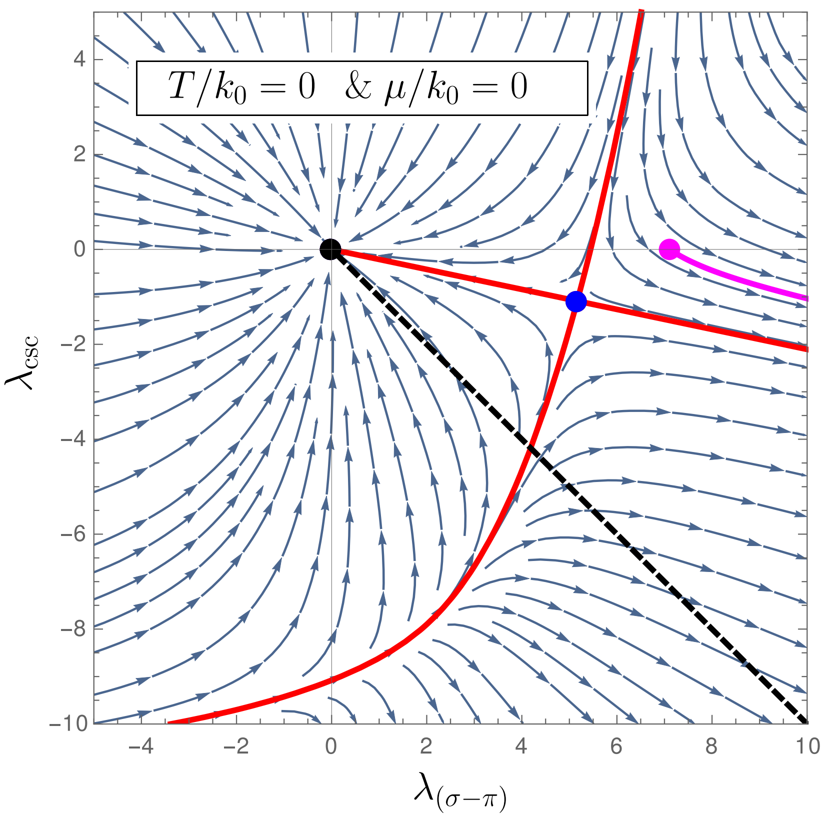
Ansatz two-channel approximation

$$\Gamma_k[\bar{\psi}, \psi] = \int_x \left\{ (\text{kinetic term}) + \frac{1}{2}\bar{\lambda}_{(\sigma-\pi)}\,(\sigma - \pi) + \frac{1}{2}\bar{\lambda}_{\text{csc}}\,(\text{csc}) \right\}$$



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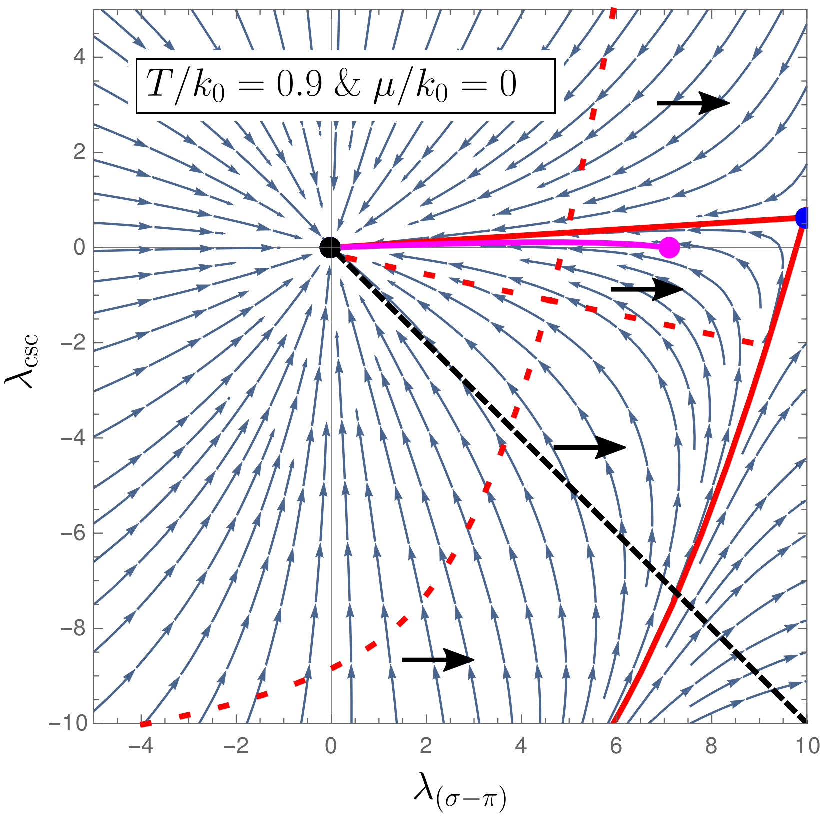

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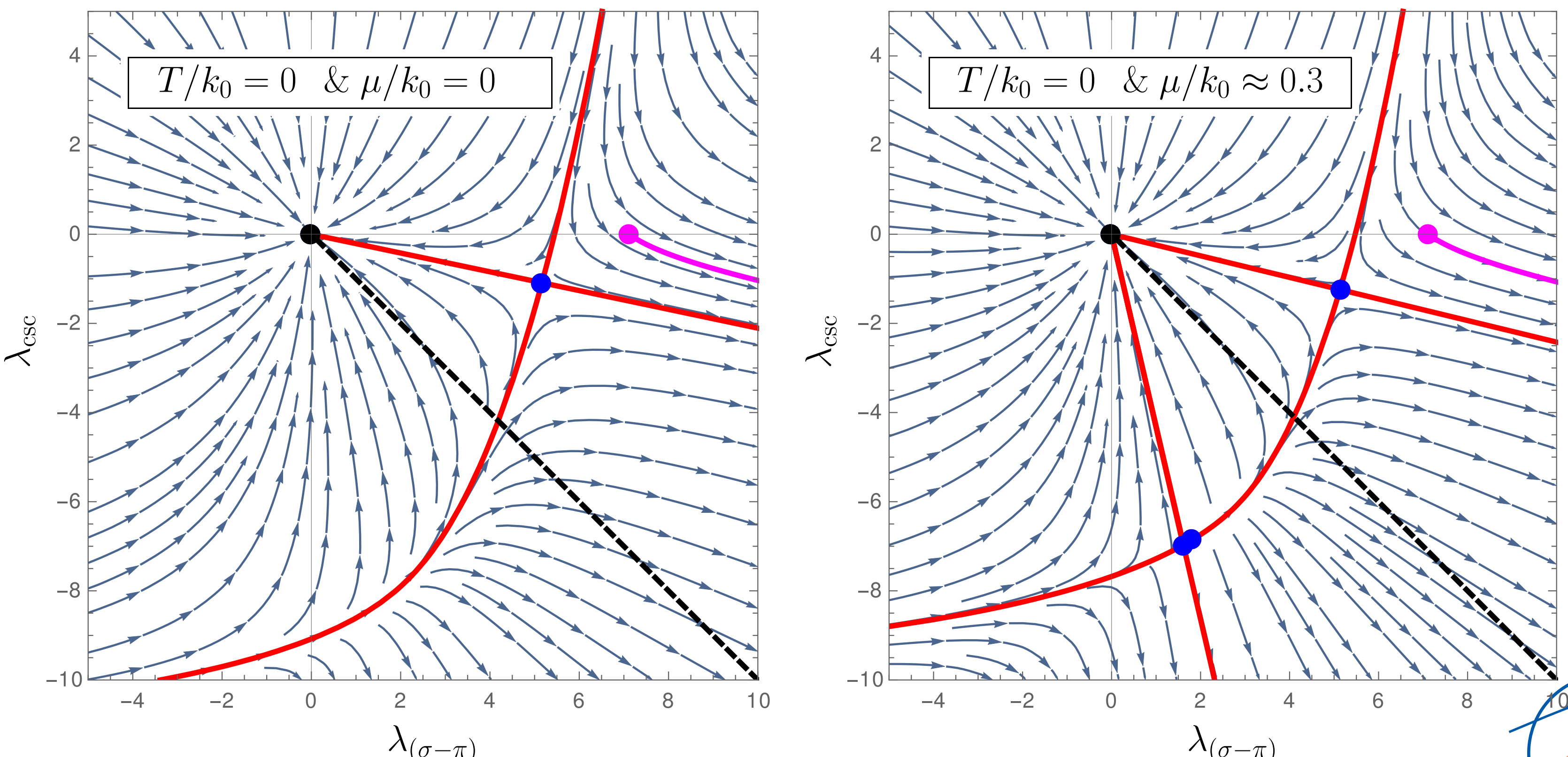

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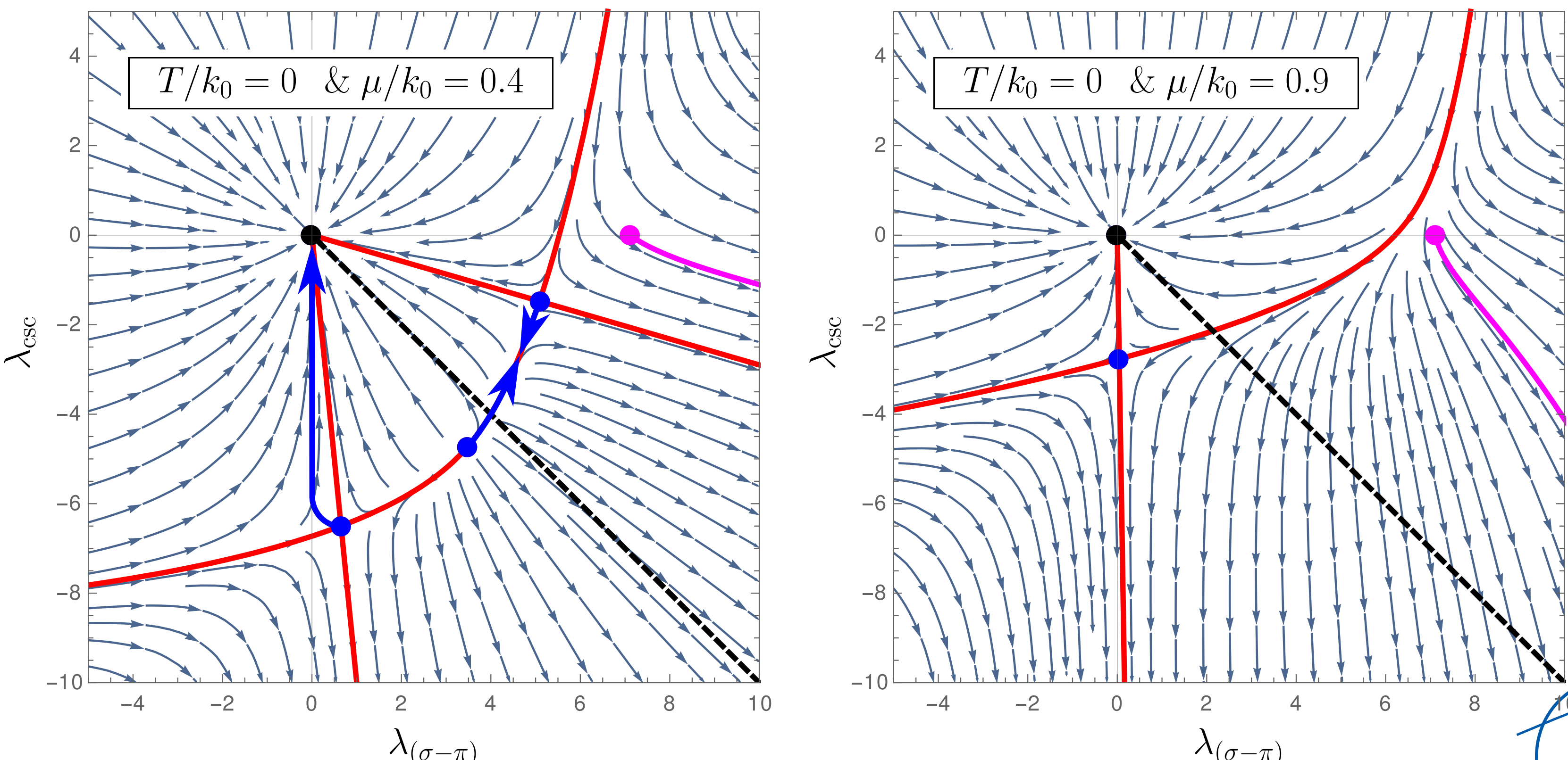

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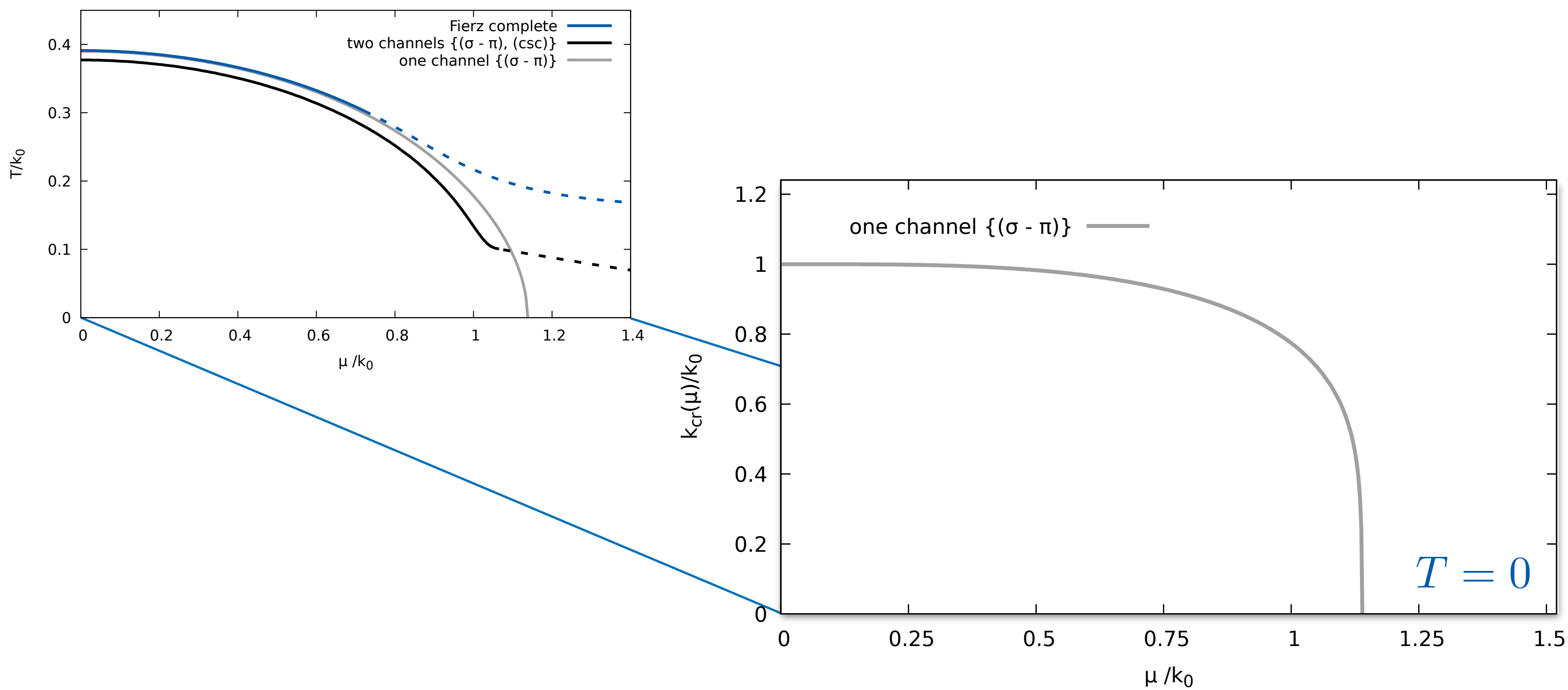

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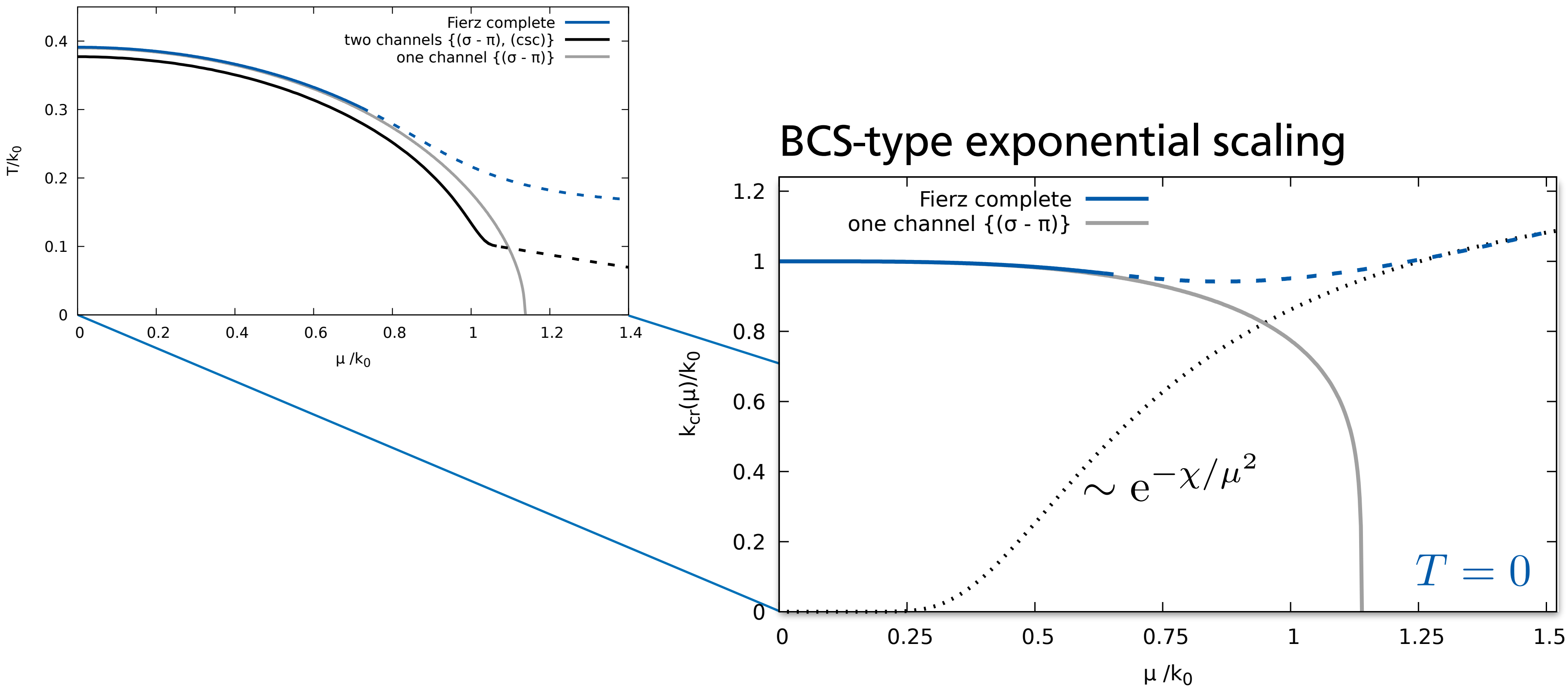

QCD phase diagram: Nuclear matter EOS and neutron stars

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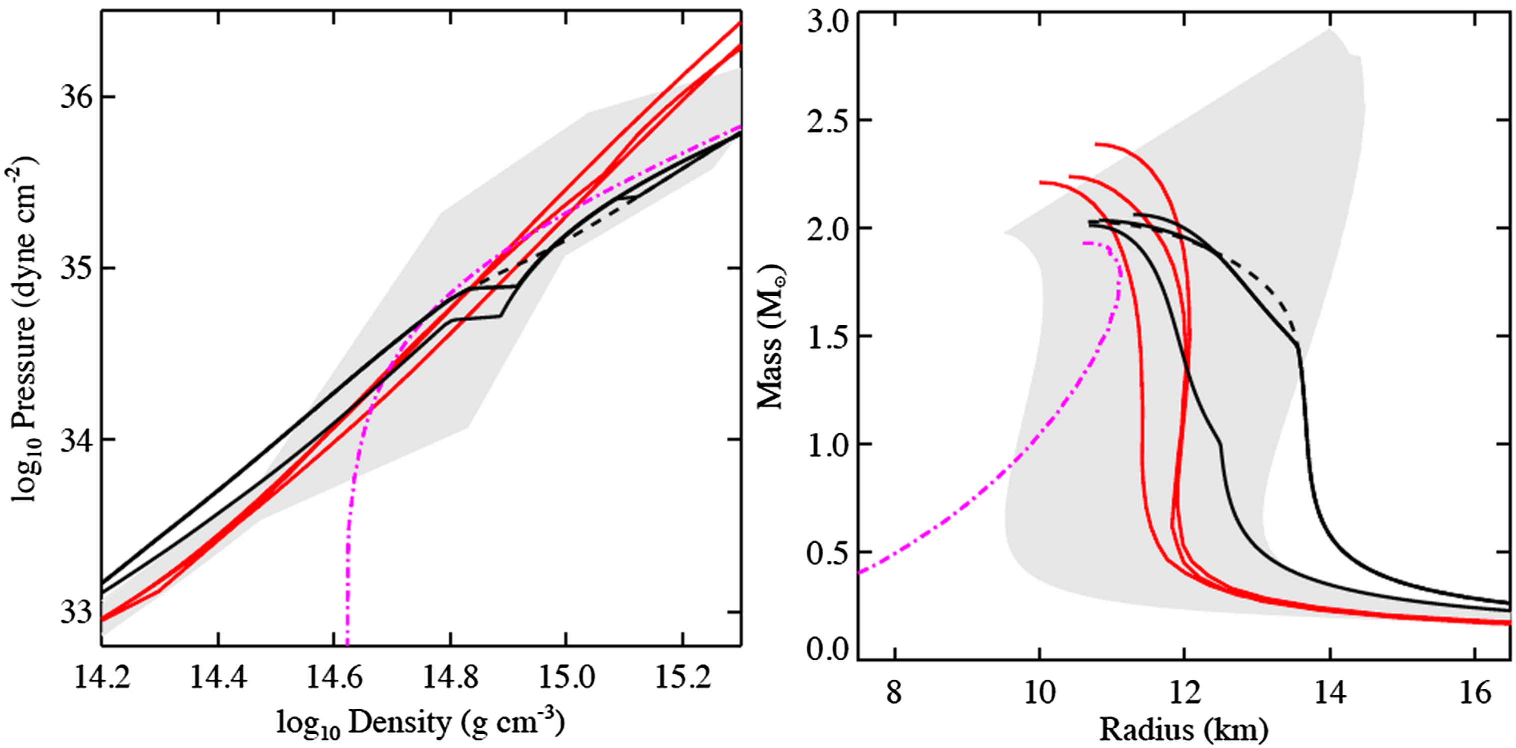
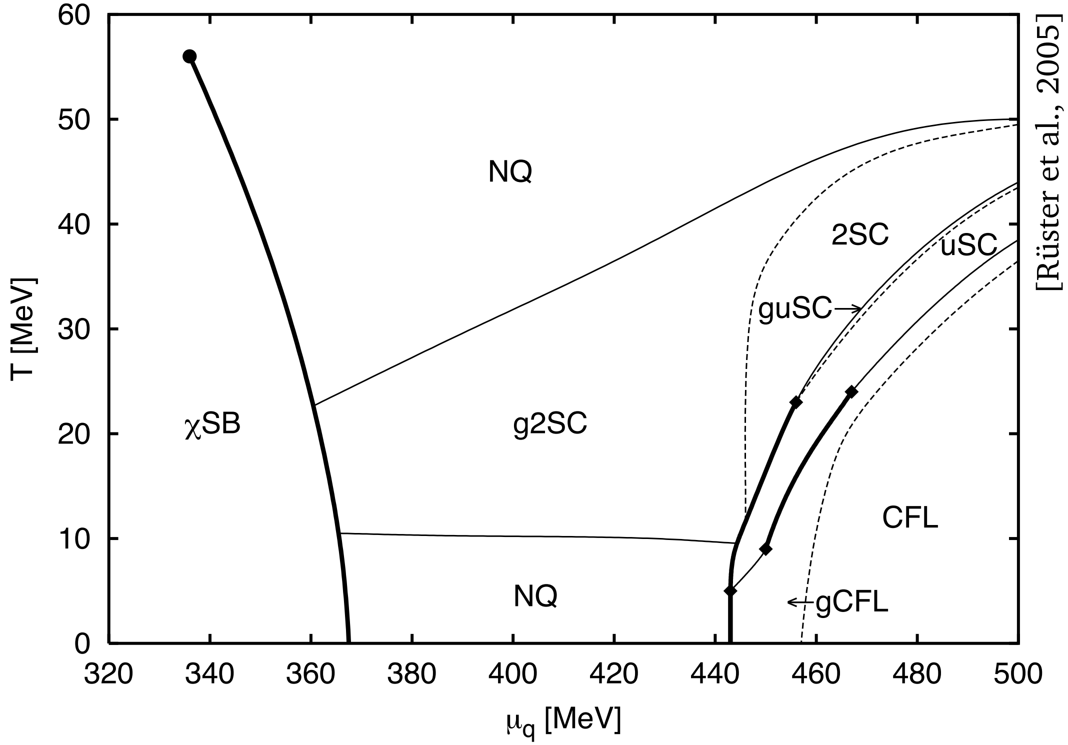


[Watts et al., 2016]

QCD phase diagram: Nuclear matter EOS and neutron stars



Symmetrie breaking and four-quark interactions in QCD

Structure of the phase boundary at finite temperature and density

TECHNISCHE UNIVERSITÄT DARMSTADT

Marc Leonhardt

Institut für Kernphysik, Technische Universität Darmstadt

with Martin Pospiech and Jens Braun

CRC 1245 Seminar
Integrated Research Training Group

SFB 1245

HIC for FAIR
Helmholtz International Center

QCD phase diagram
... and project B05

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Nuclear matter equation of state for astrophysical applications

"[...] complementary approaches, chiral effective field theory at lower densities and the functional renormalization group starting from quark-gluon dynamics at higher densities, to obtain a quantitative determination of the nuclear matter equation of state over a wide range of densities, temperatures, and proton fractions."

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Recap: QFT concepts

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All physical information is stored in correlation functions/n-point functions.

$$\langle \phi(x_1) \ldots \phi(x_n) \rangle := \mathcal{N} \int \mathcal{D}\phi \;\; \phi(x_1) \ldots \phi(x_n) \mathrm{e}^{-S[\phi]}$$

e.g. scattering amplitude (S-matrix elements) via LSZ reduction formula

	Statistical Physics	QFT
$Z[J] = \int \mathcal{D}\phi \;\; \mathrm{e}^{-S[\phi] + \int J\phi}$	Partition function	Generating functional
$W[J] = \log Z[J]$	Helmholtz free energy	Generating functional of connected diagrams
Effective action $\Gamma[\Phi]$	Gibbs free energy	Generating functional of 1PI diagrams

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Functional renormalization group (FRG)

Flow from high to low energies in QCD

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$\Gamma \xrightarrow{R_k}$ Effective average action Γ_k

[adapted from H. Gies, 2006]

Flow equation [Wetterich, 1993]

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left\{ \left[\Gamma_k^{(2)} + R_k \right]^{-1} \cdot (\partial_t R_k) \right\}$$

"RG time" $t = \ln(k/\Lambda)$

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Functional renormalization group (FRG)

Flow from high to low energies in QCD

$\Gamma \xrightarrow{R_k}$ <u>Effective average action</u> Γ_k

UV: $\Gamma_k \xrightarrow{k \to \Lambda} S$

IR: $\Gamma_k \xrightarrow{k \to 0} \Gamma$

[adapted from H. Gies, 2006]

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Functional renormalization group (FRG)

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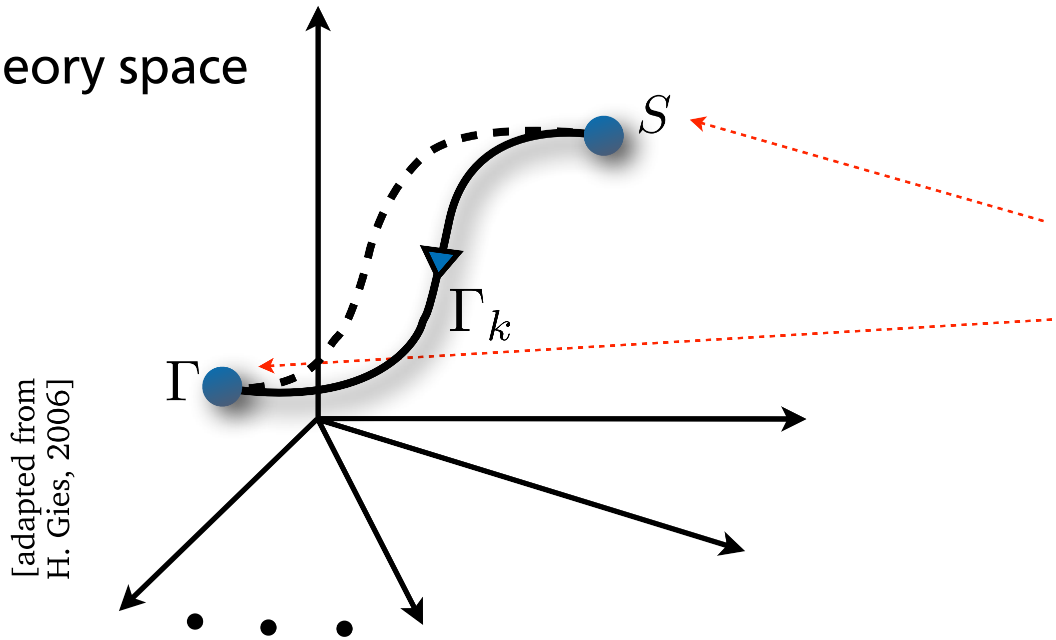
$$\Gamma \xrightarrow{R_k} \text{Effective average action } \Gamma_k$$

Theory space



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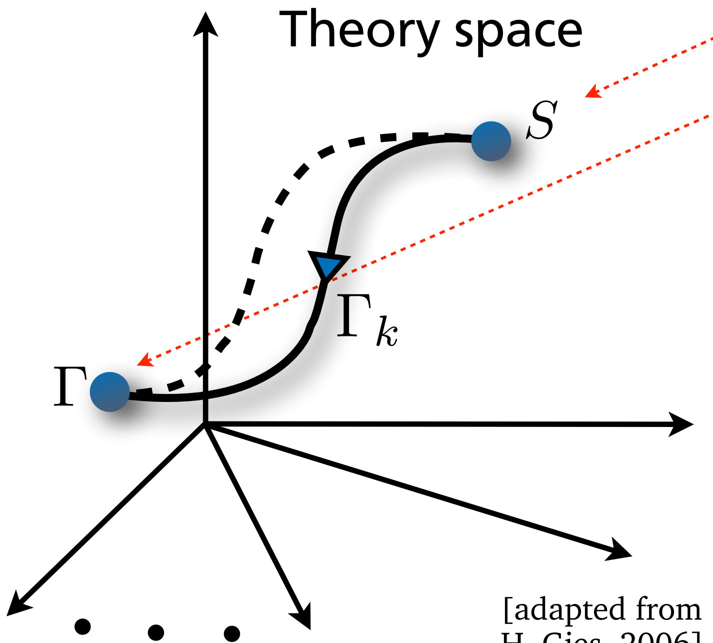

[adapted from H. Gies, 2006]

Functional renormalization group (FRG)

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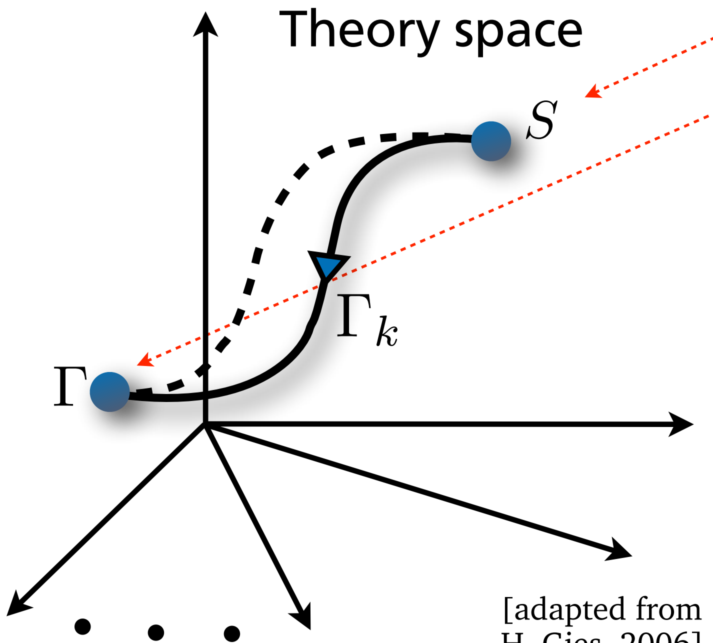

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$t = \ln(k/\Lambda)$

[C.Wetterich, *Phys. Lett. B*, 301, 1993]

Functional renormalization group (FRG)

Flow from high to low energies in QCD

$\Gamma \xrightarrow{R_k}$ Effective average action Γ_k

UV: $\Gamma_k \xrightarrow{k\to\Lambda} S$

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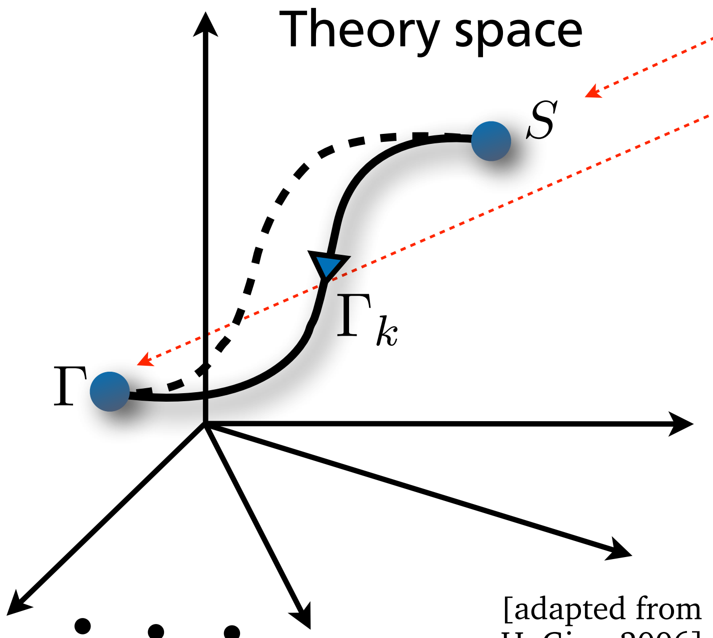

[adapted from H. Gies, 2006]

Everything that is not forbidden is allowed.

Flow equation

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\left\{\left[\Gamma_k^{(2)} + R_k\right]^{-1} \cdot (\partial_t R_k)\right\}$$

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[C.Wetterich, *Phys. Lett. B*, 301, 1993]

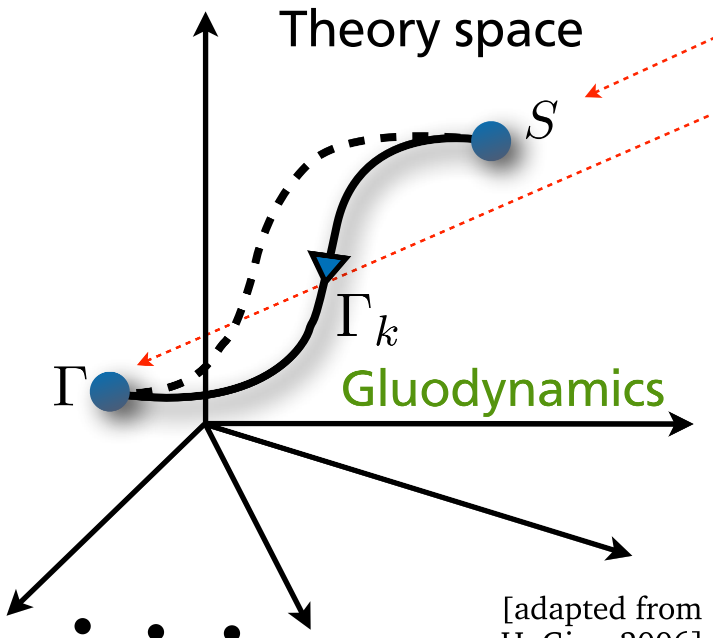
Functional renormalization group (FRG)

Flow from high to low energies in QCD

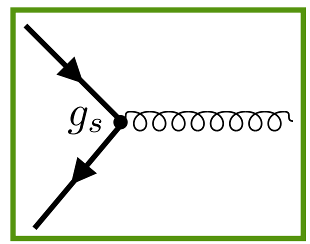
$\Gamma \xrightarrow{R_k}$ Effective average action Γ_k

UV: $\Gamma_k \xrightarrow{k \to \Lambda} S$

IR: $\Gamma_k \xrightarrow{k \to 0} \Gamma$



[adapted from H. Gies, 2006]



Everything that is not forbidden is allowed.

Flow equation

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{STr} \left\{ \left[\Gamma_k^{(2)} + R_k \right]^{-1} \cdot (\partial_t R_k) \right\}$$

$$t = \ln(k/\Lambda)$$

[C.Wetterich, *Phys. Lett. B*, 301, 1993]

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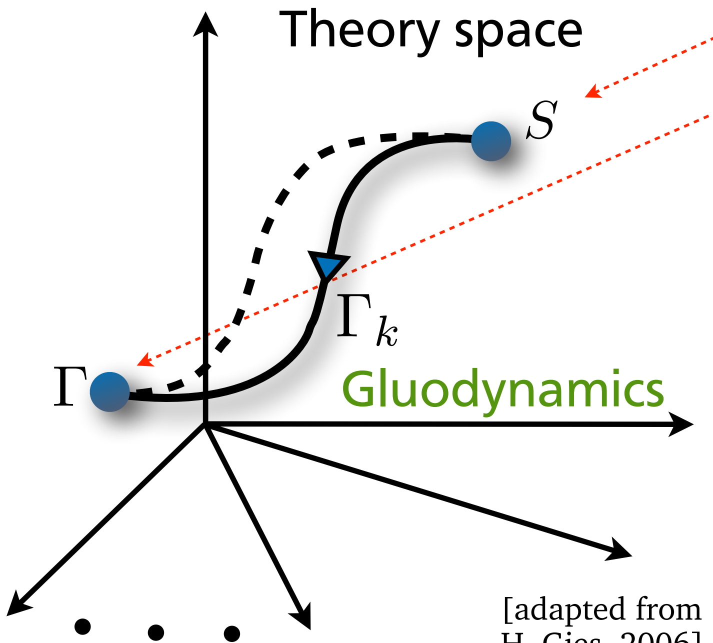

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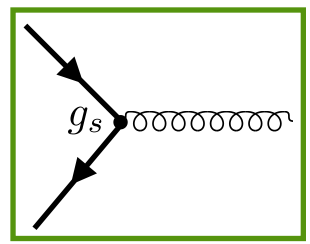

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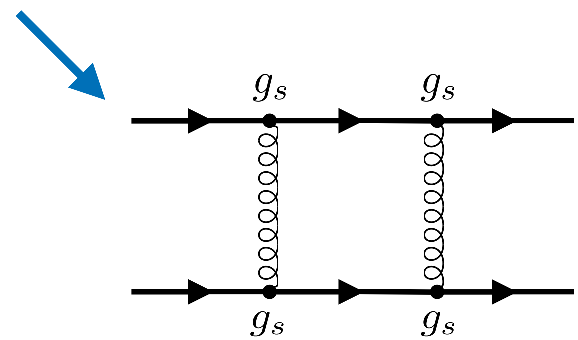

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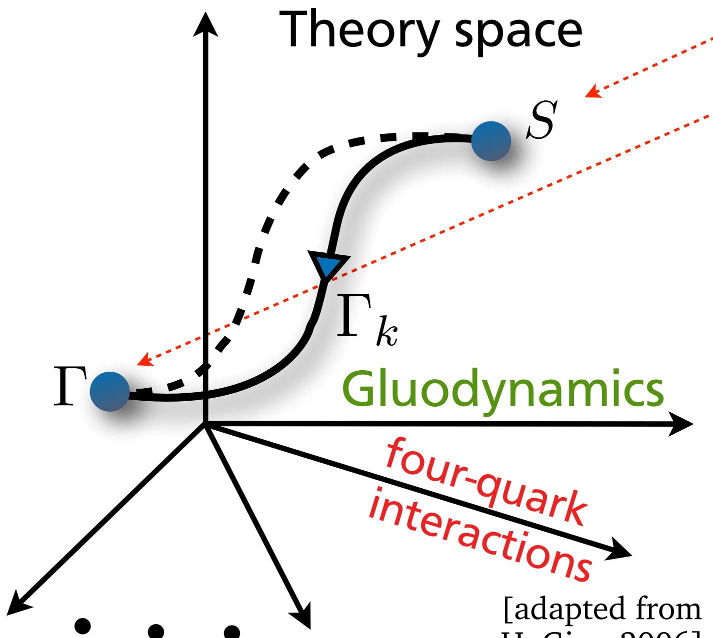

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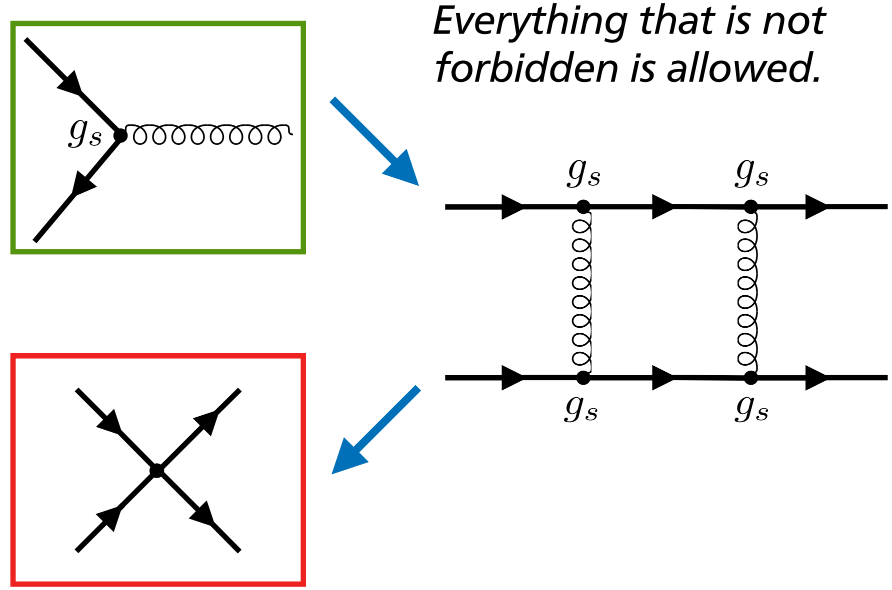

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[C.Wetterich, *Phys. Lett. B*, 301, 1993]

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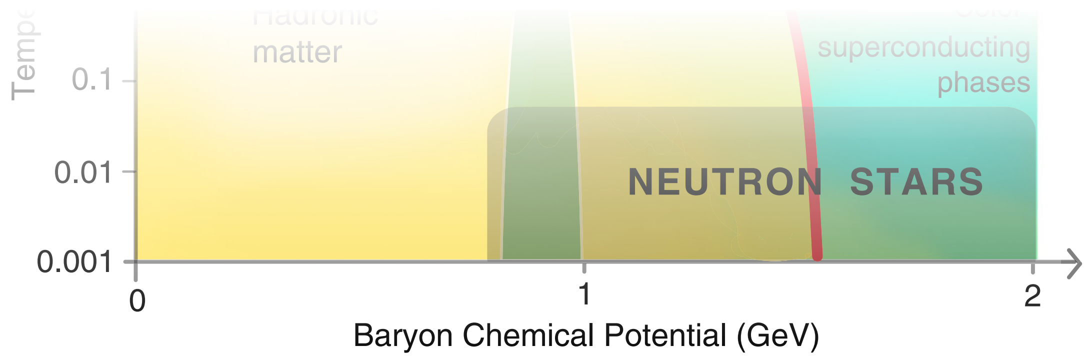
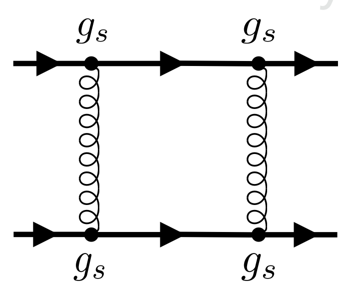
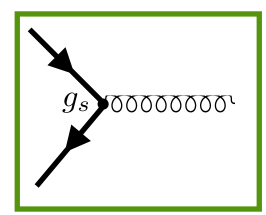
QCD phase diagram:
Neutron stars and the cold dense EoS



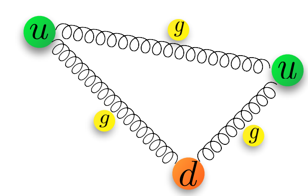
Chiral effective field theory

Functional methods

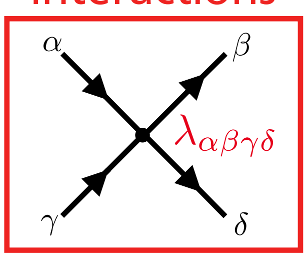




dof:



Four-quark interactions



1. Important effective interactions
2. Encode information on the ground state and SB
3. Full treatment possible and crucial

Functional renormalization group (FRG)

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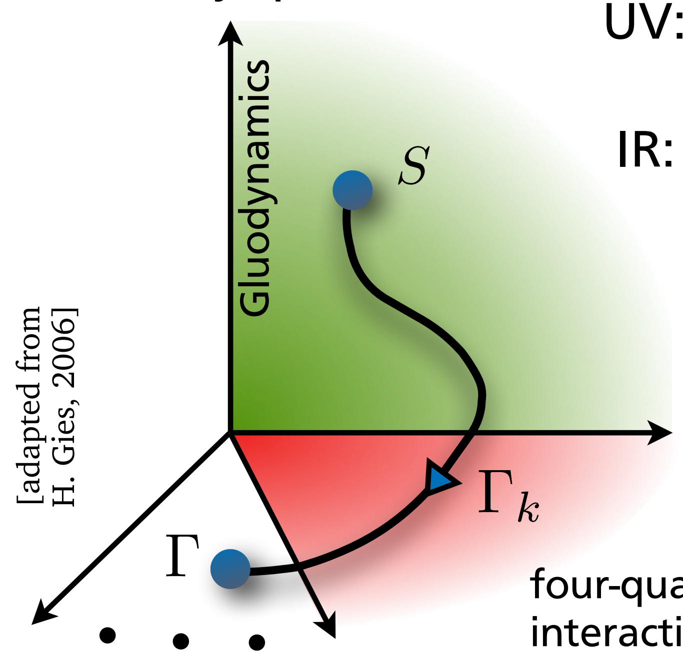
Effective average action Γ_k

Theory space



UV: $$\Gamma_{k\to\Lambda} = \int_x \left\{ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi}(\mathrm{i}\partial\!\!\!/ + \mathrm{i}g_s A\!\!\!/)\psi \right\}$$

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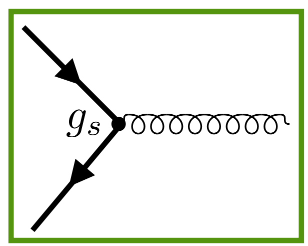
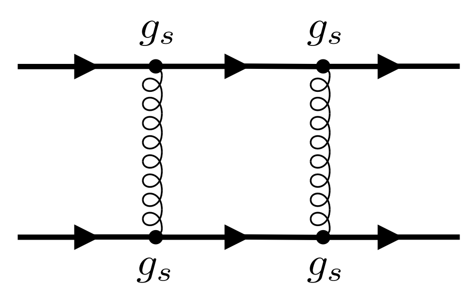
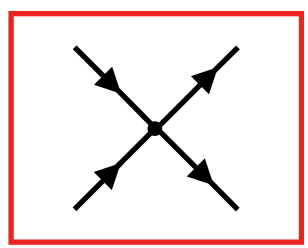




four-quark interactions



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Functional renormalization group (FRG)

Flow from high to low energies in QCD

<u>Effective average action</u> Γ_k

RG flow equation [C. Wetterich, 1993]

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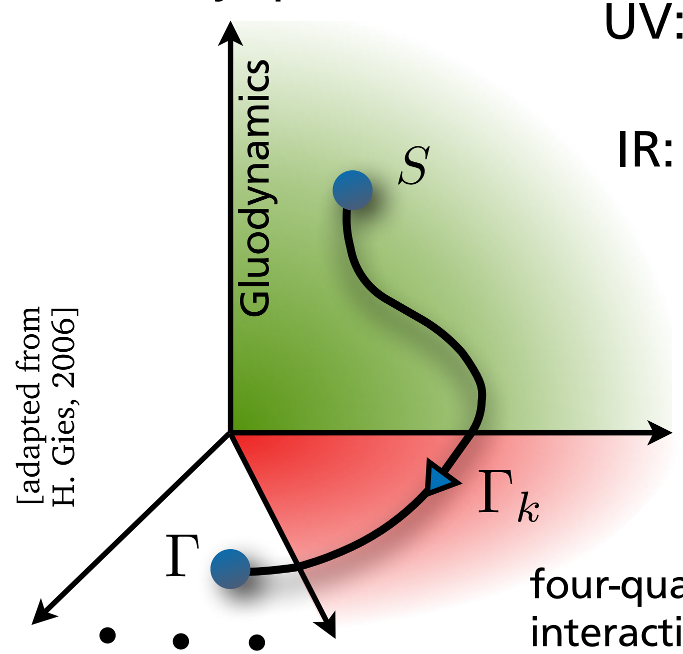

four-quark interactions

UV:

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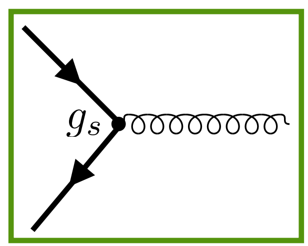

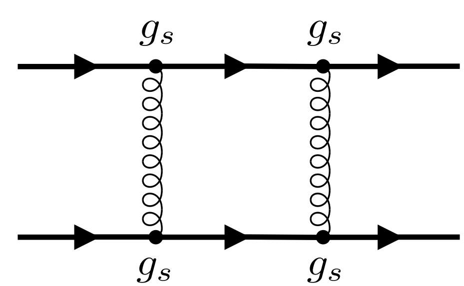

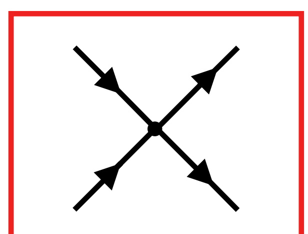

Functional renormalization group (FRG)

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$\beta = 1/T$

Partition function/ generating functional $$Z = \mathrm{tr}\left[\mathrm{e}^{-\beta \hat{H}}\right] = \int \mathcal{D}\varphi \, \mathrm{e}^{-S[\varphi]}$$

→ Effective action Γ

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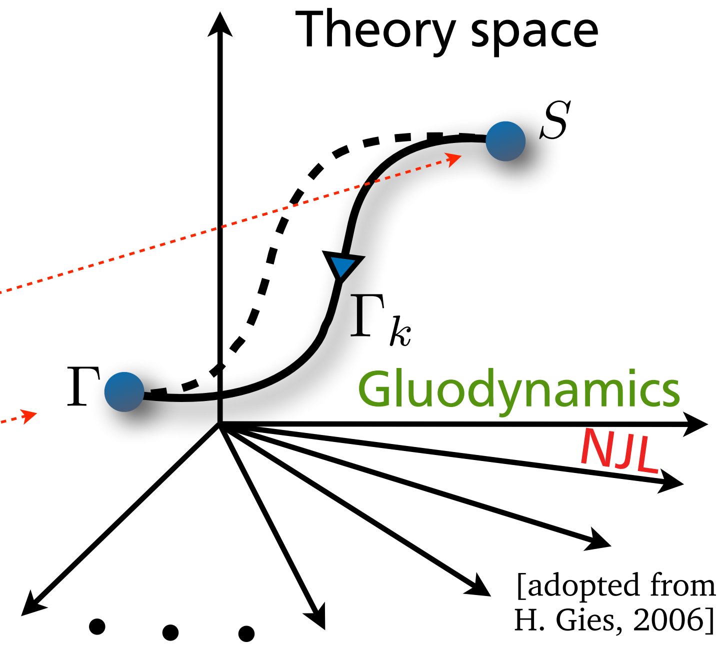

[adopted from H. Gies, 2006]

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