TECHNISCHE UNIVERSITÄT DARMSTADT

Effective field theories for heavy nuclei

Eduardo Antonio Coello Pérez

SFB 1245

Overview

Deformed nuclei (even-even nuclei)

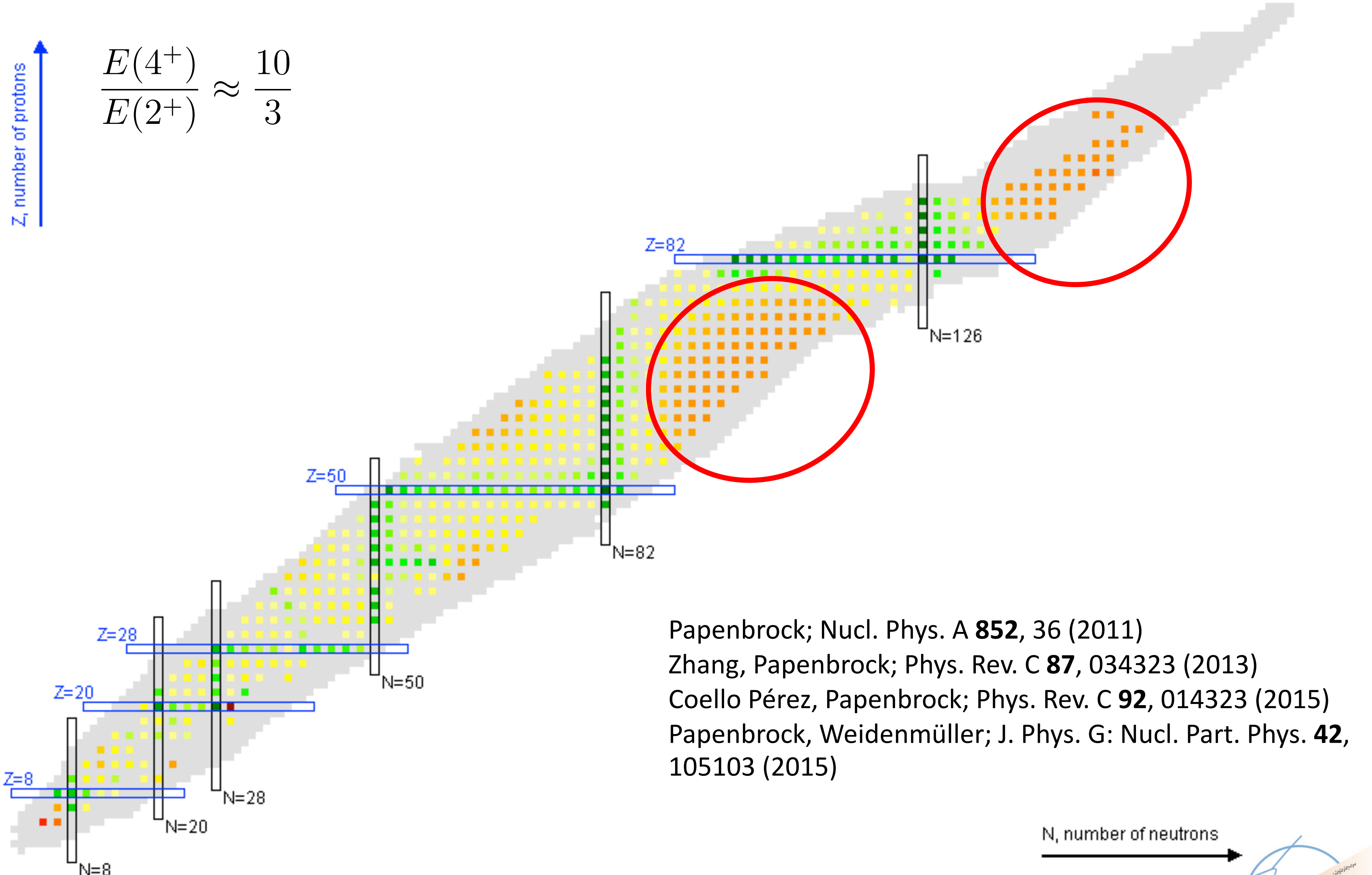
- Ground-state rotational band
  - NLO spectra and B(E2) values
- Vibrations
  - LO spectra and B(E2) values

Spherical nuclei (even-even nucleus and odd-mass neighbor)

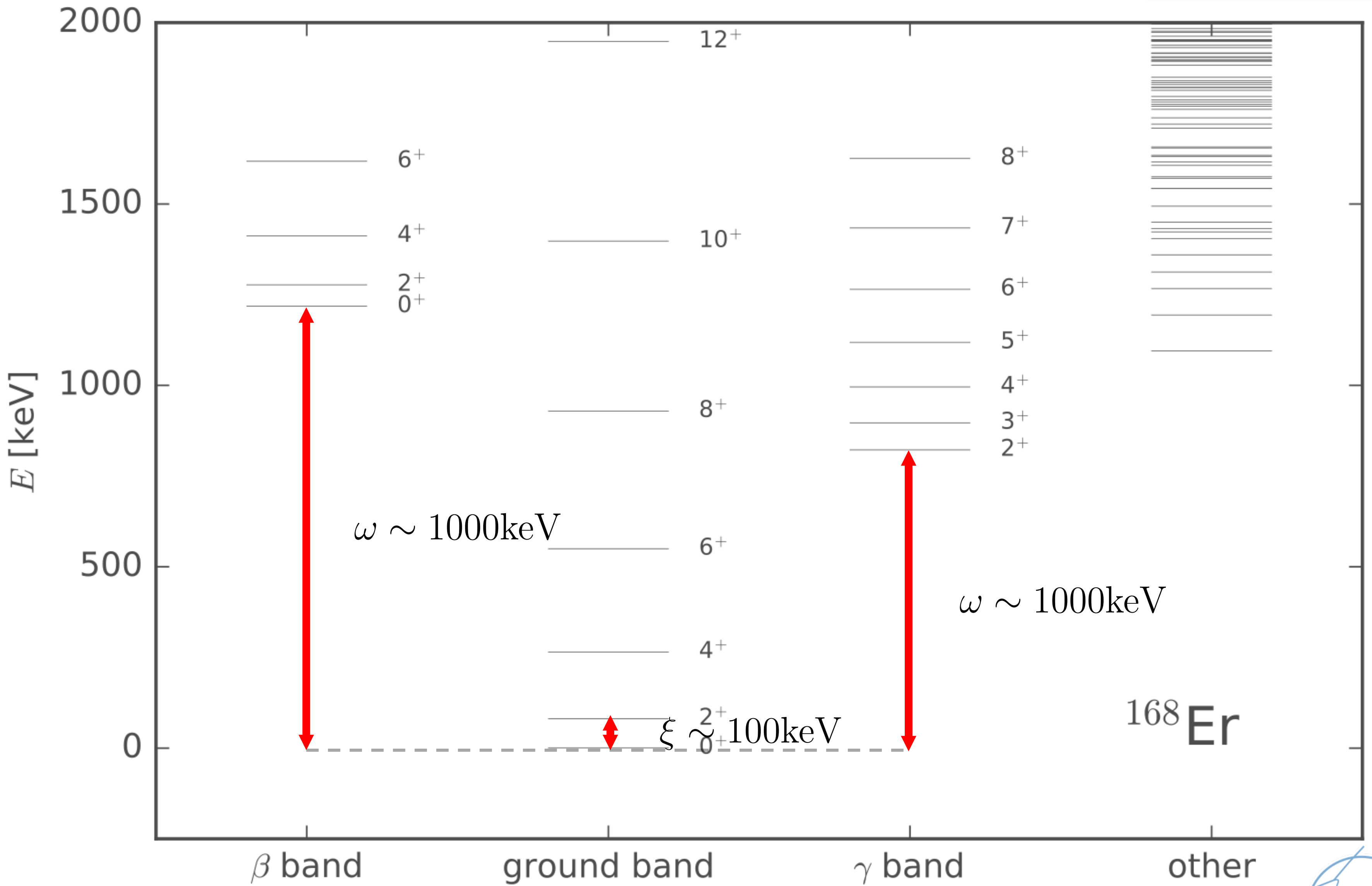
- Power counting from data
- E2 properties
  - Phonon-annihilating B(E2) values
  - LO relations between observables
- M1 properties
  - Static M1 moments and phonon-conserving transitions
  - Phonon-annihilating B(M1) values

Deformed nuclei

$$\frac{E(4^+)}{E(2^+)} \approx \frac{10}{3}$$



Papenbrock; Nucl. Phys. A **852**, 36 (2011)
Zhang, Papenbrock; Phys. Rev. C **87**, 034323 (2013)
Coello Pérez, Papenbrock; Phys. Rev. C **92**, 014323 (2015)
Papenbrock, Weidenmüller; J. Phys. G: Nucl. Part. Phys. **42**, 105103 (2015)



Baglin; Nucl. Data Sheets **111**, 1807 (2010)

The orientation angles θ and ϕ used as degrees of freedom via the building blocks

$$v_{\pm 1} \equiv \mp\sqrt{\frac{1}{2}}\left(\dot{\theta} \pm i\dot{\phi}\sin\theta\right)$$

Under an SO(3) rotation

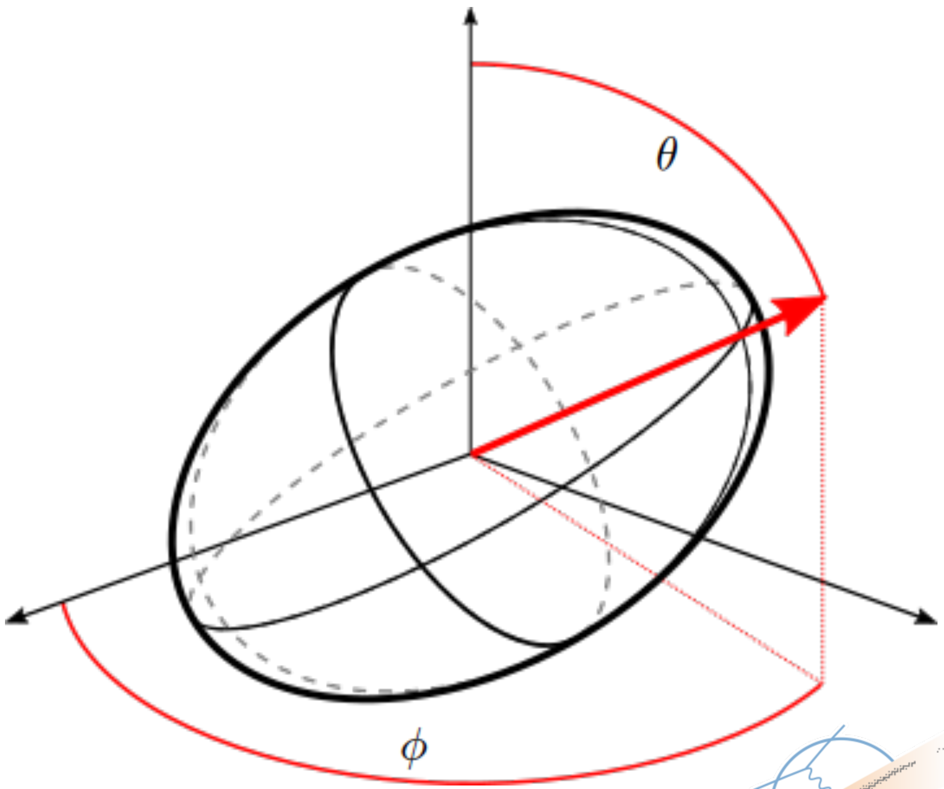
$$v_{\pm 1} \to e^{\pm i\tilde{\gamma}} v_{\pm 1}$$

Quantizing the Legendre transformation of the most simple rotationally-invariant Lagrangian yields

$$\hat{H}_{\mathrm{LO}} = \frac{1}{2C_0}\hat{I}^2$$

For a rigid rotor it is expected that

$$C_0 = \frac{3}{\xi}$$



Higher-order corrections yield a Hamiltonian equivalent to that of the variable moment of inertia model*. The NLO Hamiltonian

$$\hat{H}_{\mathrm{NLO}} = \hat{H}_{\mathrm{LO}} - \frac{C_2}{4C_0^4}\hat{I}^4$$

It is naively expected that

$$C_2 \sim \frac{C_0}{\omega^2}$$

At the breakdown scale

$$\frac{\langle \Delta\hat{H}_{\mathrm{NLO}}\rangle}{\langle \hat{H}_{\mathrm{LO}}\rangle} \sim \frac{C_2}{C_0^3} I_b^2 \sim \left(\frac{\xi}{\omega}\right)^2 I_b^2 \sim 1$$

Thus

$$I_b \sim \sqrt{\frac{C_0^3}{C_2}} \sim \frac{\omega}{\xi}$$

LO and NLO LECs compared against their naive estimates

System	$C_0\xi$	$\frac{C_2}{C_0}\omega^2$	$(\xi/\omega)^2$	C_2/C_0^3
N_2	3.00	2.1	0.000 026	0.000 006
H_2	2.99	2.2	0.0062	0.0015
^{236}U	2.99	2.3	0.0043	0.0011
^{174}Yb	2.99	3.4	0.0026	0.0010
^{168}Er	2.99	1.0	0.0094	0.0010
^{166}Er	2.98	1.6	0.011	0.0020
^{162}Dy	2.98	1.9	0.0083	0.0017
^{154}Sm	2.97	5.2	0.0056	0.0033
^{188}Os	2.91	1.5	0.06	0.012
^{154}Gd	2.88	3.3	0.033	0.013
^{152}Sm	2.88	3.5	0.032	0.013
^{150}Nd	2.85	3.6	0.037	0.017

*Mariscotti, et al.; Phys. Rev. **178**, 1864 (1969)

NLO B(E2) values for decays within the ground-state band

Coupling between the building blocks and an electric field yield the E2 operator

The NLO B(E2) values for decays within the ground-state band are

$$B(E2)_{\mathrm{NLO}} = Q_0^2 \left(C_{I_i 020}^{I_f 0}\right)^2 \left[1 + \frac{b}{a} I_i(I_i+1)\right]$$

where

$$I_f = I_i - 2$$

It is naively expected that

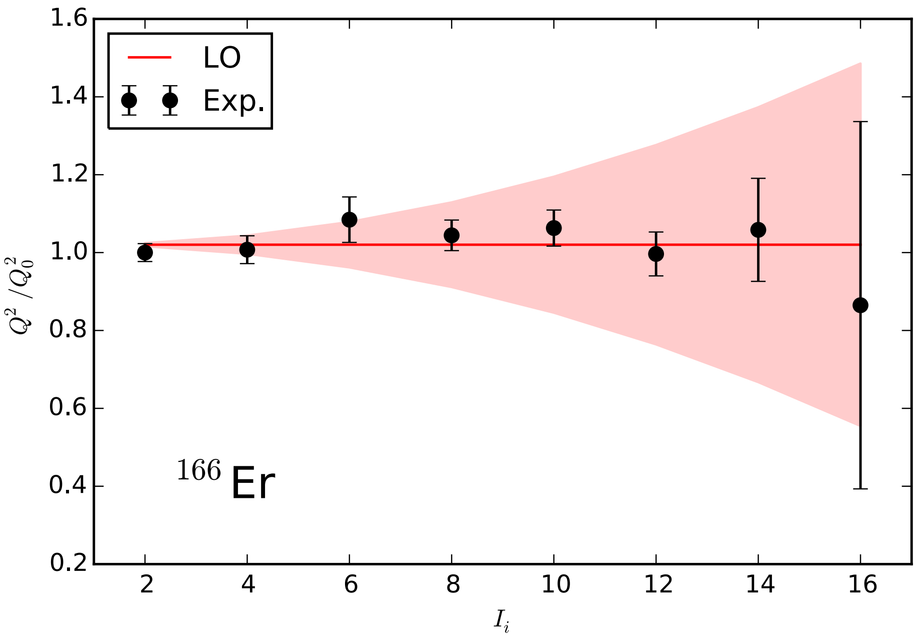
$$\frac{b}{a} \sim \left(\frac{\xi}{\omega}\right)^2 \sim \frac{C_2}{C_0^3}$$

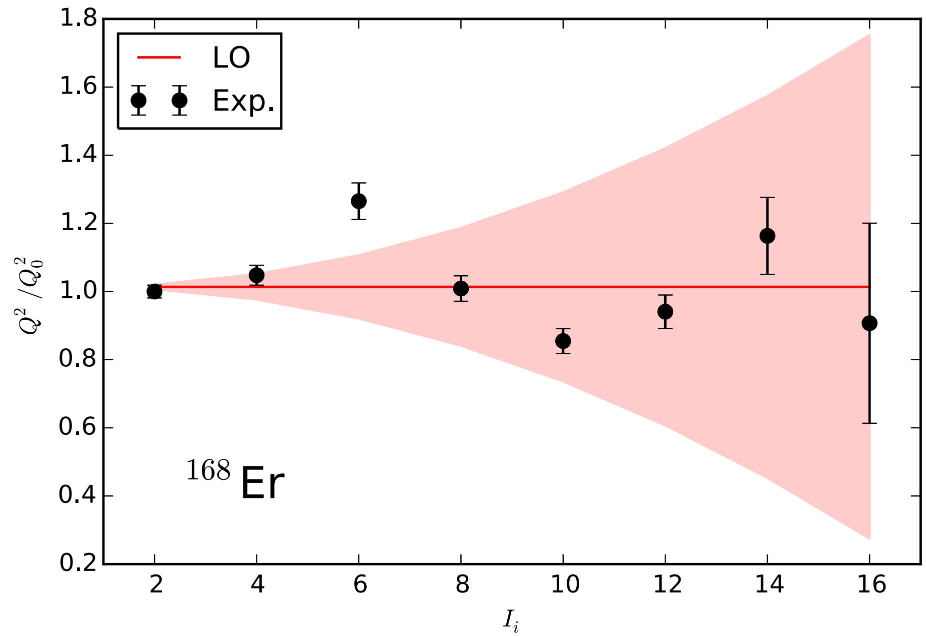
NLO LEC for B(E2) values compared against its naive estimate

System	C_2/C_0^3	b/a
N_2	0.000 006	-0.000 011
H_2	0.0015	0.0022
^{236}U	0.0011	—
^{174}Yb	0.0010	—
^{168}Er	0.0010	—
^{166}Er	0.0020	—
^{162}Dy	0.0017	—
^{154}Sm	0.0033	—
^{188}Os	0.012	0.008
^{154}Gd	0.013	0.006
^{152}Sm	0.013	0.003
^{150}Nd	0.017	0.011

Decays within the ground-state band of well-deformed nuclei

TECHNISCHE UNIVERSITÄT DARMSTADT





E2 transition moments given by

$$Q^2 \equiv \frac{B(E2)_{\rm NLO}}{\left(C_{I_i 020}^{I_f 0}\right)^2}$$

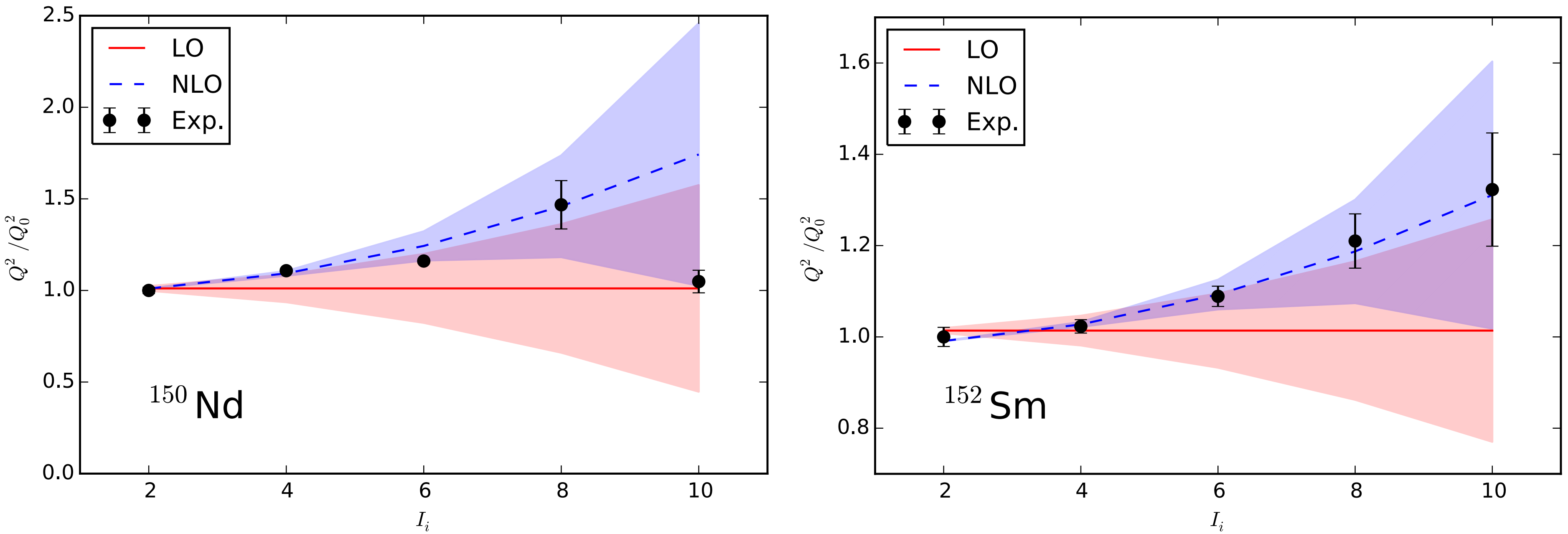
The LO description of decays within the ground-state band agrees with experimental data below the breakdown spin

Baglin; Nucl. Data Sheets **109**, 1103 (2008)

Baglin; Nucl. Data Sheets **111**, 1807 (2010)

SFB 1245

Decays within the ground-state band of transitional nuclei



E2 transition moments given by

$$Q^2 \equiv \frac{B(E2)_{\mathrm{NLO}}}{\left(C^{I_f 0}_{I_i 020}\right)^2}$$

NLO corrections are required to describe decays within the ground-state band below the breakdown spin

Krücken et al.; Phys. Rev. Lett. **88**, 232501 (2002)

Zamfir et al.; Phys. Rev. C **60**, 054312 (1999)

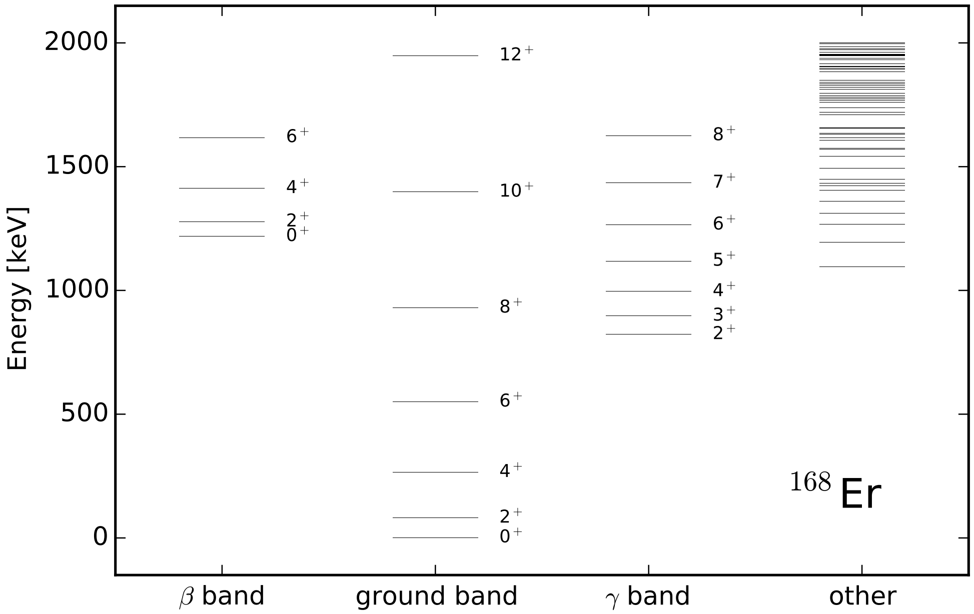
Coupling between rotations and vibrations in deformed nuclei

The vibration are described in terms of a quadrupole field. The building blocks are

$$\Psi_0 = \zeta + \psi_0 \qquad \Psi_{\pm 2} = \psi_2 e^{\pm i 2\gamma}$$

Under SO(3) rotations

$$\Psi_0 \to \Psi_0 \qquad \Psi_{\pm 2} \to e^{\pm i 2\tilde{\gamma}} \Psi_{\pm 2}$$



The NLO Hamiltonian and spectrum are

$$\hat{H}_{\rm NLO} = \frac{\hat{p_0}^2}{2} + \frac{\omega_0^2}{2}\psi_0^2 + \frac{\hat{p}_2^2}{4} + \frac{1}{4\psi_2^2}\left(\frac{\hat{p}_\gamma}{2}\right)^2 + \frac{\omega_2}{4}\psi_2^2 + \frac{1}{2C_0}\left(\hat{I}^2 - \hat{p}_\gamma^2\right)$$

$$E_{\rm NLO}(n_0, n_2, I, K) = \omega_0 n_0 + \frac{\omega_2}{2}\left(2n_2 + \frac{K}{2}\right) + \frac{I(I+1) - K^2}{2C_0}$$

LO B(E2) values for interband decays

Coupling between the building blocks and an electric field yield the E2 operator

The LO B(E2) values for interband decays are

$$B(E2, i_\beta \to f_g) = \frac{C_\beta^2}{2C_0^2\omega_0}\frac{q^2}{60}\left(C_{I_i 020}^{I_f 0}\right)^2$$

$$B(E2, i_\gamma \to f_g) = \frac{3C_\gamma^2}{2C_0^2\omega_2}\frac{q^2}{60}\left(C_{I_i 22-2}^{I_f 0}\right)^2$$

It is naively expected that

$$C_\beta \sim C_\gamma \sim \xi^{-1/2}$$

For ^{154}Sm

$$\xi^{-1/2} \approx 0.110 \text{ keV}^{-1/2}$$

$$C_\beta \approx 0.092 \text{ keV}^{-1/2} \quad C_\gamma \approx 0.181 \text{ keV}^{-1/2}$$

LO B(E2) values for interband decays in ^{154}Sm [e^2b^2].

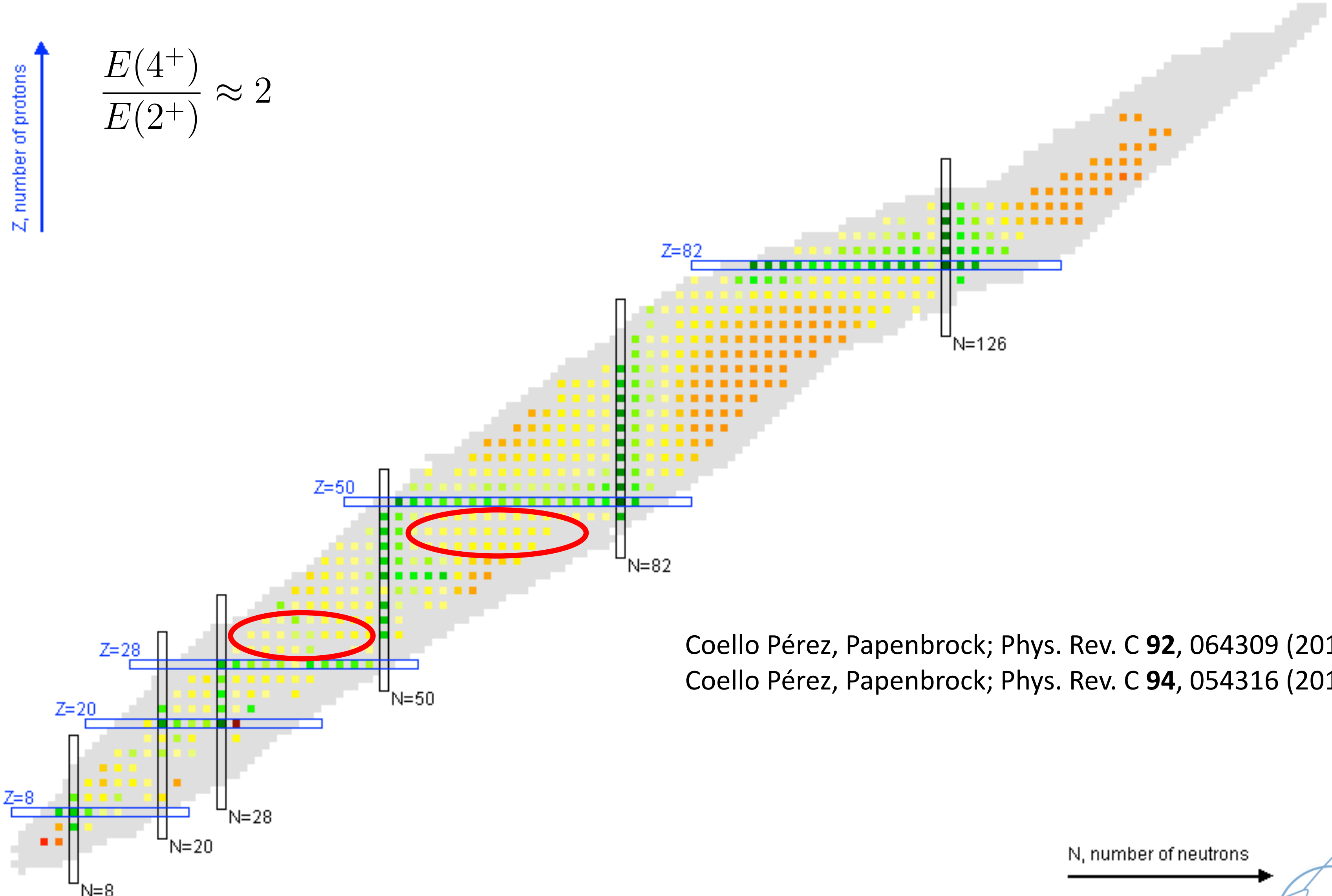
$i \to f$	$B(E2)_{\text{exp}}$	$B(E2)_{\text{ET}}$
$2_g^+ \to 0_g^+$	0.863(5)	0.863[a]
$4_g^+ \to 2_g^+$	1.201(29)	1.233(9)
$6_g^+ \to 4_g^+$	1.417(39)	1.358(23)
$8_g^+ \to 6_g^+$	1.564(83)	1.421(43)
$2_\gamma^+ \to 0_g^+$	0.0093(10)	0.0110(28)
$2_\gamma^+ \to 2_g^+$	0.0157(15)	0.0157[a]
$2_\gamma^+ \to 4_g^+$	0.0018(2)	0.0008(2)
$2_\beta^+ \to 0_g^+$	0.0016(2)	0.0025(6)
$2_\beta^+ \to 2_g^+$	0.0035(4)	0.0035[a]
$2_\beta^+ \to 4_g^+$	0.0065(7)	0.0063(16)

a) Values employed to fix the LECs
b) Experimental values from *

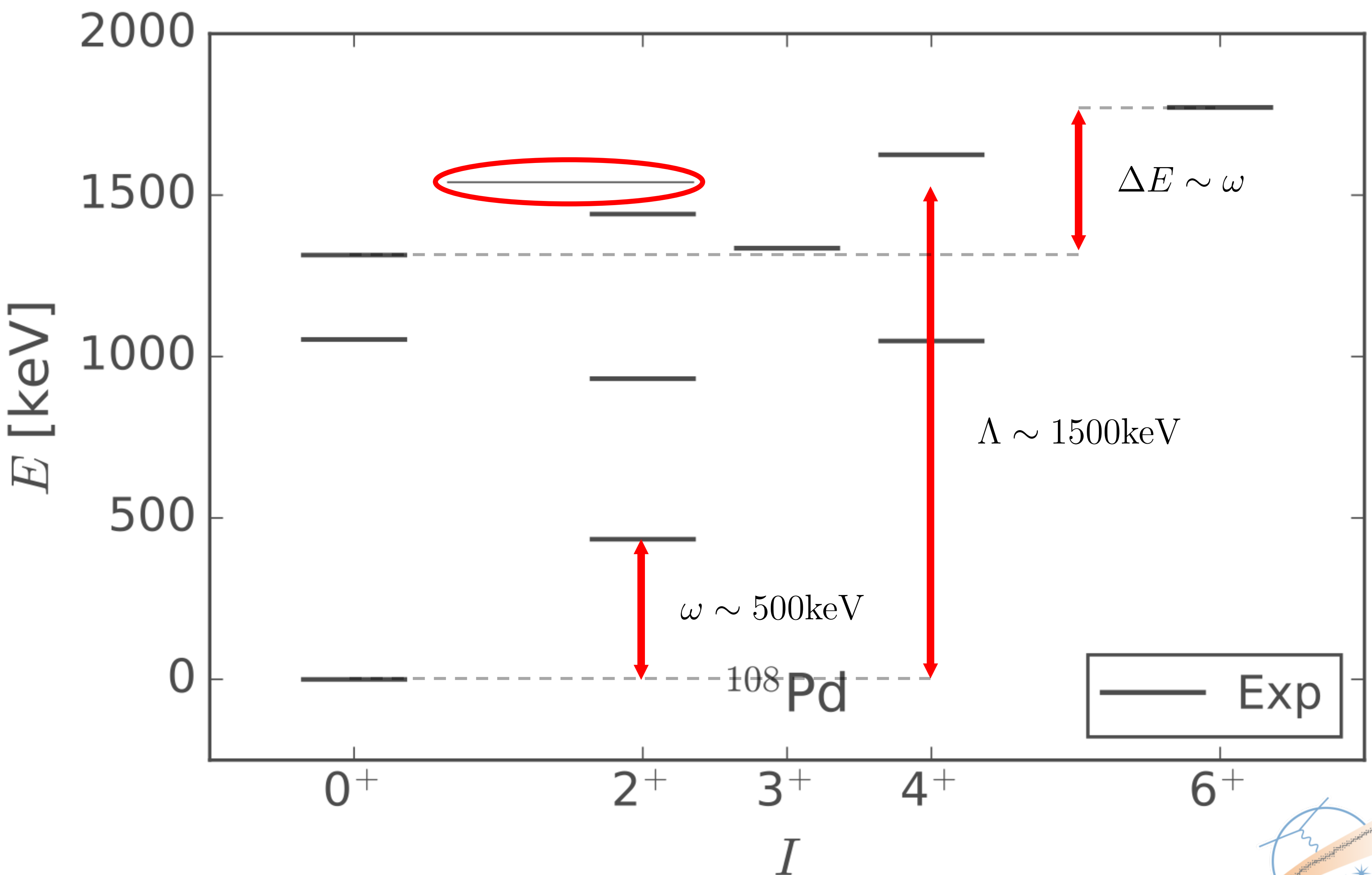
*Reich; Nuclear Data Sheets **110**, 2257 (2009)
*Möller et al.; Phys. Rev. C **86**, 031305 (2012)

Spherical nuclei

$$\frac{E(4^+)}{E(2^+)} \approx 2$$



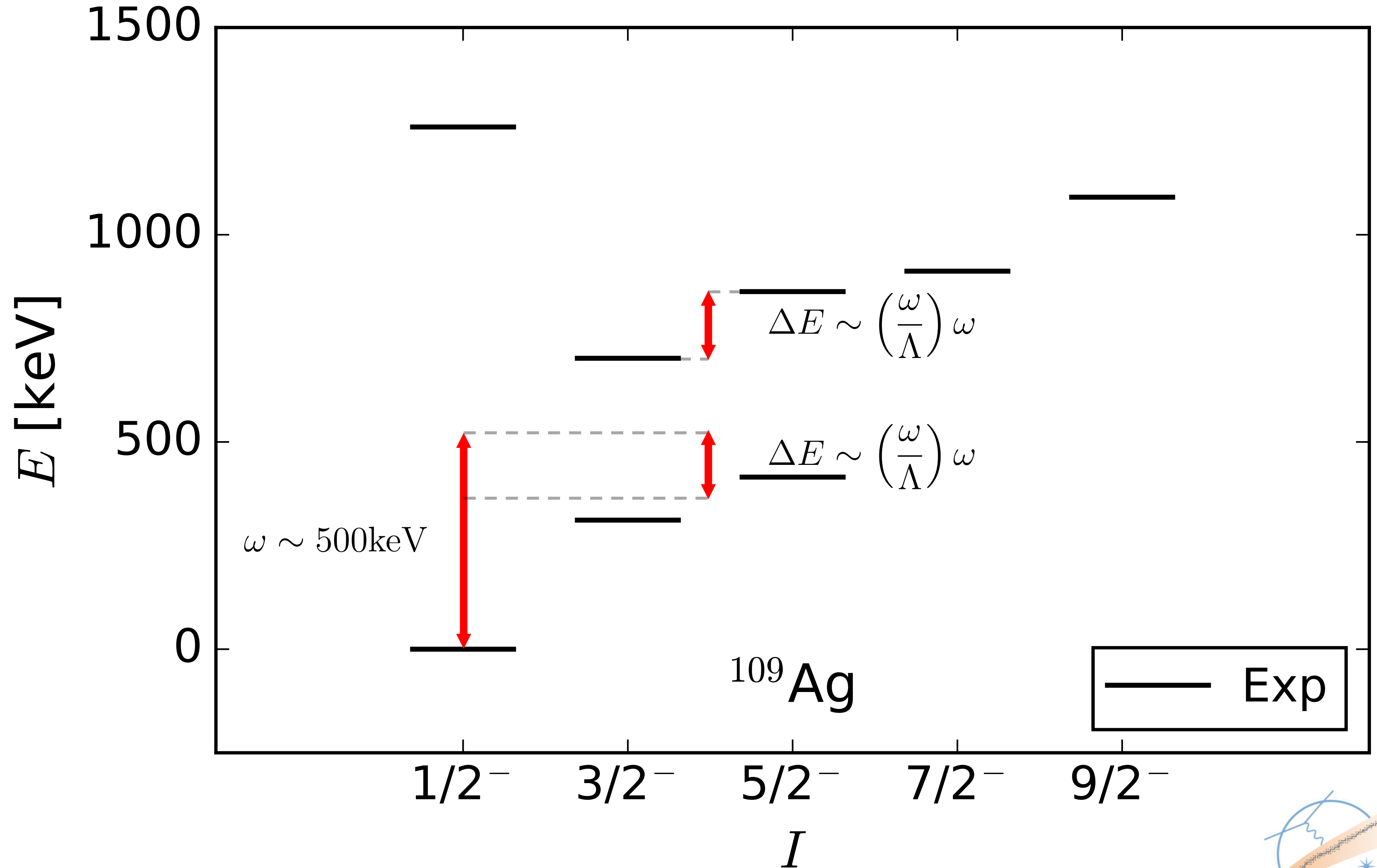
Coello Pérez, Papenbrock; Phys. Rev. C **92**, 064309 (2015)
Coello Pérez, Papenbrock; Phys. Rev. C **94**, 054316 (2016)



Blachot; Nucl. Data Sheets **91**, 135 (2000)

Relevant energy scales in odd-mass spherical nuclei with ½-ground states

TECHNISCHE UNIVERSITÄT DARMSTADT



Blachot; Nucl. Data Sheets **107**, 355 (2006)

SFB 1245

Power counting and Hamiltonian

The Hamiltonian is constructed in terms of boson quadrupole operators

$$\left[d_\mu, d_\nu^\dagger\right] = \delta_{\mu\nu}$$

and fermion operators

$$\left\{a_\mu, a_\nu^\dagger\right\} = \delta_{\mu\nu}$$

The later create and annihilate a fermion in a $j^\pi = {}^1\!/_2{}^-$ orbital

The suggested power counting based on the energy scales spherical nuclei leads to a NLO Hamiltonian with the following contributions

$$H_{\mathrm{LO}} \equiv \omega_1 \hat{N}$$

$$H_{\mathrm{NLO}} \equiv g_{Jj} \hat{\mathbf{J}} \cdot \hat{\mathbf{j}} + \omega_2 \hat{N} \hat{n}$$

$$H_{\mathrm{NNLO}} \equiv g_N \hat{N}^2 + g_v \hat{\Lambda}^2 + g_J \hat{J}^2$$

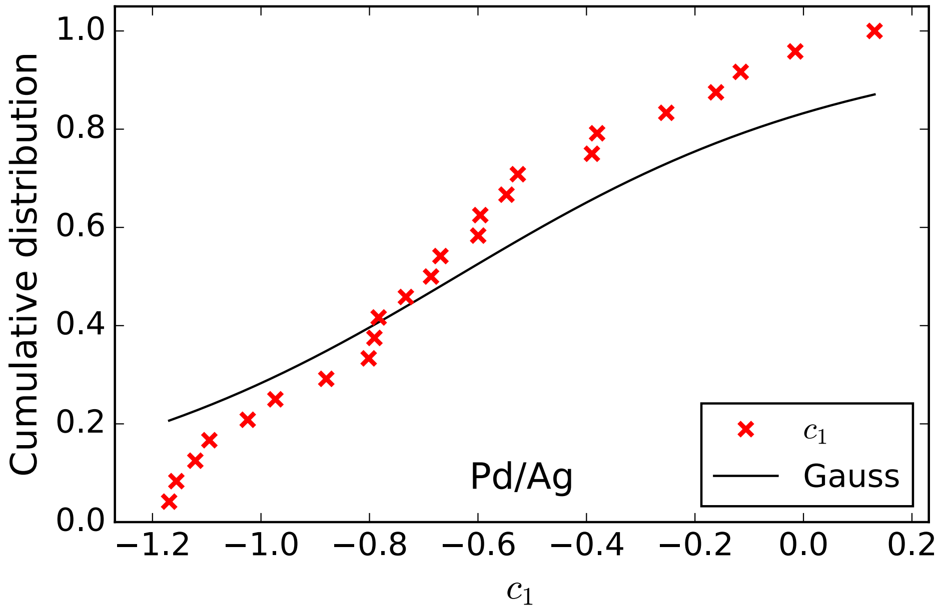
where

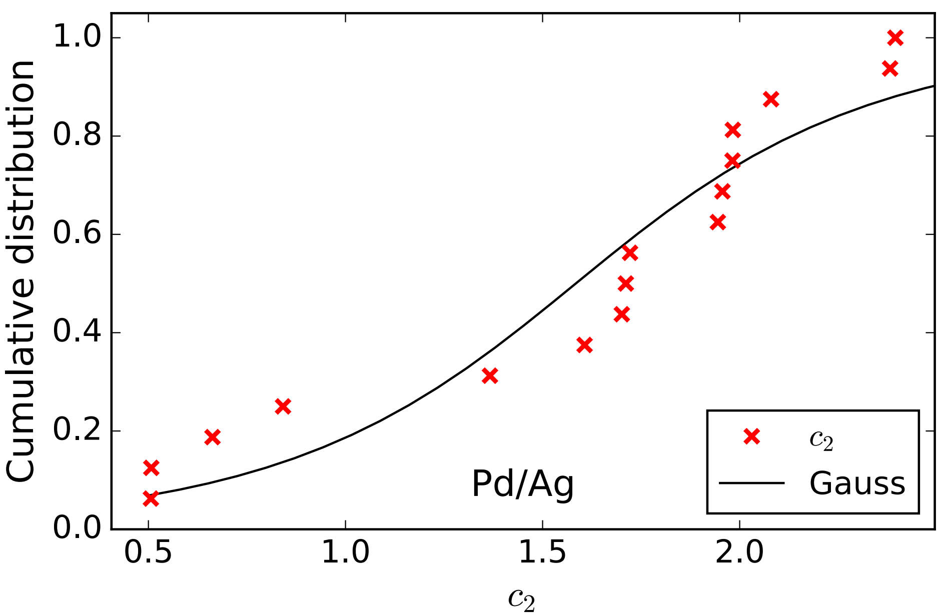
$$\hat{N} \equiv d^\dagger \cdot \tilde{d} \qquad \hat{n} \equiv a^\dagger \cdot \tilde{a}$$

$$\hat{\mathbf{J}} = \sqrt{10}\left(d^\dagger \otimes \tilde{d}\right)^{(1)} \qquad \hat{\mathbf{j}} = \frac{1}{\sqrt{2}}\left(a^\dagger \otimes \tilde{a}\right)^{(1)}$$

$$\hat{\Lambda}^2 \equiv -\left(d^\dagger \cdot d^\dagger\right)\left(\tilde{d} \cdot \tilde{d}\right) + \hat{N}^2 - 3\hat{N}$$

Testing the power counting





Observables can be written as expansions in powers of a small parameter

$$E(I^\pi) = \omega_1 \sum_i^\infty c_i(I^\pi)\varepsilon^i \qquad \varepsilon \equiv N\frac{\omega_1}{\Lambda}$$

The expansion coefficients are expected to be of order one. This expectation can be encoded into the following priors*

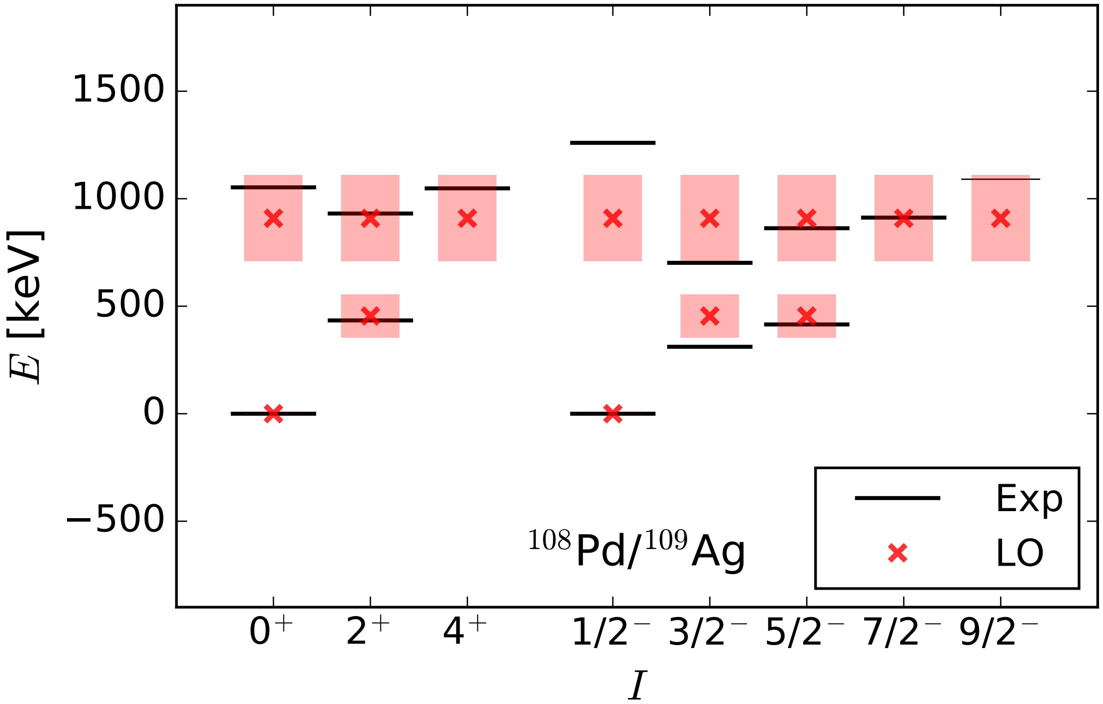
$$\mathrm{pr}^{(\mathrm{G})}(\tilde{c}_i|c) = \frac{1}{\sqrt{2\pi}sc} e^{-\frac{\tilde{c}_i^2}{2s^2c^2}}$$

$$\mathrm{pr}(c) = \frac{1}{\sqrt{2\pi}\sigma c} e^{-\frac{\log^2 c}{2\sigma^2}}$$

The cumulative distribution of the expansion coefficients agrees with the suggested power counting

Cacciari, Houdeau; Nucl. J. High Energy Phys. **09** (2011) 039
Furnstahl, et al.; J. Phys. G **42**, 034028 (2015)

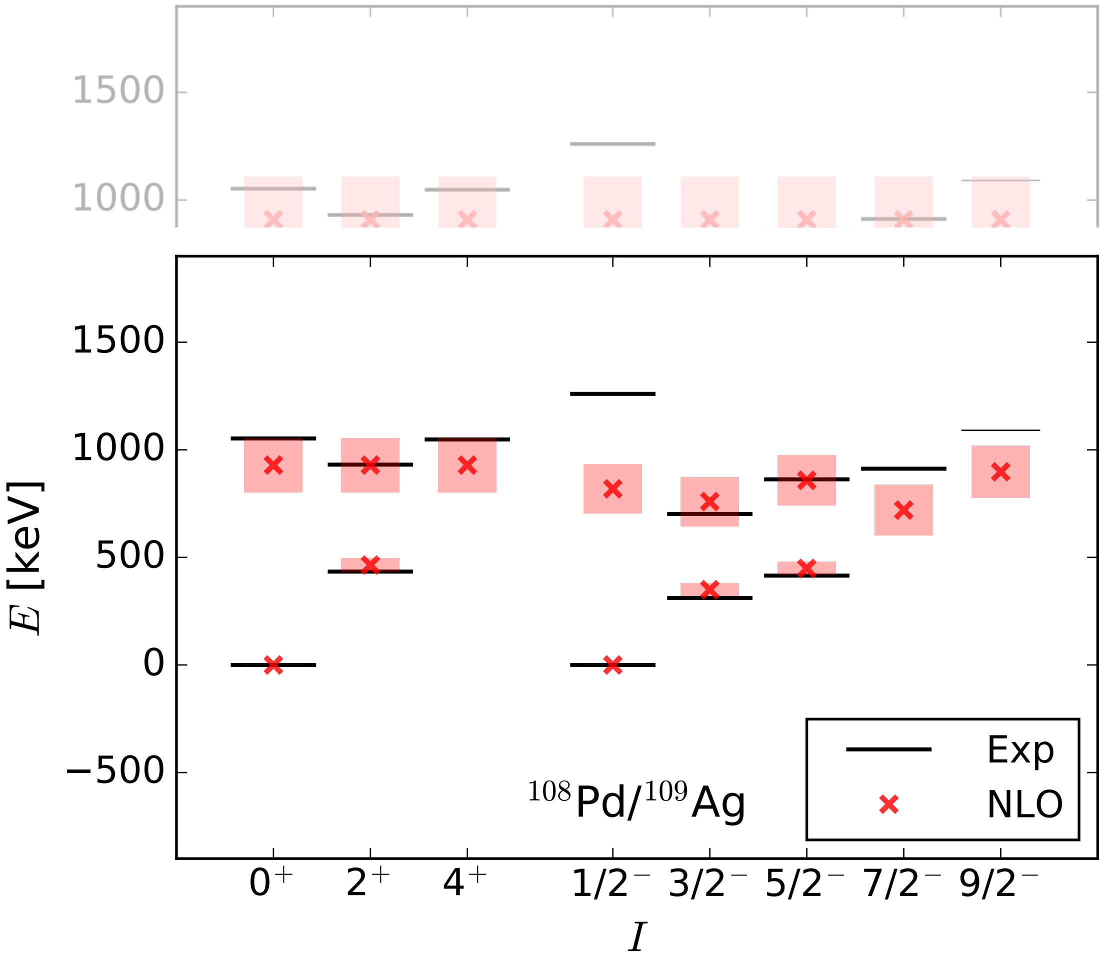
Order-by-order improvement



LO:

- One LEC
- Harmonic behavior

Order-by-order improvement



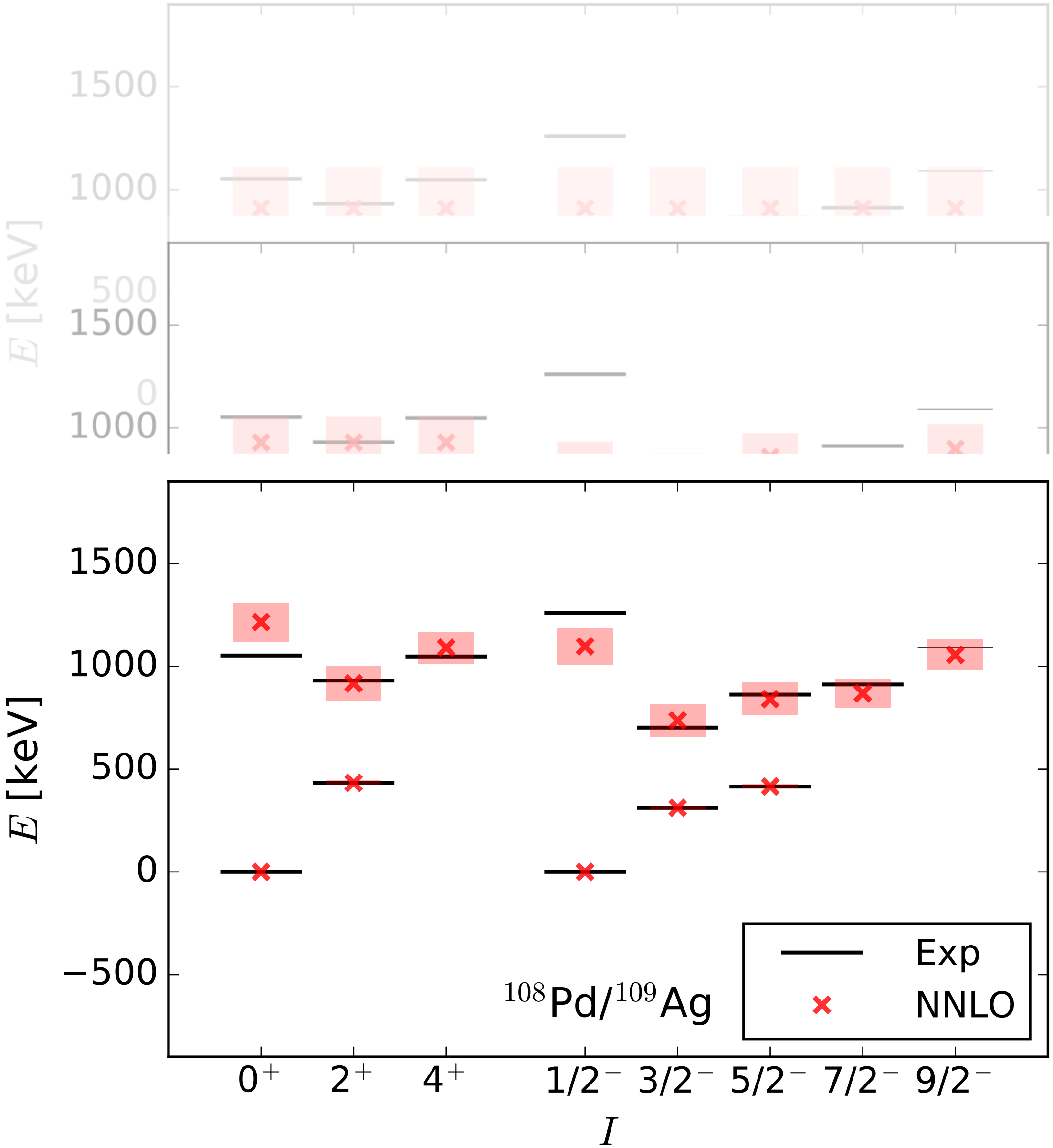
LO:

- One LEC
- Harmonic behavior

NLO:

- Two additional LECs
- Particle-core interactions

Order-by-order improvement



LO:

- One LEC
- Harmonic behavior

NLO:

- Two additional LECs
- Particle-core interactions

NNLO:

- Three additional three LECs
- Anharmonic corrections

Accuracy and precision increases order by order at the expense of reduced predictive power

LO phonon-annihilating B(E2) values

LO B(E2) values for phonon-annihilating transitions in the ^{108}Pd/^{109}Ag system [W. u.]

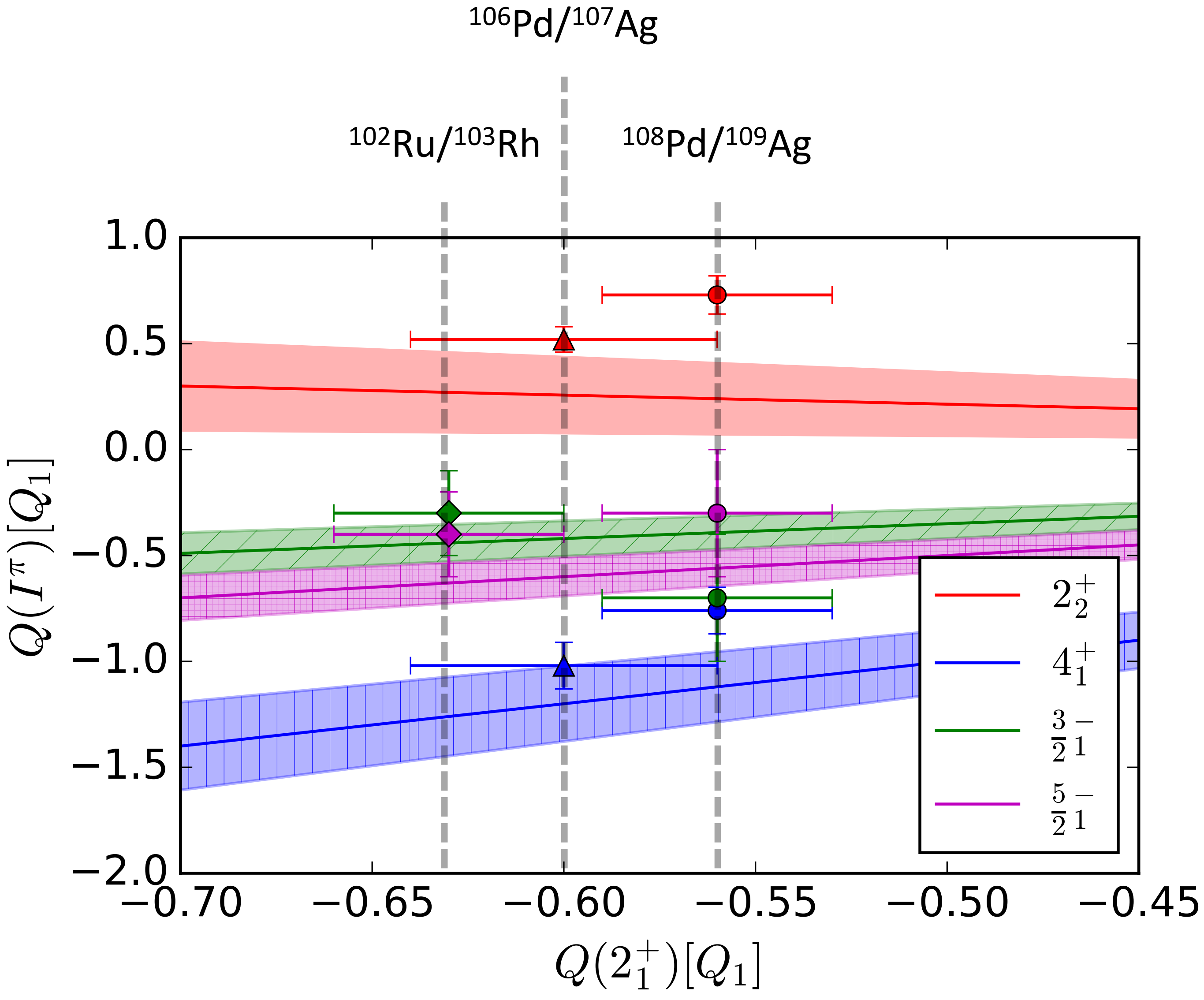
Nucleus	$I_i^\pi \to I_f^\pi$	$B(E2)_{\mathrm{exp}}$	$B(E2)_{\mathrm{EFT}}$
^{108}Pd	$2_1^+ \to 0_1^+$	49(1)	34(11)
	$0_2^+ \to 2_1^+$	52(5)	69(23)
	$2_2^+ \to 2_1^+$	71(5)	69(23)
	$4_1^+ \to 2_1^+$	73(8)	69(23)
^{109}Ag	$\frac{3}{2}_1^- \to \frac{1}{2}_1^-$	40(40)	34(11)
	$\frac{5}{2}_1^- \to \frac{1}{2}_1^-$	41(6)	34(11)
	$\frac{1}{2}_2^- \to \frac{3}{2}_1^-$		27(23)
	$\frac{1}{2}_2^- \to \frac{5}{2}_1^-$		41(23)
	$\frac{3}{2}_2^- \to \frac{3}{2}_1^-$	49(24)	47(23)
	$\frac{3}{2}_2^- \to \frac{5}{2}_1^-$		20(23)
	$\frac{5}{2}_2^- \to \frac{3}{2}_1^-$	8(4)	14(23)
	$\frac{5}{2}_2^- \to \frac{5}{2}_1^-$	10(7)	54(23)
	$\frac{7}{2}_1^- \to \frac{3}{2}_1^-$		61(23)
	$\frac{7}{2}_1^- \to \frac{5}{2}_1^-$		7(23)
	$\frac{9}{2}_1^- \to \frac{5}{2}_1^-$		68(23)

The E2 operator is constructed as the most general rank-two with positive parity

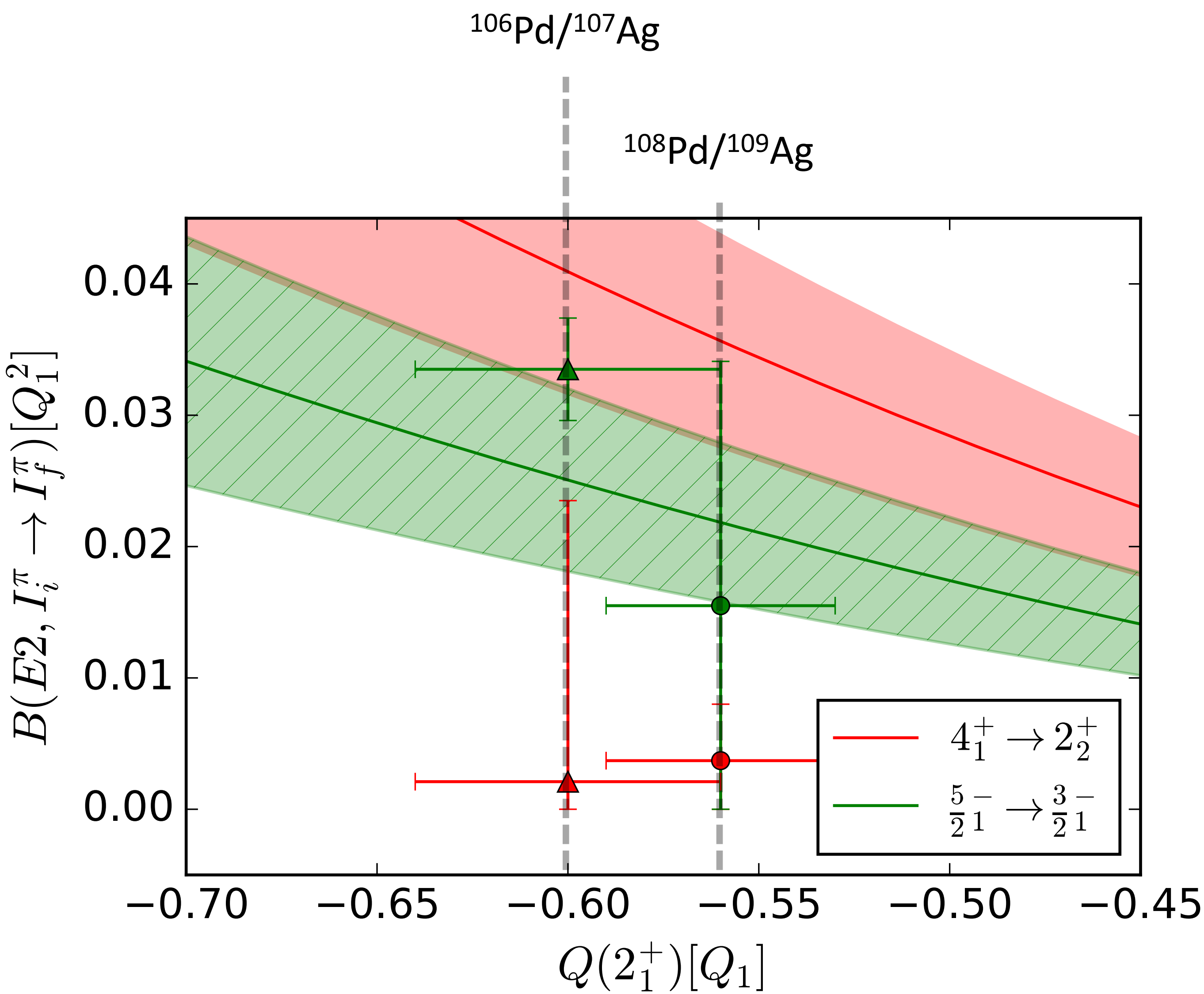
The LO B(E2) values for phonon-annihilating transitions are described in terms of one LEC

The NLO term of the E2 operator couples states with the same number of phonons. Its matrix elements enter the description of E2 static moments and B(E2) values for phonon-conserving transitions

LO relations between E2 even-even and odd-mass observables



LO relations between even-even and odd-mass E2 observables



LO M1 static moment in Pd/Ag systems [μ_N]

Nucleus	I_i^π	$\mu_{\rm exp}(I_i^\pi)$	$\mu_{\rm EFT}(I_i^\pi)$
^{106}Pd	2_1^+	$0.79(2)^*$	0.79(5)
	2_2^+	0.71(10)	0.79(10)
	4_1^+	1.8(4)	1.58(8)
^{107}Ag	$\frac{1}{2}_1^-$	-0.11^*	-0.11
	$\frac{3}{2}_1^-$	0.98(9)	0.78(5)
	$\frac{5}{2}_1^-$	1.02(9)	0.68(4)
	$\frac{7}{2}_1^-$		1.6(1)
	$\frac{9}{2}_1^-$		1.5(1)
^{108}Pd	2_1^+	$0.71(2)^*$	0.71(4)
	2_2^+		0.71(9)
	4_1^+		1.42(7)
^{109}Ag	$\frac{1}{2}_1^-$	-0.13^*	-0.13
	$\frac{3}{2}_1^-$	1.10(10)	0.72(5)
	$\frac{5}{2}_1^-$	0.85(8)	0.58(4)
	$\frac{7}{2}_1^-$		1.5(1)
	$\frac{9}{2}_1^-$		1.3(1)

LO B(M1) values for phonon-conserving transitions in Ag nuclei [W. u.]

Nucleus	$I_i^\pi \to I_f^\pi$	$B(M1)_{\rm exp}$	$B(M1)_{\rm EFT}$
^{107}Ag	$\frac{5}{2}_1^- \to \frac{3}{2}_1^-$	0.033(4)	0.036(2)
	$\frac{5}{2}_2^- \to \frac{3}{2}_2^-$		0.036(4)
	$\frac{9}{2}_1^- \to \frac{7}{2}_1^-$		0.040(2)
^{109}Ag	$\frac{5}{2}_1^- \to \frac{3}{2}_1^-$	0.043(7)	0.036(2)
	$\frac{5}{2}_2^- \to \frac{3}{2}_2^-$		0.036(3)
	$\frac{9}{2}_1^- \to \frac{7}{2}_1^-$		0.040(2)

These observables are described in terms of two LECs fixed by the static moments of low-lying states

B(M1) values for phonon-conserving transitions are predictions within the EFT

De Frenne; Nucl. Data Sheets **109**, 943 (2008)
Blachot; Nucl. Data Sheets **109**, 1383 (2008)

LO phonon-annihilating B(M1) values

TECHNISCHE UNIVERSITÄT DARMSTADT

LO B(M1) values for phonon-conserving transitions in odd-mass nuclei [W. u.]

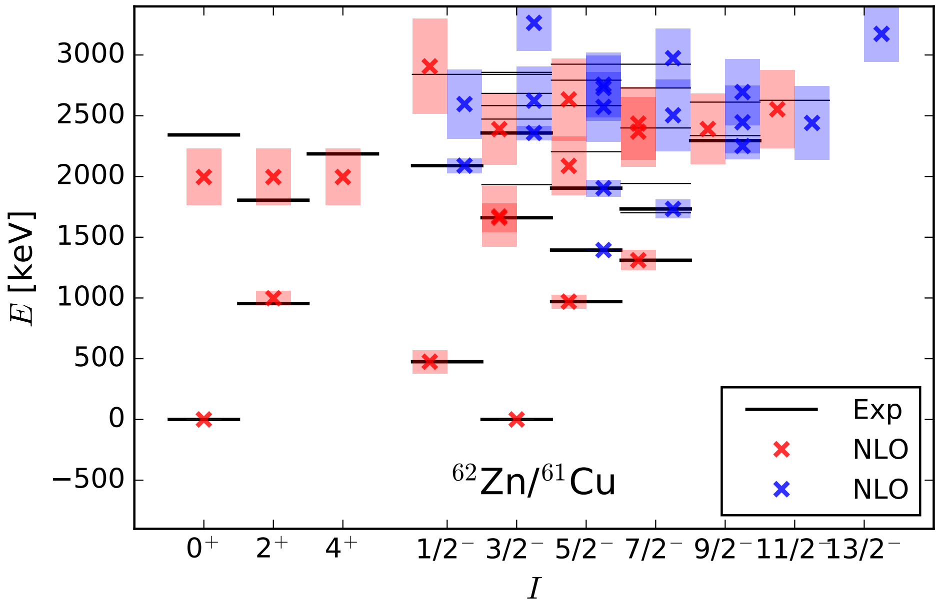
Nucleus	$I_i^\pi \to I_f^\pi$	$B(M1)_{\mathrm{exp}}$	$B(M1)_{\mathrm{EFT}}$
^{103}Rh	$\frac{3}{2}^-_1 \to \frac{1}{2}^-_1$	0.12(1)	0.10(2)
	$\frac{1}{2}^-_2 \to \frac{3}{2}^-_1$		0.08(8)
	$\frac{3}{2}^-_2 \to \frac{3}{2}^-_1$		0.10(4)
	$\frac{3}{2}^-_2 \to \frac{5}{2}^-_1$		0.03(4)
	$\frac{5}{2}^-_2 \to \frac{3}{2}^-_1$	0.014(2)	0.018(28)
	$\frac{5}{2}^-_2 \to \frac{5}{2}^-_1$	0.020(3)	0.023(28)
	$\frac{7}{2}^-_1 \to \frac{5}{2}^-_1$		0.17(2)
^{109}Ag	$\frac{3}{2}^-_1 \to \frac{1}{2}^-_1$	0.117(15)	0.122(27)
	$\frac{1}{2}^-_2 \to \frac{3}{2}^-_1$		0.10(11)
	$\frac{3}{2}^-_2 \to \frac{3}{2}^-_1$	0.16(7)	0.07(5)
	$\frac{3}{2}^-_2 \to \frac{5}{2}^-_1$		0.05(5)
	$\frac{5}{2}^-_2 \to \frac{3}{2}^-_1$	0.036(16)	0.033(36)
	$\frac{5}{2}^-_2 \to \frac{5}{2}^-_1$	0.10(4)	0.07(4)
	$\frac{7}{2}^-_1 \to \frac{5}{2}^-_1$		0.22(3)

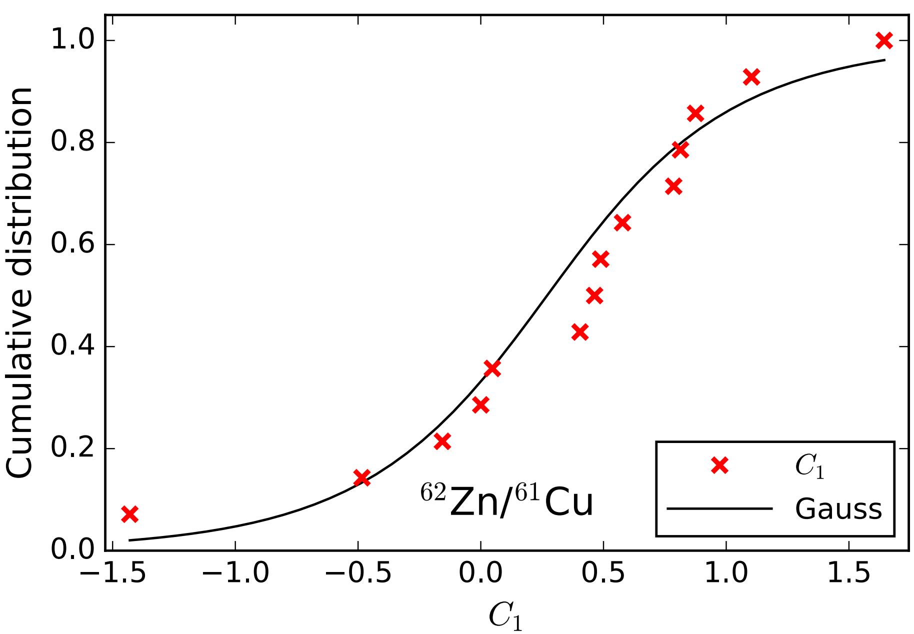
The LO B(M1) values for phonon-annihilating transitions are described in terms of two LECs

The LO B(M1) values are in agreement with the scarce experimental data

De Frenne; Nucl. Data Sheets **110**, 2081 (2009)

SFB 1245





LO B(E2) values in the ^{62}Zn/^{61}Cu system [W. u.]

Nucleus	$I\ N\ J$	$\rightarrow$	$I\ N\ J$	$B(E2)_{\mathrm{exp}}$	$B(E2)_{\mathrm{EFT}}$
^{62}Zn	2 1 2	$\rightarrow$	0 0 0	17(1)	13(4)
	0 2 0	$\rightarrow$	2 1 2		25(8)
	2 2 2	$\rightarrow$	2 1 2	18(4)	25(8)
	4 2 4	$\rightarrow$	2 1 2	26(12)	25(8)
^{61}Cu	$\frac{1}{2}$ 1 2	$\rightarrow$	$\frac{3}{2}$ 0 0		13(4)
	$\frac{3}{2}$ 1 2	$\rightarrow$	$\frac{3}{2}$ 0 0	1(1)	13(4)
		$\rightarrow$	$\frac{5}{2}$ 1 2	0(4)	0(1)
	$\frac{5}{2}$ 1 2	$\rightarrow$	$\frac{3}{2}$ 0 0	7(2)	13(4)
		$\rightarrow$	$\frac{1}{2}$ 1 2	17(7)	9(1)
	$\frac{7}{2}$ 1 2	$\rightarrow$	$\frac{3}{2}$ 0 0	18(3)	13(4)
		$\rightarrow$	$\frac{5}{2}$ 1 2	0(1)	3(1)
	$\frac{3}{2}$ 2 0	$\rightarrow$	$\frac{7}{2}$ 1 2		10(8)
		$\rightarrow$	$\frac{1}{2}$ 1 2	0(0)	2(8)
	$\frac{1}{2}$ 2 2	$\rightarrow$	$\frac{3}{2}$ 1 2		18(8)
		$\rightarrow$	$\frac{1}{2}$ 1 2	1(1)	0(8)
	$\frac{3}{2}$ 2 2	$\rightarrow$	$\frac{5}{2}$ 1 2	1(1) or 2(6)	14(8)
	$\frac{5}{2}$ 2 2	$\rightarrow$	$\frac{7}{2}$ 1 2		11(8)
	$\frac{7}{2}$ 2 2	$\rightarrow$	$\frac{7}{2}$ 1 2	24(12)	16(8)
		$\rightarrow$	$\frac{5}{2}$ 1 2	1(0)	8(8)
	$\frac{5}{2}$ 2 4	$\rightarrow$	$\frac{1}{2}$ 1 2	3(1)	15(8)
		$\rightarrow$	$\frac{7}{2}$ 1 2	0(0)	0(8)
		$\rightarrow$	$\frac{5}{2}$ 1 2	0(0)	1(8)
	$\frac{7}{2}$ 2 4	$\rightarrow$	$\frac{3}{2}$ 1 2		16(8)
	$\frac{9}{2}$ 2 4	$\rightarrow$	$\frac{5}{2}$ 1 2	15(2)	20(8)
		$\rightarrow$	$\frac{7}{2}$ 1 2	1(1)	5(8)
	$\frac{11}{2}$ 2 4	$\rightarrow$	$\frac{7}{2}$ 1 2	<27	25(8)

The spectra and electromagnetic properties of heavy nuclei were studied within an effective field theory approach

The systematic construction of the operators allows for the estimation of:

- the scale of the LECs that must be fitted to experimental data and
- theoretical uncertainties.

Deformed nuclei

Spectra and B(E2) values for decays within the ground-state rotational band are consistent with data below the breakdown scale even in transitional nuclei

B(E2) values for decays between states in different bands are reproduced for LECs of natural size

Spherical nuclei

Anharmonicities in the spectra and static E2 moments in these systems scale as expected based on the power counting

E2 and M1 observables are reproduced within the EFT

Relations between observables in the even-even and odd-mass systems are fulfilled within theoretical uncertainties

Thanks