Nuclear physics around the unitarity limit

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SFB 1245 Workshop 2017

Mainz

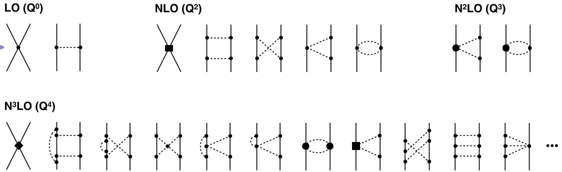
October 5, 2017

SK, H.W. Grießhammer, H.-W. Hammer, U. van Kolck, PRL **118** 202501 (2017)
SK, J Phys. G **44** 064007 (2017)

Prelude

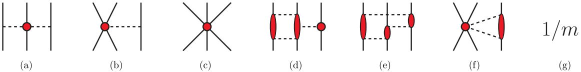
Typical nuclear *ab initio* calculation

chiral potential → SRG → many-body method → result



Epelbaum *et al.*, EPJA **51** 53 (2015)

starting at $\mathcal{O}(Q^3)$:



Hebeler *et al.*, PRC **91** 044001 (2015)

- power counting → hierarchy of forces: $Q \sim m_\pi \ll M_{\text{QCD}}$
- **Weinberg approach:** diagrams → potential → iterate. . .

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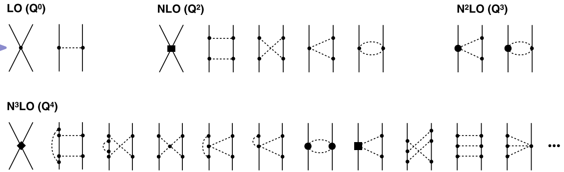

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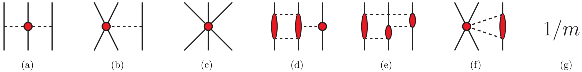

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- power counting → hierarchy of forces: $Q \sim m_\pi \ll M_{\text{QCD}}$
- **Weinberg approach:** diagrams → potential → iterate...
- **Note: need at least $\mathcal{O}(Q^3)$ for reasonable triton!**

Prelude

hierarchy of forces (natural in EFT)

many-body forces $\leftrightarrow$ two-body off-shell tuning

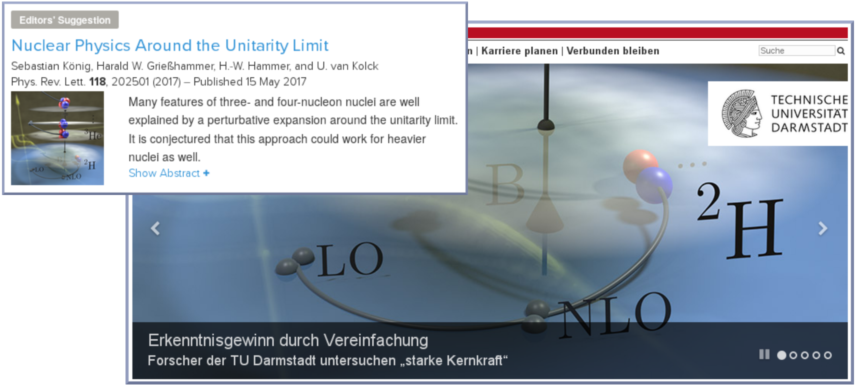
Various approaches depart from focusing on two-body input...

- **JISP16** Shirokov *et al.*, PLB **644** 33 (2007)
  $\hookrightarrow$ two-body only, but input from nuclei up to ^{16}O
- **N2LO$_{\text{opt}}$, N2LO$_{\text{sat}}$** Ekstöm *et al.*, PRL **110** 192502 (2013), PRC **91** 051301 (2015)
  simultaneous fit to NN + light nuclei, saturation properties
- **SRG-evolved 2N + N2LO 3N** Simonis *et al.*, PRC **93** (2016)
  $\hookrightarrow$ predict realistic saturation properties
- **nuclear lattice calculations** Elhatisari *et al.*. PRL **117** 132501 (2016)
  $\hookrightarrow$ use input from α-α scattering
- ...

Prelude

Novel approach to few-nucleon systems

SK *et al.*, PRL **118** 202501 (2017)



- suggests a paradigm shift away from two-body precision
- establishes feasibility of perturbative few-body calculations

Outline

The unitarity expansion

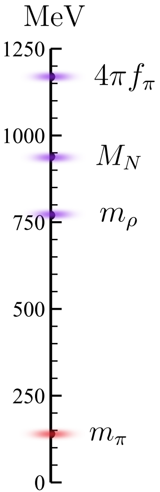
Bound states

Resonances and currents

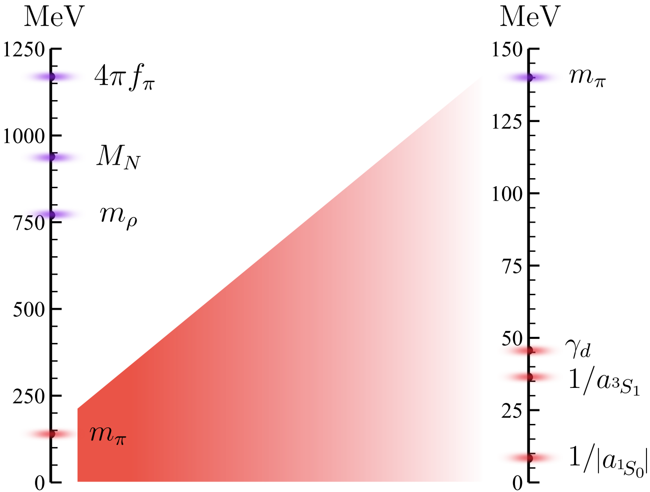
(Second order)

Summary

Nuclear scales



Nuclear scales



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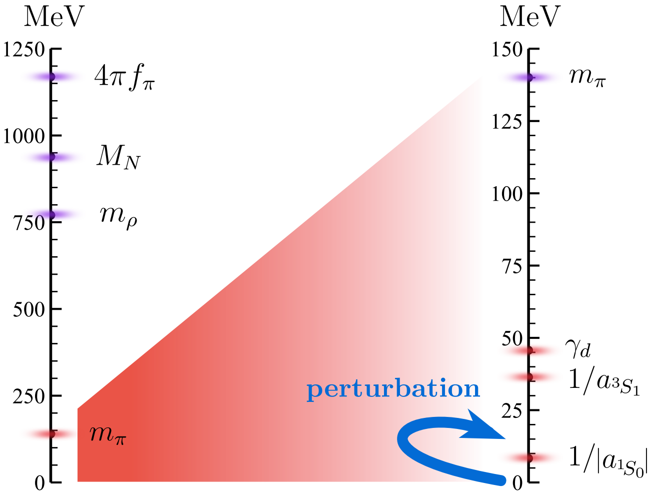
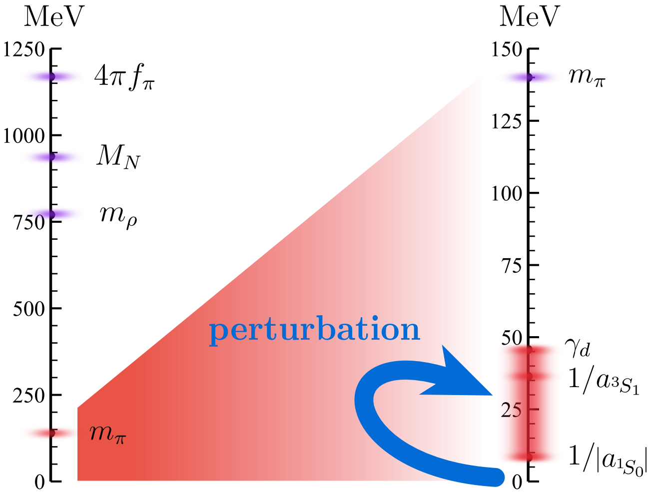


unitarity limit: $1/a = 0$

SK *et al.* JPG **43** 055106 (2016)

Nuclear scales



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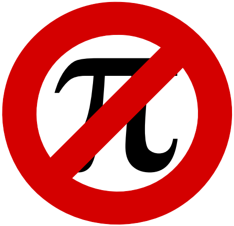
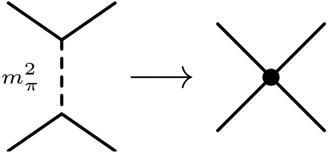
SK *et al.* PRL **118** 202501 (2017)

The unitarity expansion



$q^2 \ll m_\pi^2$



Basic setup

- two-body physics (LECs) $\leftrightarrow$ effective range expansion
- assume $a_{s={}^1S_0,t={}^3S_1} = \infty \iff 1/a_{s,t} = 0$ at leading order
- **need pionless LO three-body force!**
  $\hookrightarrow$ reproduce triton energy exactly
- finite a, Coulomb, ranges $\rightarrow$ perturbative corrections!

Capture gross features at leading order, build up the rest as perturbative "fine structure!"

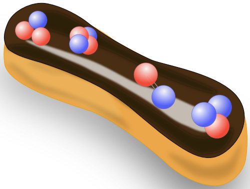
- **shift focus away from two-body details**
- **note:** zero-energy deuteron at LO and NLO
- exact $SU(4)_W$ symmetry at LO *cf.* Vanasse+Phillips, FB Syst. **58** 26 (2017)
- universality regime: Efimov effect, bosonic clusters, ...

Conjecture

Nuclear sweet spot

$$1/a_{s,t} < Q_A < 1/R$$

$$Q_A \sim \sqrt{2M_N B_A/A}$$

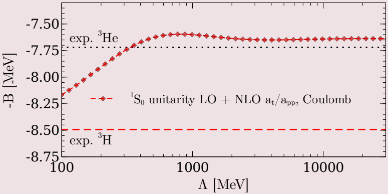


original eclair by Herve1729 (via Wikimedia Commons)

Helium results

^{3}He at $^1S_0$ and full unitarity

- good NLO established for $^1S_0$ unitarity



SK, Hammer, Grießhammer, van Kolck (2015/16, 2016/17)

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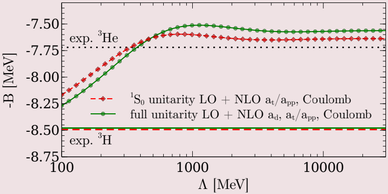

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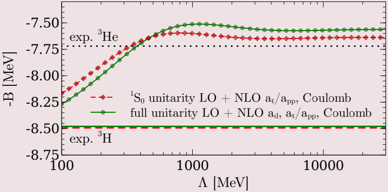
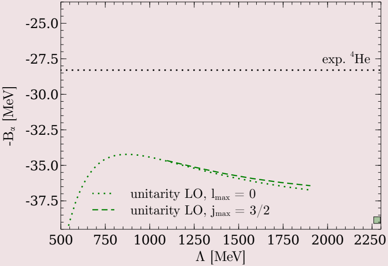


^{4}He (zero-range, no Coulomb)



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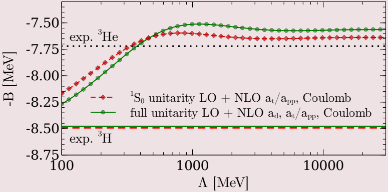

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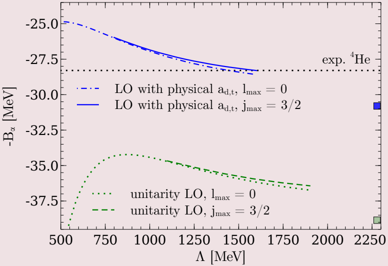

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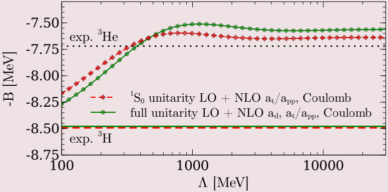

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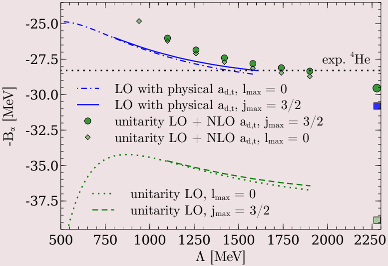

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Some details

- binding energies at LO: find zeros of $\det(\mathbf{1} - K(E))$,
  $K(E)$ = Faddeev(-Yakubowsky) kernel
- NLO energy shift: $\Delta E = \langle\Psi|V^{(1)}|\Psi\rangle$, $|\Psi\rangle$ = LO wavefunction

$$|\Psi\rangle = (\mathbf{1} - P_{34} - PP_{34})(1 + P)\,|\psi_A\rangle + (\mathbf{1} + P)(\mathbf{1} + \tilde{P})\,|\psi_B\rangle$$

wavefunction convergence slower than eigenvalue convergence!

$\hookrightarrow$ need more mesh points and partial-wave components. . .

Energy balance

- sample calculation with physical scattering lengths at LO:

$\Lambda\,/\,\mathrm{MeV}$	800	1000	1200	1400
$E_{\text{kin}}\,/\,\mathrm{MeV}$	113.67	140.58	168.44	197.09
$E_{\text{pot}}\,/\,\mathrm{MeV}$	-139.77	-167.41	-195.76	-224.62

- E_{kin} and E_{pot} not observable
- sum converges as cutoff is increased, individual values do not!

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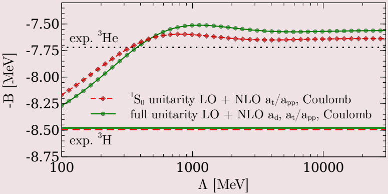

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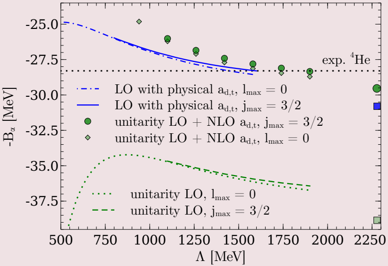

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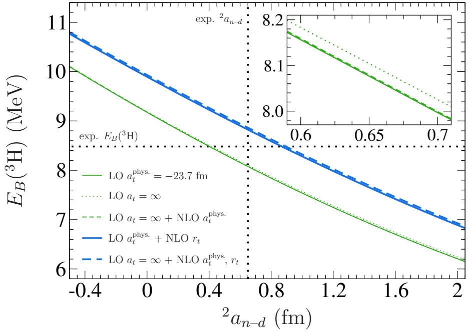
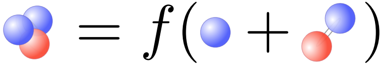
Few-nucleon correlations



Phillips line



($^1S_0$ unitarity only)

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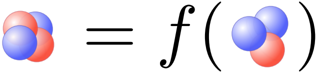

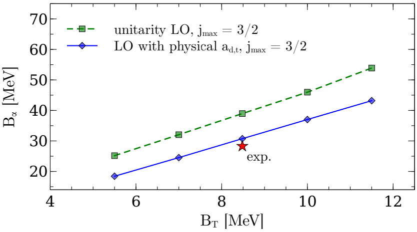

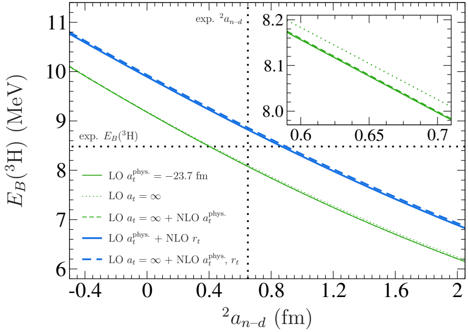

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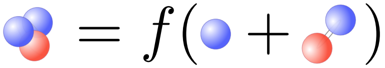

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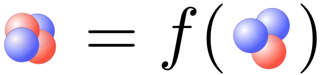

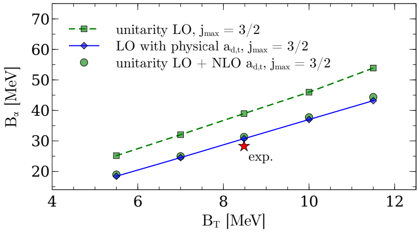

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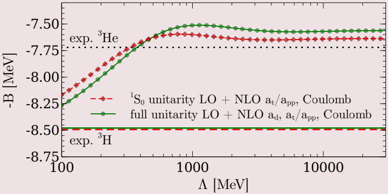

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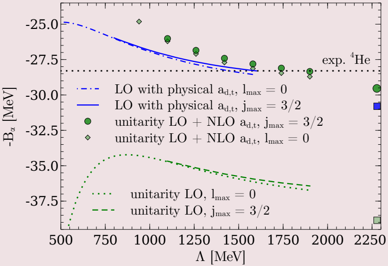
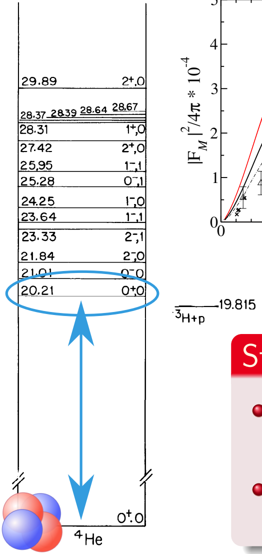


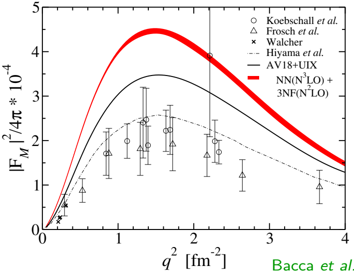
- $^4\mathrm{He}$ resonance state $\sim 0.3\ \mathrm{MeV}$ above $^3\mathrm{H} + p$ threshold
- just below threshold at unitarity LO
- boson calculations with nuclear scales
  $\rightsquigarrow$ shift by about $0.2 - 0.5\ \mathrm{MeV}$

SK, Hammer, Grießhammer, van Kolck (2016/17)
cf. also Platter (2004)

^{4}He monopole resonance



TUNL nuclear data



Bacca *et al.*, PRL **110** 042503 (2013)

theory$_A$ ↯ theory$_B$

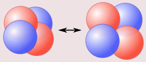
↯ experiment!

“a prism to nuclear Hamiltonians”

Structure of the 0^+ resonance

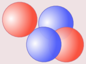
- suggested to be a “breathing mode”

  Bacca *et al.*, PRC **91** 024303 (2015)



- indications for $p+{}^3\mathrm{H}$ cluster structure

  this work



Current work

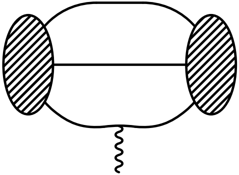
in progress: look at observables beyond binding energies

$\hookrightarrow$ **radii and charge form factors**

Point charge radius

$$F_C(q^2) = \langle\Psi|J_0(q^2)|\Psi\rangle$$

$$\langle r^2\rangle = -\frac{1}{6}\frac{\mathrm{d}}{\mathrm{d}(q^2)}F_C(q^2)\Big|_{q^2=0}$$



—preliminary—

	unit.	phys. $a_{s,t}$	exp.
^{2}H	—	1.91	1.98
^{3}H	0.99	1.09	1.60
^{4}He	1.06	1.26	1.22

fixed $\Lambda = 800$ MeV, no extrapolation

Atomic Data and Nucler Data Tables **99** 69 (2013)

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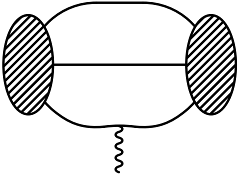

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Atomic Data and Nucler Data Tables **99** 69 (2013)

^{3}H calculation up to N^{2}LO (a_s, $B(^2\mathrm{H})$):

$1.14(19)\ \mathrm{fm} \to 1.59(8)\ \mathrm{fm} \to 1.62(3)\ \mathrm{fm}$

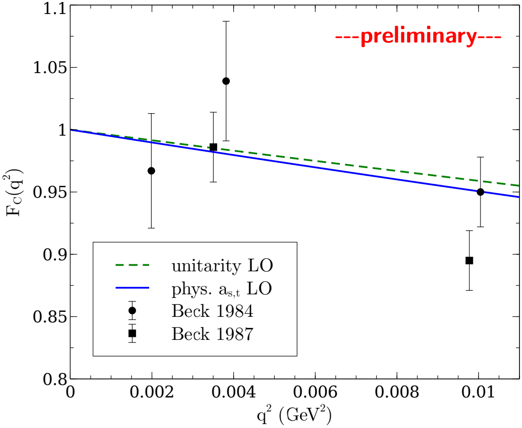
Vanasse, PRC **95** 024002 (2017)

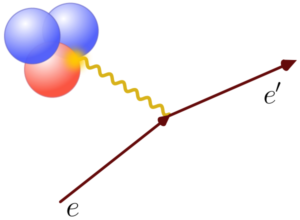
LO unitarity trinucleon @ 7.62 MeV:

$1.10\ \mathrm{fm}$

Vanasse+Phillips, Few-Body Syst. **58** 26 (2017)

Triton form factor





data from Beck, PRC **30** 1403 (1984)
Beck *et al.*, PRL **59** 1537 (1987)

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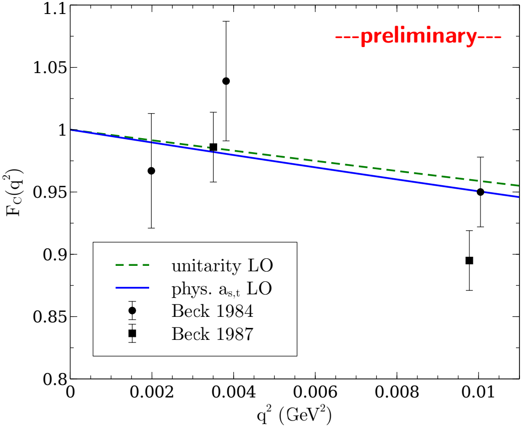

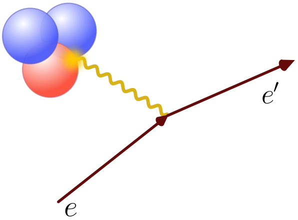

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Beck *et al.*, PRL **59** 1537 (1987)

Next steps: perturbation theory, range corrections, Coulomb corrections

Also: work on chiral two-body currents, led by Rodric Seutin

Unitarity expansion(s) at second order

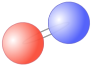
Various contributions at N^{2}LO...

SK, J Phys. G **44** 064007 (2017)

1. quadratic scattering-length corrections
   - at NLO, the deuteron remains at zero energy...
   - ...but it moves to $\kappa^{(1)} = 1/a_t$ at N^{2}LO

$$B_0 = \frac{(\kappa^{(0)})^2}{M_N} \;,\; B_1 = \frac{2\kappa^{(0)}\kappa^{(1)}}{M_N} \;,\; B_2 = \frac{(\kappa^{(1)})^2}{M_N} \;,\; \kappa^{(0)} \to 0$$

   - **expansion in momentum, not energy**



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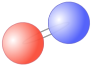

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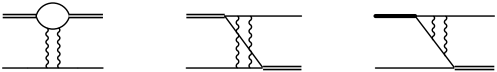
2. two-photon exchange

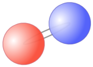


3. quadratic range corrections
4. isospin-breaking effective ranges: $r_{pp} \neq r_{np}$

Various contributions at N^{2}LO...

SK, J Phys. G **44** 064007 (2017)

1. quadratic scattering-length corrections
   - at NLO, the deuteron remains at zero energy...
   - ...but it moves to $\kappa^{(1)} = 1/a_t$ at N^{2}LO



$$B_0 = \frac{(\kappa^{(0)})^2}{M_N} \;,\; B_1 = \frac{2\kappa^{(0)}\kappa^{(1)}}{M_N} \;,\; B_2 = \frac{(\kappa^{(1)})^2}{M_N} \;,\; \kappa^{(0)} \to 0$$

   - **expansion in momentum, not energy**
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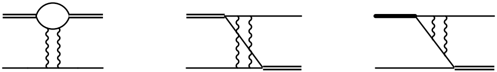
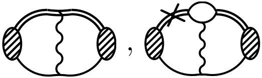


3. quadratic range corrections
4. isospin-breaking effective ranges: $r_{pp} \neq r_{np}$
5. mixed Coulomb and range corrections!   e.g.

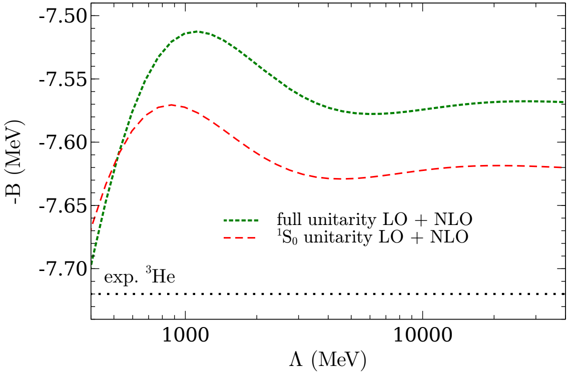


$\rightsquigarrow$ **log. divergence, new pd counterterm!**

More ^{3}He results

SK, J Phys. G **44** 064007 (2017)

- with range corrections, there is a new pd three-body force at N^{2}LO...
- ...but the convergence of the unitarity expansions can be checked for the zero-range case



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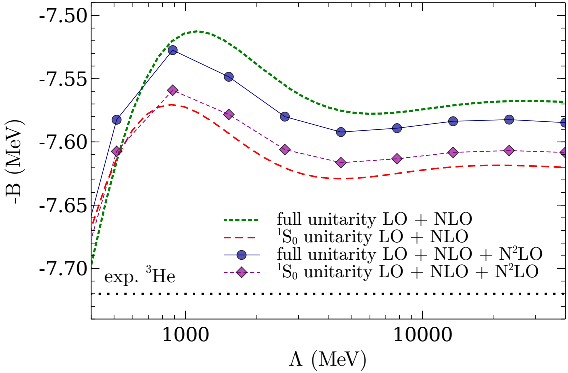
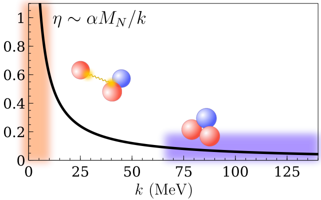


$\hookrightarrow$ **good convergence of half- and full-unitarity expansions**

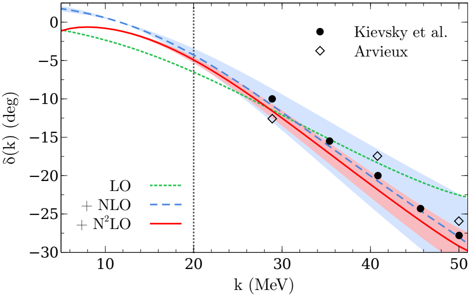
Perturbative p-d phase shifts

At intermediate energies, Coulomb is perturbative for pp/pd scattering!

SK *et al.* (2015); SK (2017)



$\eta \leq 1/3$ for $k \geq 20$ MeV



Perturbative subtracted phase shifts

$$\delta(k) \equiv \delta_{\text{full}}(k) - \delta_{\text{c}}(k)$$

$$= \delta_{\text{full}}^{(0)}(k) - \cancel{\delta_{\text{c}}^{(0)}(k)} + \delta_{\text{full}}^{(1)}(k) - \delta_{\text{c}}^{(1)}(k) + \delta_{\text{full}}^{(2)}(k) - \delta_{\text{c}}^{(2)}(k) + \cdots$$

cf. also SK, Hammer (2014)

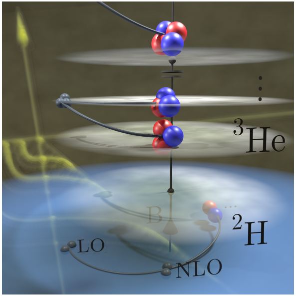
Unitarity expansion summary

Novel approach to few-nucleon systems

SK *et al.*, PRL **118** 202501 (2017)

	LO	NLO	N^2LO	exp.
^{2}H	0	0	1.41	2.22
^{3}H	8.48	8.48	8.48	8.48
^{3}He	8.48	7.56		7.72
^{4}He	38.86	29.50		28.30

four-body: no Coulomb, zero-range
NLO uncertainties: 0.2 MeV (^{3}He), 9 MeV (^{4}He)



- emphasize three-body sector over two-body precision
- enhanced symmetry and only one parameter at leading order
- **conjecture:** unitarity expansion useful beyond four nucleons

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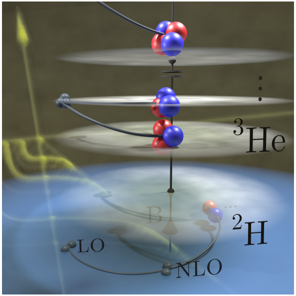

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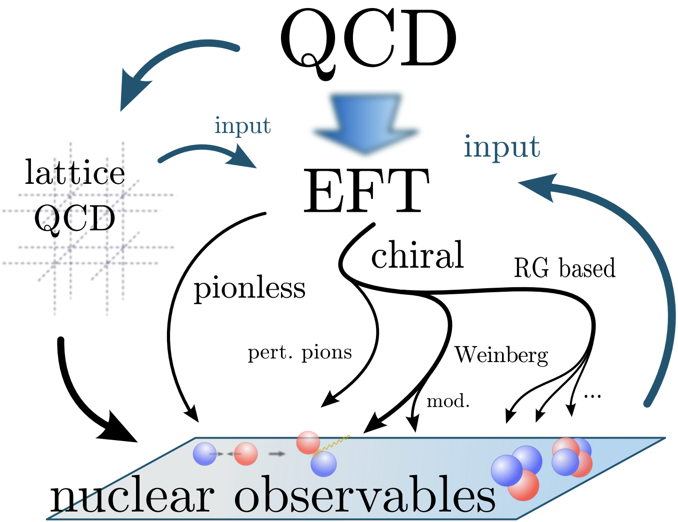
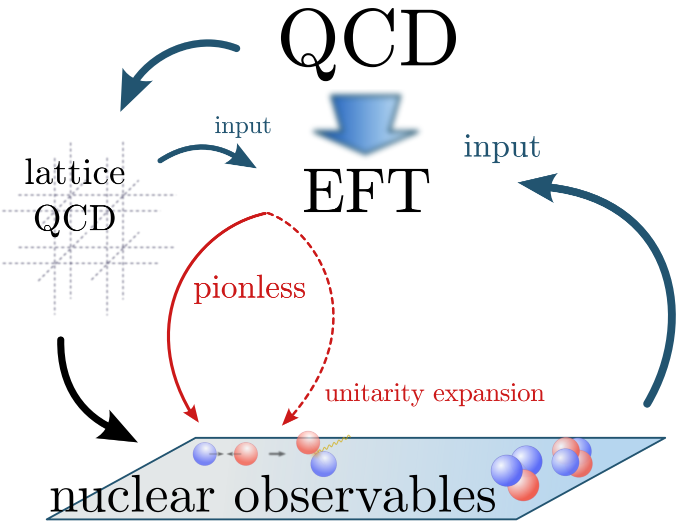
***** Thank you! *****

Backup slides

Furher outlook



Furher outlook



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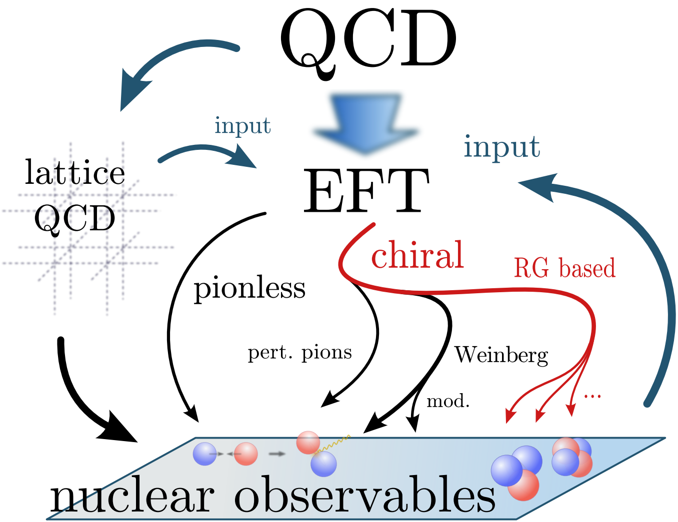


LECs for unitarity expansion

$$\mathcal{L} = N^\dagger \left(\mathrm{i}\mathcal{D}_0 + \frac{\boldsymbol{\mathcal{D}}^2}{2M_N} \right) N + \sum_{\mathbf{i}} C_{0,\mathbf{i}} \left(N^T P_{\mathbf{i}} N \right)^\dagger \left(N^T P_{\mathbf{i}} N \right) + D_0 \left(N^\dagger N \right)^3 + \cdots$$

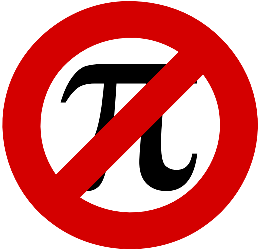
$$V_2^{(0)} = \sum_{\mathbf{i}} C_{0,\mathbf{i}}^{(0)} \; |\mathbf{i}\rangle \, |g\rangle \, \langle g| \, \langle \mathbf{i}| \quad , \quad V_3^{(0,1)} = D_0^{(0,1)} \, \left|{}^3\mathrm{H}\right\rangle |\xi\rangle \, \langle \xi| \left\langle {}^3\mathrm{H}\right|$$

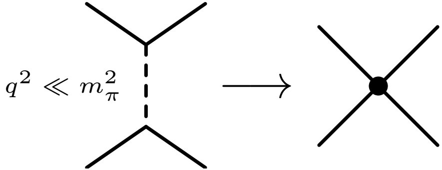
$$C_{0,\mathbf{i}}^{(0)} = \frac{-2\pi^2}{M_N \Lambda} \theta^{-1} \quad , \quad C_{0,\mathbf{i}}^{(1)} = \frac{M_N}{4\pi a_{\mathbf{i}}} C_{0,\mathbf{i}}^{(0)2}$$

$$D^{(0)}(\Lambda) \propto \frac{1}{\Lambda^4} \frac{\sin\left(s_0 \log(\Lambda/\Lambda_*) - \arctan s_0^{-1}\right)}{\sin\left(s_0 \log(\Lambda/\Lambda_*) + \arctan s_0^{-1}\right)}$$

No pions at low energy!

- derivative coupling of pions!
  $\hookrightarrow$ no one-pion exchange contribution to NN scattering lengths!
- chiral power counting designed for momenta $Q \sim m_\pi$
- relevant symmetries: spatial (rot., Galilei boost), discrete, isospin
- only contact interactions left (plus Coulomb)





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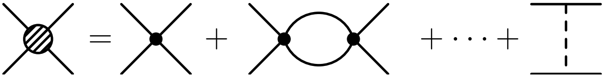
Perturbative pions

- possible to use pionless EFT + perturbative pions. . .

  Kaplan, Save, Wise NPA **478** 629 (1996), NPB **534** 329 (1998), PLB **424** 390 (1998)

- . . . but fails in channels with **attractive singular tensor force!**

  Fleming, Mehen, Stewart, NPA **677** 313 (2000)



Open questions to be studied:

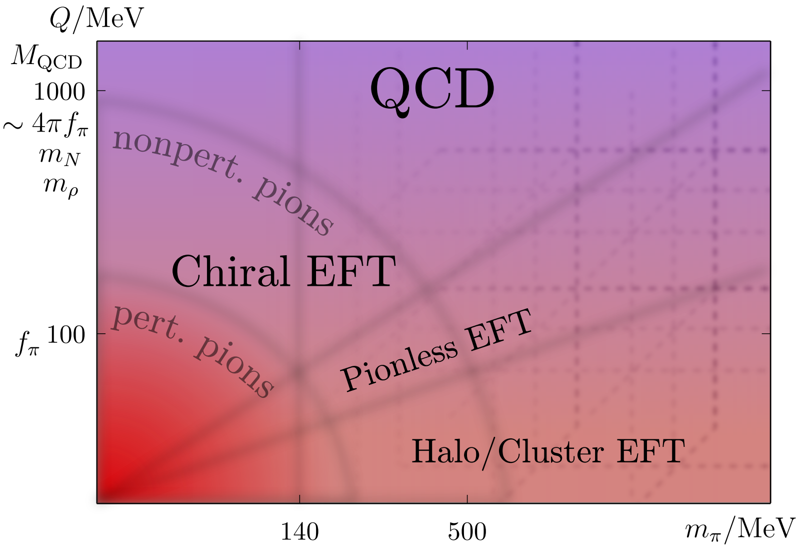
1. (non-)perturbativeness
2. long-range forces (Coulomb)
3. renormalization and regulators

All these questions are relevant for chiral EFT as well!

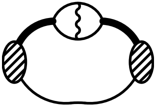
Why pionless EFT?

- conceptually clean and (reasonably) simple
- allows for a fully perturbative treatment of higher orders
- cutoff can be made arbitrarily large
- still clearly connected to QCD!

EFT Landscape



Coulomb bubble divergence



- an additional diagram is logarithmically divergent. . .
- . . . but this divergence comes from the photon-bubble subdiagram!

Strategy

SK, Grießhammer, Hammer, van Kolck, JPG **43** 055106 (2016), 1508.05085 [nucl-th]

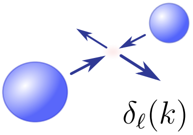
1. isolate divergence: 
2. **take the leading-order $^1S_0$ in the unitarity limit!**

$$a_{^1S_0} = -23.7 \approx \infty \rightsquigarrow 1/a_{^1S_0} \approx 0$$

3. include divergent diagram together with finite $a_{^1S_0}$

 $+$  $=$ finite

ERE and EFT



Effective range expansion (ERE)

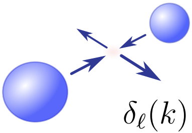
$$k^{2\ell+1}\cot\delta_\ell(k) = -\frac{1}{a_\ell} + \frac{1}{2}r_\ell\,k^2 + \cdots$$

- a_ℓ – scattering length
- r_ℓ – effective range

physical properties

$\leftrightarrow$ small number of low-energy parameters

ERE and EFT



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Effective field theory (EFT)

- identify relevant degrees of freedom
- exploit separation of scales $\longrightarrow$ expansion parameter
- symmetries restrict possible terms
- order by size $\longrightarrow$ **power counting!**

Perturbative vs. nonperturbative schemes

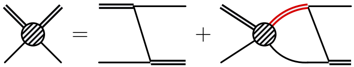
chiral EFT: Weinberg counting

- expand potential: $V = V_{\mathrm{LO}} + V_{\mathrm{NLO}} + \cdots$
- solve Lippmann–Schwinger equation: $T = V + V G_0 T$

pionless EFT: partial-resummation approach Bedaque *et al.* NPA **714** 589 (2003)

- include range corrections in NLO propagator:

- then insert into LS equation: $\rightarrow$



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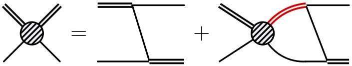
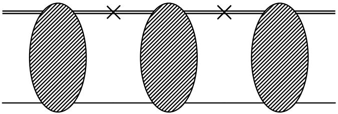


Resums certain higher-order corrections!

So why do it?

$\hookrightarrow$ need full-off shell amplitudes otherwise...



N^{2}LO is expensive...

Perturbative vs. nonperturbative schemes

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Perturbative vs. nonperturbative schemes

N^2LO used to be expensive...

$\hookrightarrow$ new elegant approach to fully perturbative calculations

Vanasse, PRC **88** 044001 (2013)

- re-shuffle diagrams to inhomogeneous term in LS equation
- actually slightly cheaper than partial resummation!

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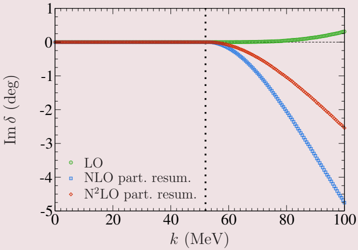
Example: n-d quartet scattering



unitarity violation!

$$s_0(k) = \mathrm{e}^{\mathrm{i}\delta_0(k)}$$
$$= \mathrm{e}^{\mathrm{iRe}\,\delta_0(k)}\mathrm{e}^{-\mathrm{Im}\,\delta_0(k)}$$

$$|s_0(k)| \leq 1$$

Perturbative vs. nonperturbative schemes

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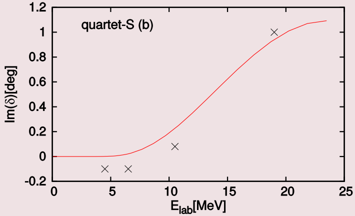
problem fixed with fully perturbative calculation:



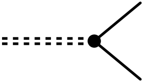
Vanasse, PRC **88** 044001 (2013)

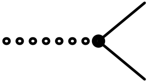
Effective Lagrangian

$$\mathcal{L} = \underline{N^\dagger \left(\mathrm{i}D_0 + \frac{\vec{D}^2}{2M_N} \right) N} + \mathcal{L}_{\mathrm{photon}} + \mathcal{L}_3$$

$$- d^{i\dagger} \left[\sigma_d + \ldots \right] d^i \quad - t^{A\dagger} \left[\sigma_t + \ldots \right] t^A$$

$$- y_d \left[d^{i\dagger} \left(N^T P_d^i N \right) + \mathrm{h.c.} \right] - y_t \left[t^{A\dagger} \left(N^T P_t^A N \right) + \mathrm{h.c.} \right]$$

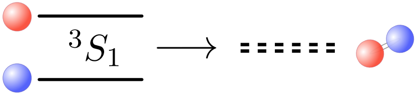




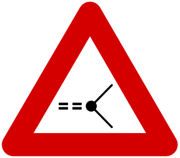
- **nucleon** field N, doublet in spin and isospin space
- auxiliary **dibaryon** fields d^i $(^3S_1,\ I = 0)$ and t^A $(^1S_0,\ I = 1)$
  $\leftrightarrow$ channels in N–N scattering
- **coupling constants** $y_{d,t}$ and $\sigma_{d,t}$
- dibaryon propagators are just constants at leading order

Two-body sector

Introduce dibaryon fields...



Bedaque *et al.* (1998)



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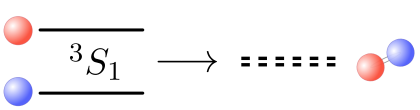

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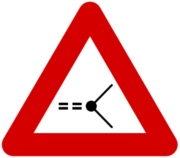
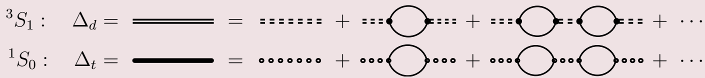
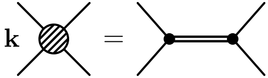


. . . and resum bubble-insertions to all orders!

Full dibaryon propagators



$$\Delta_d(k) \sim \frac{\mathrm{i}}{\underbrace{k\cot\delta_d}_{=-\gamma_d+\frac{\rho_d}{2}(k^2+\gamma_d^2)+\cdots} - \mathrm{i}k}$$



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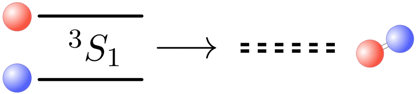

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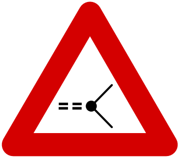

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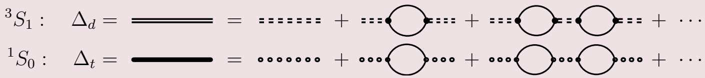


$$\Delta_d(k) \sim \frac{\mathrm{i}}{\underbrace{k\cot\delta_d}_{=-\gamma_d+\frac{\rho_d}{2}(k^2+\gamma_d^2)+\cdots} - \mathrm{i}k}$$

$$\gamma_d\rho_d \sim Q/\Lambda_{\not\pi} = \mathcal{O}(1/3)$$

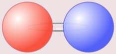
Three-body sector

Nucleon

- spin $1/2$
- isospin $1/2$

Deuteron

- spin 1
- isospin 0



$\hookrightarrow$ **two S-wave channels:**

$$\mathbf{1}\otimes\frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{3}}{\mathbf{2}}\left(\sim \uparrow\uparrow\uparrow\right) \oplus \overbrace{\frac{\mathbf{1}}{\mathbf{2}}\left(\sim \uparrow\uparrow\downarrow + \cdots\right)}^{\text{spin doublet} \to {}^3\mathrm{H}, {}^3\mathrm{He}}$$

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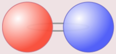

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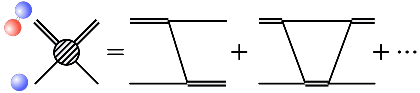
—quartet channel—



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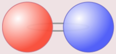
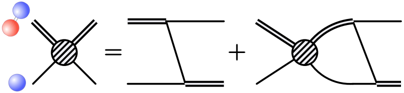


$\hookrightarrow$ **two S-wave channels:**

$$1 \otimes \frac{1}{2} = \frac{3}{2}\left(\sim \uparrow\uparrow\uparrow\right) \oplus \overbrace{\frac{1}{2}\left(\sim \uparrow\uparrow\downarrow + \cdots\right)}^{\text{spin doublet} \to {}^3\mathrm{H}, {}^3\mathrm{He}}$$

—quartet channel—



$\hookrightarrow$ **solve integral equations to get phase shifts and binding energies**

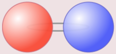
Three-body sector

Nucleon

- spin $^1/_2$
- isospin $^1/_2$

Deuteron

- spin 1
- isospin 0



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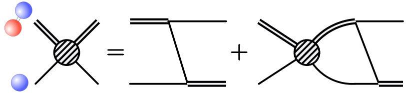
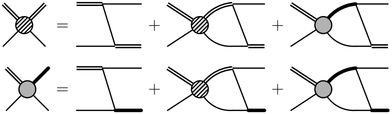
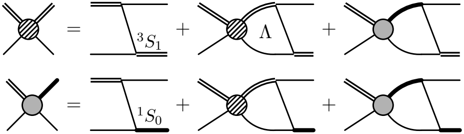


—doublet channel—



$\hookrightarrow$ **solve integral equations to get phase shifts and binding energies**

The triton



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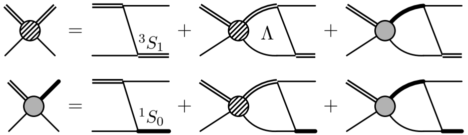
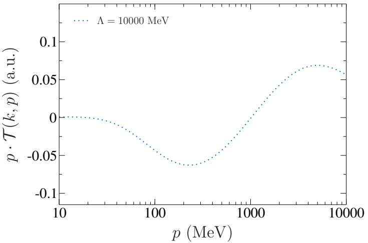
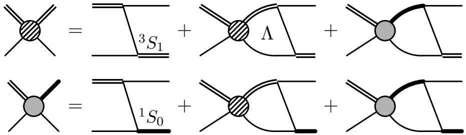


=
$^3S_1$
+
Λ
+
=
$^1S_0$
+
+

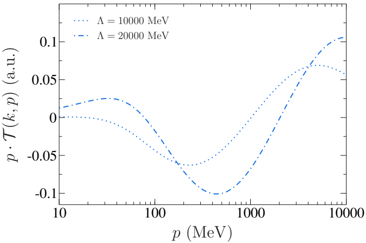


Λ = 10000 MeV
$p \cdot \mathcal{T}(k,p)$ (a.u.)
0.1
0.05
0
-0.05
-0.1
10
100
1000
10000
p (MeV)

The triton

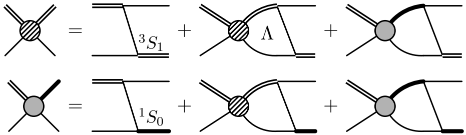


$^3S_1$
Λ
$^1S_0$

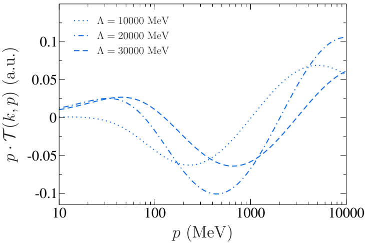


$\Lambda = 10000$ MeV
$\Lambda = 20000$ MeV
0.1
0.05
0
-0.05
-0.1
$p \cdot \mathcal{T}(k,p)$ (a.u.)
10
100
1000
10000
p (MeV)

The triton

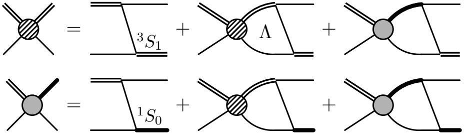


$=$
$^3S_1$
$+$
Λ
$+$
$=$
$^1S_0$
$+$
$+$

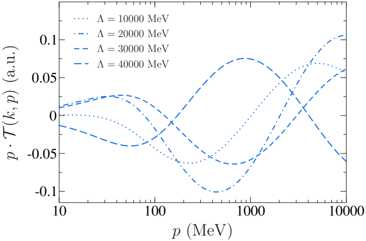


$\Lambda = 10000$ MeV
$\Lambda = 20000$ MeV
$\Lambda = 30000$ MeV
0.1
0.05
0
-0.05
-0.1
10
100
1000
10000
p (MeV)
$p \cdot \mathcal{T}(k,p)$ (a.u.)

The triton



$^3S_1$
Λ
$^1S_0$



$\Lambda = 10000$ MeV
$\Lambda = 20000$ MeV
$\Lambda = 30000$ MeV
$\Lambda = 40000$ MeV
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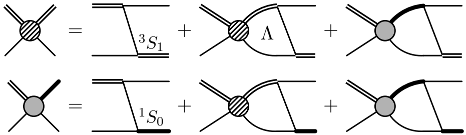
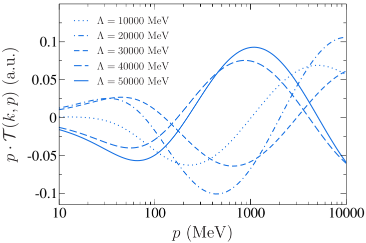
0.1
0.05
0
-0.05
-0.1
10
100
1000
10000
p (MeV)

The triton

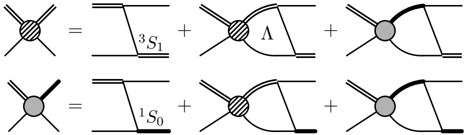


$=$
${}^3S_1$
$+$
Λ
$+$
$=$
${}^1S_0$
$+$
$+$



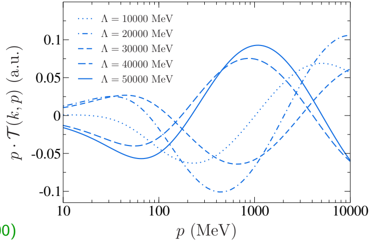
Λ = 10000 MeV
Λ = 20000 MeV
Λ = 30000 MeV
Λ = 40000 MeV
Λ = 50000 MeV
$p \cdot \mathcal{T}(k,p)$ (a.u.)
0.1
0.05
0
-0.05
-0.1
10
100
1000
10000
p (MeV)

The triton

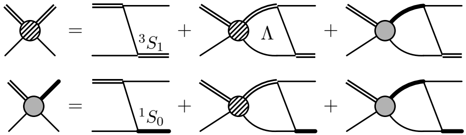


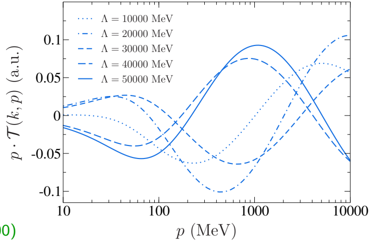
amplitude has no well-defined limit!

Bedaque *et al.*, NPA **676** 357 (2000)



The triton





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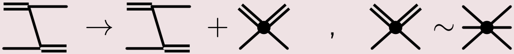
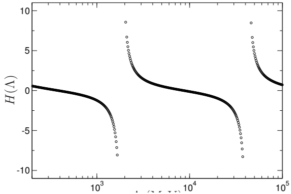
Three-body force promotion

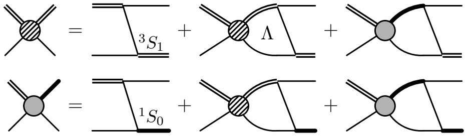
already at LO:

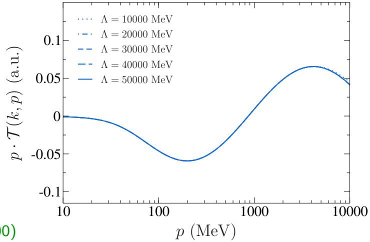




- independent of spin and isospin $\rightarrow SU(4)$-symmetry
- RG limit cycle $\leftrightarrow$ Efimov effect
- **makes amplitude cutoff-independent**

The triton





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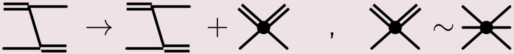
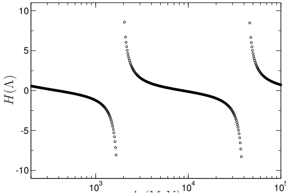
already at LO: 



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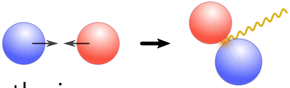
Some applications

1. **capture reactions:** $np \to d\gamma$



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- pionless gives precise prediction: error $< 4\%$. . . Chen, Savage, PRC **60** 065205 (1999)
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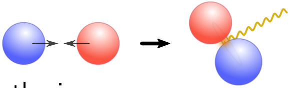

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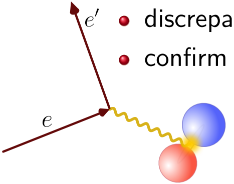
Christlmeier, Grießhammer, PRC **77** 064001 (2008)
Ryezayeva *et al.*, PRL **100** 172501 (2008)



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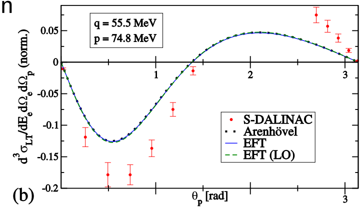
- confirm potential-model calculation



(b)

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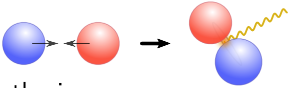

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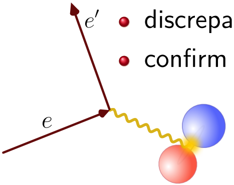

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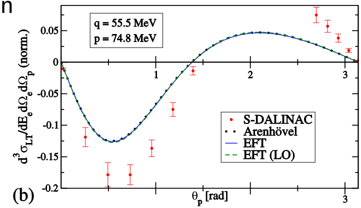

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- heavy-pion pionless EFT
- $np \to d\gamma$

Barnea *et al.* PRL **114** 052501 (2015)
Kirscher *et al.* PRC **92** 054002 (2015)

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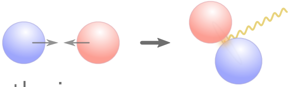

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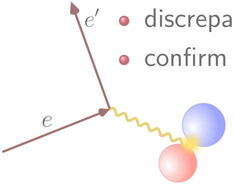

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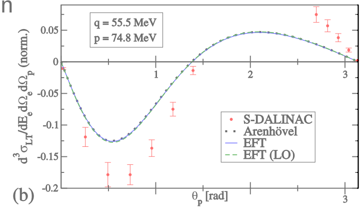

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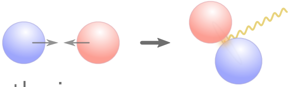

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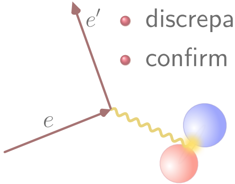

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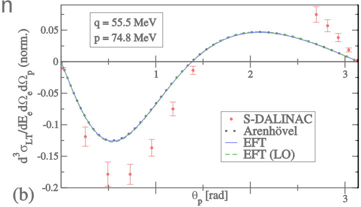

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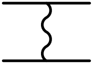
Long-range forces

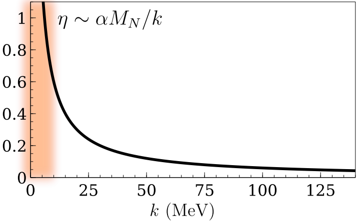
- most nuclear systems involve charged particles $\rightarrow$ include photons!
- long (infinite) range $\rightarrow$ **nonperturbative** at small momentum transfer!

Coulomb photons



$$\sim \; (\mathrm{ie}) \, \frac{\mathrm{i}}{\mathbf{k}^2 + \lambda^2} \, (\mathrm{ie})$$

$\hookrightarrow$ p-d scattering length with consistent screening



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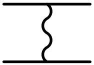

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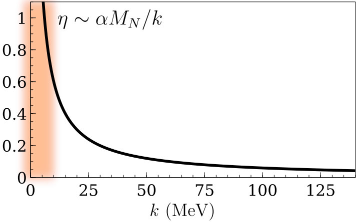


Coulomb-dressed propagator

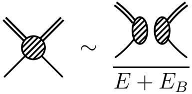
$$\Delta_{t,pp}(p) \sim \frac{1}{-1/a_{p\text{-}p} - \alpha M_N H(\eta)} \;,\; \eta = \alpha M_N / \left(2\mathrm{i}\sqrt{\mathbf{p}^2/4 - M_N p_0 - \mathrm{i}\varepsilon}\right)$$

$\hookrightarrow$ **Coulomb-modified effective range expansion**

Kong, Ravndal (1999)
Bethe (1949)
cf. Ando, Birse (2010)

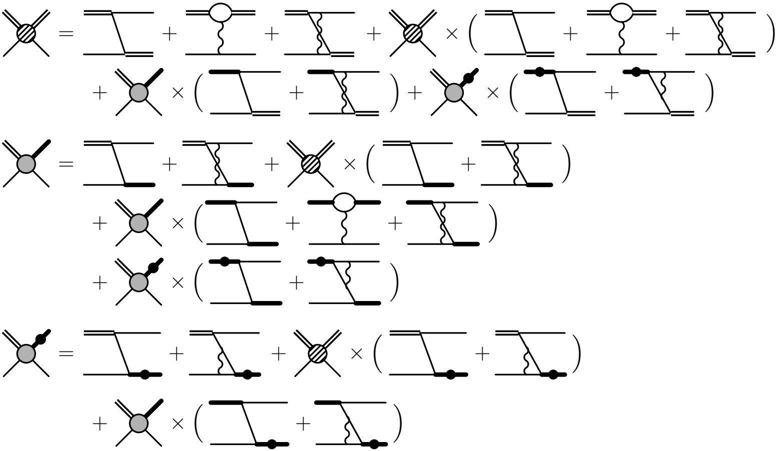
He-3 binding energy

bound-state $\leftrightarrow$ pole!

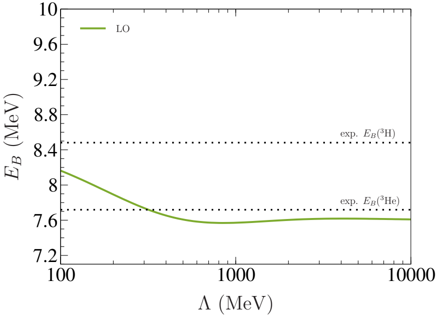


$+$ regular terms

$\hookrightarrow$ calculate ^{3}He binding energy!



He-3 beyond leading order



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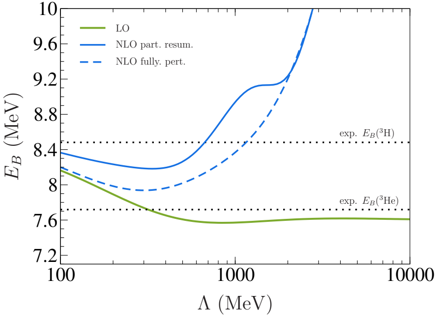
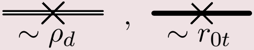
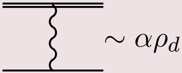


NLO corrections

- effective ranges



- dibaryon-photon coupling



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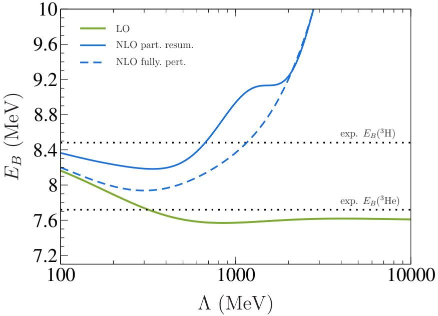

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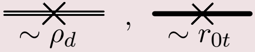

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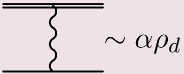

- NLO result is not cutoff stable $\hookrightarrow$ incomplete renormalization!
- refitting the three-body force to $E_B(^3\text{He})$ gives stable p-d phase shifts!

SK, Ph.D. thesis (2013)
SK, Grießhammer Hammer, JPG **42** 045101 (2015)

- form of new p-d specific counterterm can be derived analytically!
  $\rightsquigarrow$ three body-force $H(\Lambda) = H_{0,0}(\Lambda) + H_{0,1}(\Lambda) + H_{0,1}^{(\alpha)}(\Lambda)$

Vanasse, Egolf, Kerin, SK, Springer, PRC **89** 064003 (2014)

Counterterm controversy

A recent paper does not find a new counterterm at NLO!

Kirscher, Gazit 1510.00118 [nucl-th]

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Kirscher and Gazit

- configuration-space (R)RGM
- Gaussian regulators
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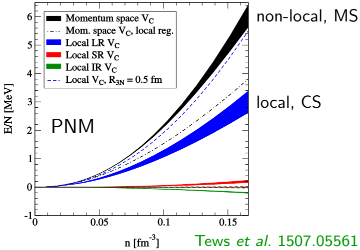
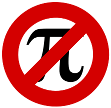
similar issues in chiral EFT!



Tews *et al.* 1507.05561 [nucl-th]



pionless EFT can study this question in a clean scenario without other complications!

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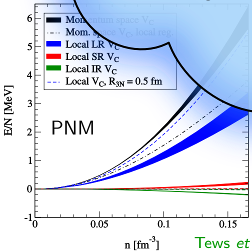
coming up:

EMMI RRTF Workshop to investigate the issue

- co-organized by D. Gazit, H. Grießhammer, SK, J. Vanasse
- meetings at TU Darmstadt in January and May/June 2016

power counting ↔ regulators?

similar issues in chiral EFT!



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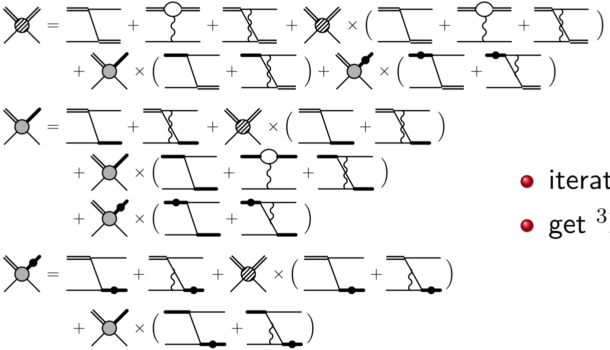
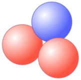
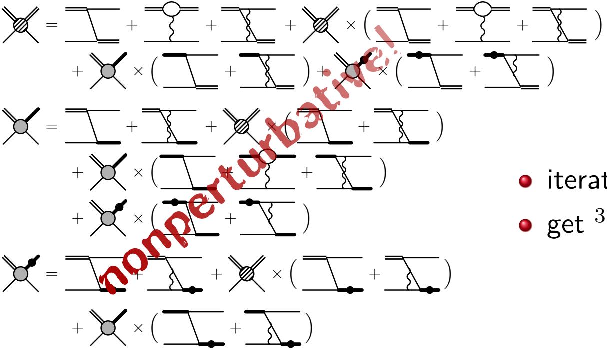
Nonperturbative vs. perturbative and helium

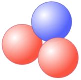




- iterate $\mathcal{O}(\alpha)$ diagrams...
- get $^{3}\mathrm{He}$ pole directly

Nonperturbative vs. perturbative and helium





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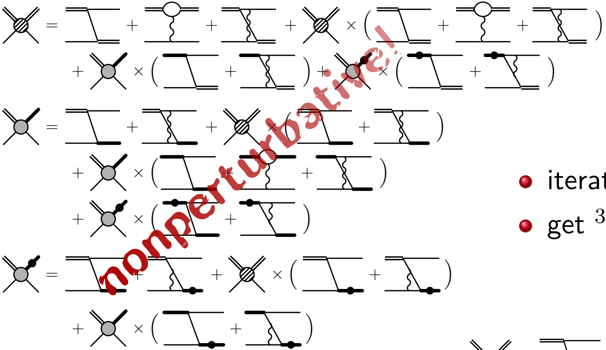
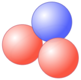
Nonperturbative vs. perturbative and helium





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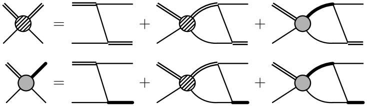
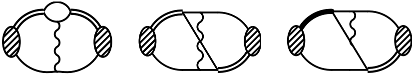
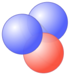
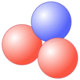


$$\Delta E = \langle\psi|V_C|\psi\rangle$$



- use trinucleon wavefunctions
- fully perturbative in α!

ΔE :  $-$ 

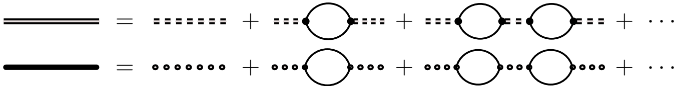
SK *et al.*, J. Phys. G **42** 045101 (2015)

Dibaryon propagators

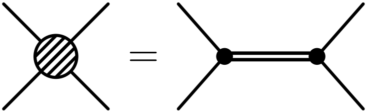
Bubble chains

$^3S_1: \quad \Delta_d =$

$^1S_0: \quad \Delta_t =$



Fix parameters from N-N scattering!



$$\mathrm{i}\mathcal{A}_{d,t}(k) = -y_{d,t}^2 \Delta_{d,t}\left(p_0 = \frac{\mathbf{k}^2}{2M_N}, \mathbf{p} = 0\right) = \frac{4\pi}{M_N} \frac{\mathrm{i}}{k \cot \delta_{d,t} - \mathrm{i}k}$$

- $k \cot \delta_d(k) = -\gamma_d + \frac{\rho_d}{2}(k^2 + \gamma_d^2) + \ldots \longrightarrow y_d, \sigma_d$
- $k \cot \delta_t(k) = -\gamma_t + \frac{r_{0t}}{2} k^2 + \ldots$ with $\gamma_t \equiv \frac{1}{a_t} \longrightarrow y_t, \sigma_t$

Range corrections

Dibaryon kinetic-energy terms

 $\sim \; \mathrm{i}\Delta_d^{\mathrm{LO}}(p) \times (-\mathrm{i}) \left(p_0 - \frac{\mathbf{p}^2}{4M_N}\right) \times \mathrm{i}\Delta_d^{\mathrm{LO}}(p)$

$\hookrightarrow$ effective-range corrections

$$\Delta_d(p) \sim \frac{1}{-\gamma_d + \sqrt{\frac{\mathbf{p}^2}{4} - M_N p_0 - \mathrm{i}\varepsilon} - \frac{\rho_d}{2}\left(\frac{\mathbf{p}^2}{4} - M_N p_0 - \gamma_d^2\right)}$$

$\boxed{\mathcal{O}(Q/\Lambda) \sim \mathcal{O}(\gamma_d \rho_d)}$ expand in ρ_d, r_{0t} $\to$ NLO, N^{2}LO, ...

$$D_d(E;q) = D_d^{(0)}(E;q) + D_d^{(1)}(E;q) + \cdots$$

$$= -\frac{4\pi}{M_N y_d^2} \frac{1}{-\gamma_d + \sqrt{3q^2/4 - M_N E - \mathrm{i}\varepsilon}} \times \left[1 + \frac{\rho_d}{2} \frac{\left(3q^2/4 - M_N E - \gamma_d^2\right)}{-\gamma_d + \sqrt{3q^2/4 - M_N E - \mathrm{i}\varepsilon}} + \cdots\right]$$

Coulomb photons

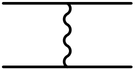
$$\mathcal{L}_{\rm photon} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}\Big(\underbrace{\partial_\mu A^\mu - \eta_\mu\eta_\nu\partial^\nu A^\mu}_{=\nabla\cdot\mathbf{A}\quad \text{for}\quad \eta^\mu=(1,0,0,0)}\Big)^2 - \mathrm{e}\, j_\mu A^\mu$$

→ **quantization in Coulomb gauge**

- field component A_0 does not propagate
  ↪ eliminate with equation of motion

$$\Delta A^0 = -\mathrm{e}\, j^0 \iff (\mathrm{i}\mathbf{k})^2 A^0 = -\mathrm{e}\, j^0$$

- re-insert into Lagrangian → $\mathrm{i}\mathcal{L}_{\rm int}(\mathbf{k}) \supset (\mathrm{ie})\, j_0(\mathbf{k})\, \frac{\mathrm{i}}{\mathbf{k}^2}\, (\mathrm{ie})\, j_0(\mathbf{k})$



$$\sim \quad \mathrm{ie}^2 \frac{1}{(\mathrm{i}\mathbf{k})^2} = (\mathrm{ie})\, \frac{\mathrm{i}}{\mathbf{k}^2}\, (\mathrm{ie})$$

→ **exchange of Coulomb photons**

Transverse photons are suppressed by powers of momenta and/or α/M_N.

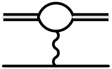
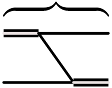
Coulomb diagrams

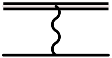
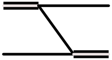
Coulomb effects $\sim \alpha M_N/p$ are dominant at very low momenta!

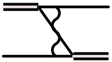
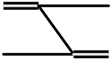
$\rightarrow$ we can no longer assume $p \sim \gamma_d, \gamma_t \sim Q$

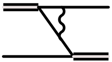
Need simultaneous expansion in Q/Λ and $p/(\alpha M_N)$!

Rupak, Kong (2003)

 $\sim \dfrac{\Lambda}{Q}\dfrac{\alpha}{p^2}$ = $\sim\Lambda/(M_N Q^2)$  $\times \dfrac{\alpha M_N}{p}\dfrac{Q}{p}$

 $\sim \dfrac{\alpha}{p^2}$ =  $\times \dfrac{\alpha M_N}{p}\dfrac{Q}{p} \times \dfrac{Q}{\Lambda}$

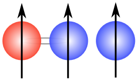
 $\sim \dfrac{\Lambda}{Q}\dfrac{\alpha}{Q^2}$ =  $\times \dfrac{\alpha M_N}{Q}$

 $\sim$ same as above

Quartet and doublet channel

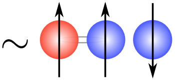
$$\mathbf{1} \otimes \frac{\mathbf{1}}{\mathbf{2}} = \frac{\mathbf{3}}{\mathbf{2}} \oplus \frac{\mathbf{1}}{\mathbf{2}}$$

Quartet channel - couple to spin ${}^3\!/_2$



- all three nucleon spins aligned $\rightarrow$ Pauli principle
- not very sensitive to short-range physics
- no bound state

Doublet channel - couple to spin ${}^1\!/_2$



- no Pauli principle
- ${}^1S_0$-dibaryon can appear $\rightarrow$ coupled channels
- leading-order $3N$-interaction

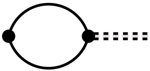
Power counting

Scales & scaling

- low-energy scale $Q \sim \mathcal{O}(\gamma_d) \sim p$
- cut-off $\Lambda \sim \mathcal{O}(m_\pi) \sim 1/R$
- nucleon mass M_N
- assume $y^2 \sim \Lambda/M_N^2$ and $\sigma \sim Q\Lambda/M_N$

Consequences

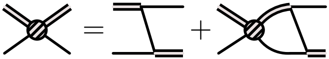
- integration measure $\int \mathrm{d}^3q\mathrm{d}q_0 \sim Q^5/M_N$
- nucleon propagator $\sim M_N/Q^2$
- leading-order dibaryon propagator $= -\mathrm{i}/\sigma \sim M_N/(Q\Lambda)$



$$\sim \frac{\Lambda}{M_N^2} \times \frac{Q^5}{M_N} \times \left(\frac{M_N}{Q^2}\right)^2 \times \frac{M_N}{Q\Lambda} = \mathcal{O}(1)$$

$\hookrightarrow$ **re-sum propagators!**

Bound-state equation



$$\mathcal{T}(E;k,p) = K(E;k,p) + \int \mathrm{d}q\, q^2\, K(E;q,p) \times D(E;q)\, \mathcal{T}(E;k,q)$$

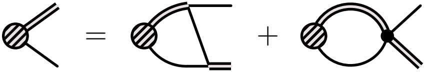
bound state

$\rightarrow$ pole in $\mathcal{T}$-matrix

$$\mathcal{T}(E;k,p) \sim \frac{\mathcal{B}(k)\mathcal{B}(p)}{E+E_B} \quad \text{as} \quad E \rightarrow -E_B$$

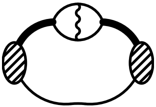
$\lim_{E\rightarrow -E_B}(E+E_B)K(E,k,p) = 0$ $\rightarrow$ **homogeneous equation!**

$$\mathcal{B}(E,p) = \int_0^\Lambda \mathrm{d}q\, q^2 \left[K(E;q,p) + \frac{2H(\Lambda)}{\Lambda^2}\right] D(E;q)\, \mathcal{B}(E,q)$$



To determine $3N$-force, fix $E = -E_B^{^3\mathrm{H}}$ and cut-off Λ, find suitable $H(\Lambda)$

Coulomb bubble divergence



- The additional diagram is logarithmically divergent!
- But this divergence comes from the photon-bubble subdiagram!

$\hookrightarrow$ **determine counterterm from p–p scattering!**

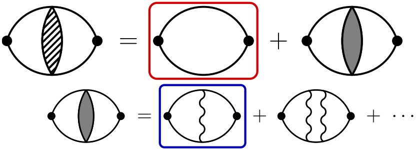
$$\Delta_{t,pp}(p_0,\mathbf{p}) = \frac{-\mathrm{i}}{\underbrace{\sigma_{t,pp} - \frac{2\Lambda}{\pi} + \alpha M_N\left(\log\frac{2\Lambda}{\alpha M_N} - C_E\right)}_{=1/a_C} - \alpha M_N H(\eta)}$$

cf. Kong, Ravndal (1999)

Important to isolate divergence for consistent renormalization!

Dressed bubble integral

$$J_0(k) = G_C(k^2/M_N;\mathbf{0},\mathbf{0})$$



$$\hat{G}_C = \hat{G}_0^{(+)} + \hat{G}_0^{(+)}\,\hat{V}_C\,\hat{G}_C = \hat{G}_0^{(+)} + \hat{G}_0^{(+)}\,\hat{T}_C\,\hat{G}_0^{(+)}$$

$$\hat{T}_C = \hat{V}_C + \hat{V}_C\,\hat{G}_0^{(+)}\,\hat{T}_C$$

Doublet-channel phase shift