

# Few-Neutron Resonances From Chiral Effective Field Theory

1<sup>st</sup> Workshop of the SFB 1245



TECHNISCHE  
UNIVERSITÄT  
DARMSTADT



Hessisches Kompetenzzentrum  
für Hochleistungsrechnen

Joel E. Lynn in collaboration with  
S. Gandolfi, H.-W. Hammer, P. Klos, and A. Schwenk

November 23, 2016

# A Recent History

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2002

2003

2005

Experiment  
Theory

F. M. Marqués et al. Phys. Rev. C **65**, 052501.

Experimental claim of a bound tetraneutron from detection of neutron clusters from  $^{14}\text{Be}$  fragmentation.

~6 events!

2002

2003

2005

Experiment  
Theory

# Experiment Theory

2002      2003      2005

1) C. A. Bertulani and V. Zelevinsky, J. Phys. G **29**, 2431 (2003),  
2) N. K. Timofeyuk J. Phys. G **29**, L9 (2003).  
No bound tetraneutron using 1) a dineutron-dineutron molecule  
model and 2) a toy *NN* potential.

# Experiment Theory

2002

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- 1) C. A. Bertulani and V. Zelevinsky, J. Phys. G **29**, 2431 (2003),
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No bound tetraneutron using 1) a dineutron-dineutron molecule model and 2) a toy  $NN$  potential.

S. C. Pieper Phys. Rev. Lett. **90**, 252501.

Modern nuclear Hamiltonians cannot tolerate a bound tetraneutron.

But...

*"This suggests that there might be a  ${}^4n$  resonance near 2 MeV"*

# Experiment Theory

2002      2003      2005

R. Lazauskas and J. Carbonell, *Phys. Rev. C* **72**, 034003.  
Complex scaling w/ Reid 93 potential (*NN* only!)  
Low-lying  $^4n$  resonance not seen.

2002

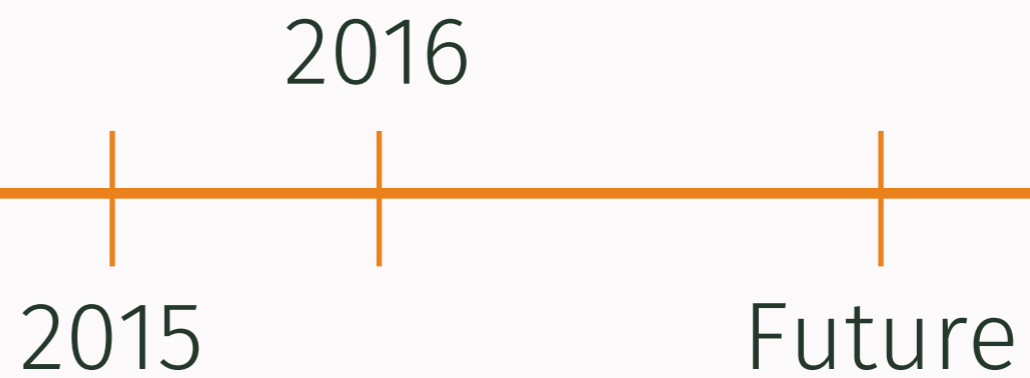
2003

2005

Experiment  
Theory



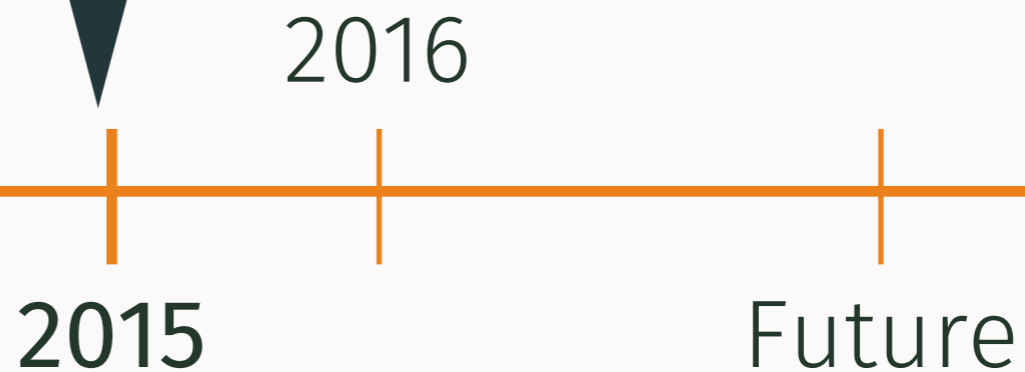
Experiment  
Theory



K. Kisamori et al., Phys. Rev. Lett. **116**, 044006.

A recent double-charge-exchange reaction  ${}^8_2\text{He} + {}^4_2\text{He} \rightarrow {}^8_4\text{Be} + 4n$  measurement at the RIKEN radioactive ion beam factory (RIBF) suggests a tetraneutron resonance at  $0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst})$  MeV.

Experiment  
Theory



# Experiment Theory



E. Hiyama, R. Lazauskas, J. Carbonell, and M. Kamimura, *Phys. Rev. C* **93**, 044004.

Complex scaling w/ $AV8'$  potential + toy  $T = 3/2$   $3N$  interaction. Low-lying  ${}^4n$  resonance only possible if other well-known resonance structure in light nuclei are strongly perturbed.

# Experiment Theory



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Complex scaling w/ $AV8'$  potential + toy  $T = 3/2$   $3N$  interaction. Low-lying  ${}^4n$  resonance only possible if other well-known resonance structure in light nuclei are strongly perturbed.

A. M. Shirokov, G. Papadimitriou, A. I. Mazur, R. Roth, J. P. Vary, *Phys. Rev. Lett.* **117**, 1825022.

No-Core Shell Model + Single-State Harmonic Oscillator Representation of Scattering equations. Compelling confirmation of a  ${}^4n$  resonance at 0.8 MeV with **JISP**  $NN$  interaction.

T. Aumann, D. Rossi, S. Shimoura, S. Paschalis et al., RIBF Experimental Proposal **SFB 1245 A06**

NP1406-SAMURAI19.



Experiment  
Theory

2015

2016

Future

T. Aumann, D. Rossi, S. Shimoura, S. Paschalis et al., RIBF Experimental Proposal [SFB 1245 A06](#)

NP1406-SAMURAI19.  
 ${}^8_2\text{He}(p, p\alpha)^4n$

K. Kisamori et al., RIKEN-RIBF proposal

“Many-neutron systems: search for superheavy  ${}^7\text{H}$  and its tetraneutron decay,” NP-1512-SAMURAI34.

Experiment  
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T. Aumann, D. Rossi, S. Shimoura, S. Paschalis et al., RIBF Experimental Proposal **SFB 1245 A06**

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S. Shimoura et al., RIKEN-RIBF proposal “Tetraneutron resonance produced by exothermic double-charge exchange reaction,” NP1512-SHARAQ10.

Experiment  
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NP1406-SAMURAI19.



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What's still missing?

An *ab initio* calculation with chiral *NN* and *3N* interactions.

Initial efforts using Quantum Monte Carlo calculations with chiral interactions.  
(This talk!)



# Outline

- Quantum Monte Carlo Methods
- Chiral EFT
  - Three-Nucleon Interactions
  - Fitting  $c_D$  and  $c_E$
- Few-body resonances

# Quantum Monte Carlo (QMC) Methods

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QMC methods in two lines:

$$H |\Psi\rangle = E |\Psi\rangle$$
$$\lim_{T \rightarrow \infty} e^{-HT} |\Psi_T\rangle \rightarrow |\Psi_0\rangle$$

QMC methods in more than two lines:

J. Carlson et al, RMP **87**, 1067 (2015).

# QMC Methods - Variational Monte Carlo (VMC) Method

1. Guess a trial wave function  $\Psi_T$  and generate a random position:  $\mathbf{R} = \mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_A$ .
2. Use the Metropolis algorithm to generate new positions  $\mathbf{R}'$  based on the probability  $P = \frac{|\Psi_T(\mathbf{R}')|^2}{|\Psi_T(\mathbf{R})|^2}$ .  
(Yields a set of “walkers” distributed according to  $|\Psi_T|^2$ ).
3. Invoke the variational principle:  $E_T = \frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} > E_0$ .

# QMC Methods - Diffusion Monte Carlo Method

- The wave function is imperfect:  $|\Psi_T\rangle = \sum_{i=0}^{\infty} \alpha_i |\Psi_i\rangle$  .
- Propagate in imaginary time to project out the ground state  $|\Psi_0\rangle$  .

$$\begin{aligned} |\Psi(\tau)\rangle &= e^{-(H-E_T)\tau} |\Psi_T\rangle \\ &= e^{-(E_0-E_T)\tau} \left[ \alpha_0 |\Psi_0\rangle + \sum_{i \neq 0} \alpha_i e^{-(E_i-E_0)\tau} |\Psi_i\rangle \right]. \end{aligned}$$

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$$|\Psi(\tau)\rangle \xrightarrow{\tau \rightarrow \infty} |\Psi_0\rangle.$$

# The Hamiltonian

Of course, the nuclear Hamiltonian is complicated.

$$H = \sum_{i=1}^A \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i<j}^A V_{ij} + \sum_{i<j<k}^A V_{ijk} + \dots$$

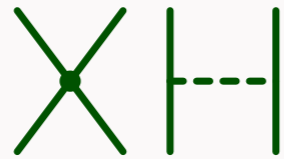
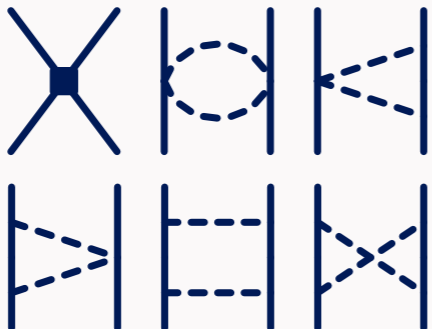
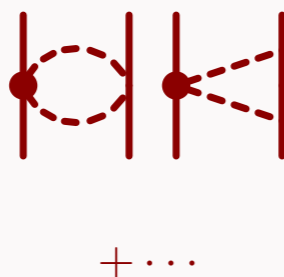
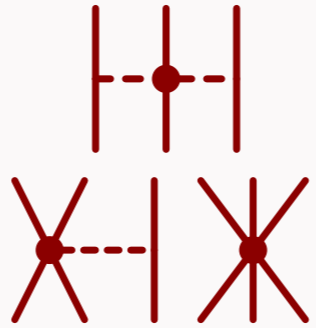

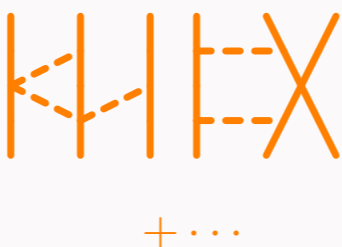
Where should it come from?

# Chiral EFT

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# Chiral EFT

		NN	NNN
LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^0$		-
NLO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^2$		-
N <sup>2</sup> LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^3$	 + ...	
N <sup>3</sup> LO	$\mathcal{O}\left(\frac{Q}{\Lambda_b}\right)^4$	 + ...	 + ...

- Chiral EFT: Expand in powers of  $Q/\Lambda_b$ .  
 $Q \sim m_\pi \sim 100 \text{ MeV}$   
 $\Lambda_b \sim 500 \text{ MeV}$
- Long-range physics:  $\pi$  exchanges.
- Short-range physics: Contacts  $\times$  LECs.
- Many-body forces & currents enter systematically.

Local construction possible<sup>1</sup> up to N<sup>2</sup>LO.

Definitions.

$$\mathbf{q} = \mathbf{p} - \mathbf{p}', \quad \mathbf{k} = \mathbf{p} + \mathbf{p}'$$

Regulator:

$$f(p, p') = e^{-(p/\Lambda)^n} e^{-(p'/\Lambda)^n}$$

Contacts:

$$\propto \mathbf{q} \text{ and } \mathbf{k}$$

<sup>1</sup>A. Gezerlis et al, PRL **111** 032501 (2013); JEL et al, PRL **113** 192501 (2014); A. Gezerlis et al, PRC **90** 054323 (2014)

# Chiral EFT

Local construction possible<sup>1</sup> up to N<sup>2</sup>LO.

Definitions.

$$\mathbf{q} = \mathbf{p} - \mathbf{p}', \mathbf{k} = \mathbf{p} + \mathbf{p}'$$

Regulator:

~~$$f(\mathbf{p}, \mathbf{p}') = e^{-(\mathbf{p}/\Lambda)^n} e^{-(\mathbf{p}'/\Lambda)^n}$$~~

$$\rightarrow f_{\text{long}}(r) = 1 - e^{-(r/R_0)^4} : R_0 = 1.0, 1.1, 1.2 \text{ fm.}$$

Contacts:

~~$$\propto \mathbf{q} \text{ and } \mathbf{k}$$~~

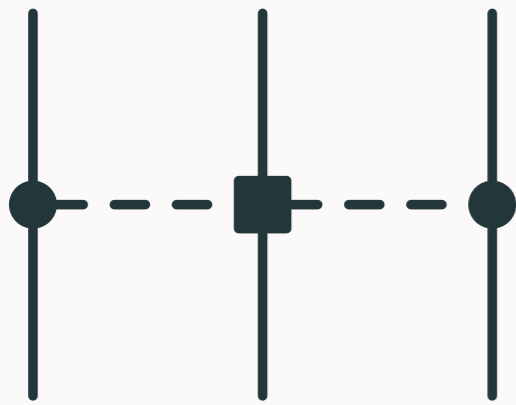
→ Choose contacts  $\propto \mathbf{q}$  (As much as possible!)

<sup>1</sup>A. Gezerlis et al, PRL **111** 032501 (2013); JEL et al, PRL **113** 192501 (2014); A. Gezerlis et al, PRC **90** 054323 (2014)

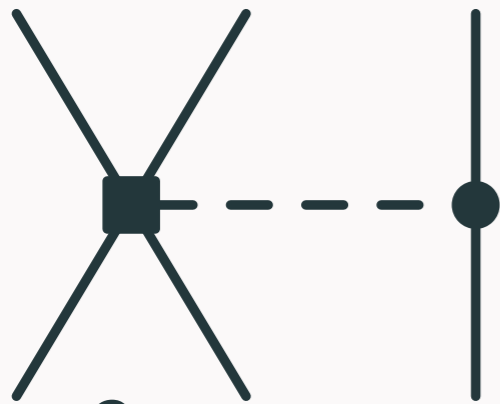
# Three-Nucleon Interactions

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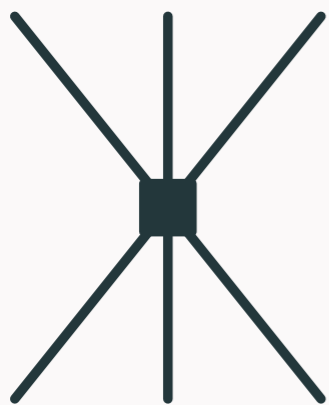
# Three-Nucleon Interaction



$C_1, C_3, C_4$

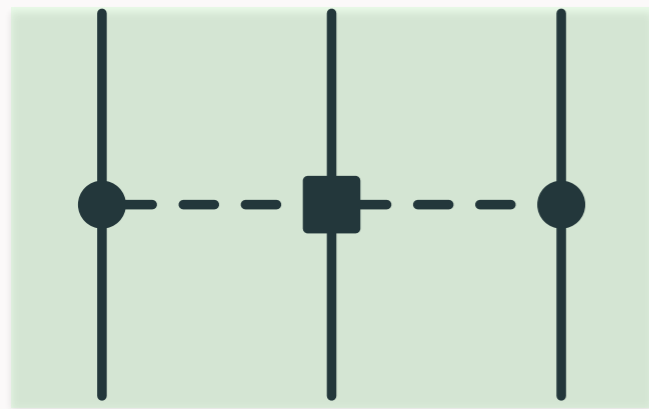


$C_D$



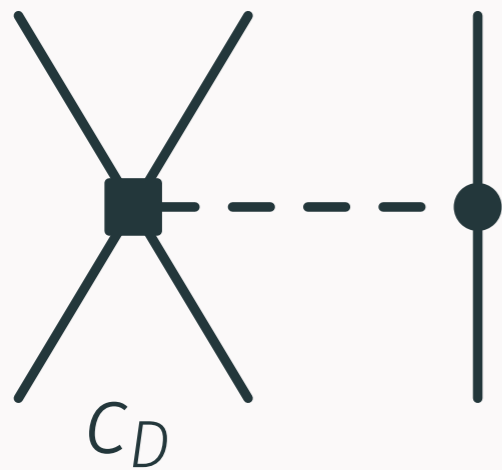
$C_E$

# Three-Nucleon Interaction



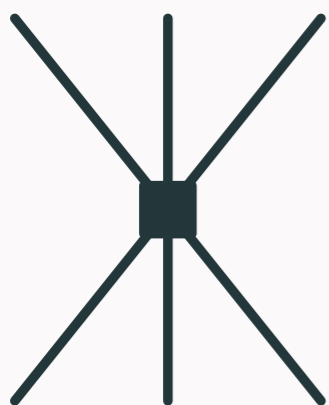
$C_1, C_3, C_4$

$$\mathcal{F} \left\{ \begin{array}{c} \bullet \text{---} \blacksquare \text{---} \bullet \\ | \quad | \quad | \\ C_1 \end{array} \right\} \rightarrow \sim \text{Tucson-Melbourne } a' \text{ Term}$$



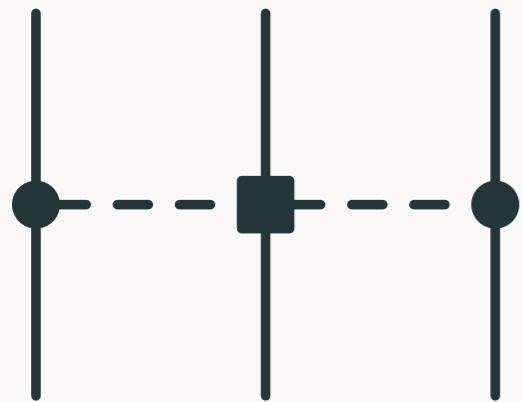
$C_D$

$$\mathcal{F} \left\{ \begin{array}{c} \bullet \text{---} \blacksquare \text{---} \bullet \\ | \quad | \quad | \\ C_3, C_4 \end{array} \right\} \rightarrow \sim \text{Fujita-Miyazawa}$$



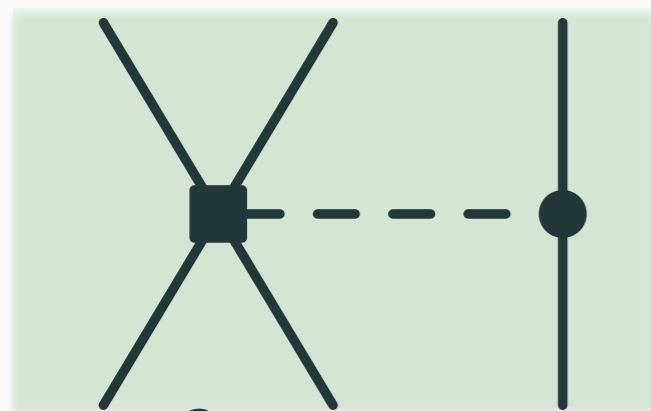
$C_E$

# Three-Nucleon Interaction



$C_1, C_3, C_4$

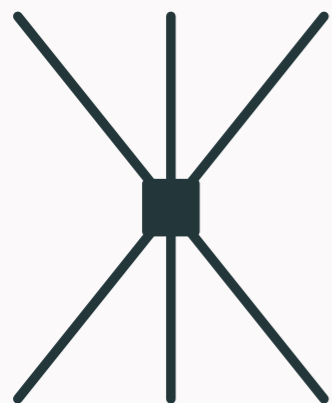
$$\mathcal{F} \left\{ \begin{array}{c} \text{---} \bullet \text{---} \blacksquare \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{---} \end{array} \right\} \rightarrow \sim \text{Tucson-Melbourne } a' \text{ Term}$$



$C_D$

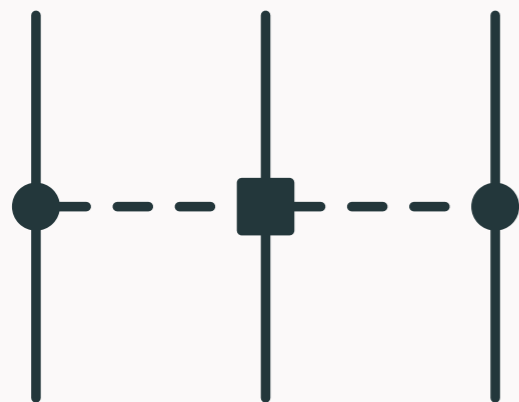
$$\mathcal{F} \left\{ \begin{array}{c} \text{---} \bullet \text{---} \blacksquare \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{---} \end{array} \right\} \rightarrow \sim \text{Fujita-Miyazawa}$$

$$\mathcal{F} \left\{ \begin{array}{c} \diagdown \quad \diagup \\ \blacksquare \text{---} \bullet \text{---} \\ \diagup \quad \diagdown \\ | \quad | \\ \text{---} \end{array} \right\} \rightarrow 1\pi\text{-Exchange} + \text{Contact}$$



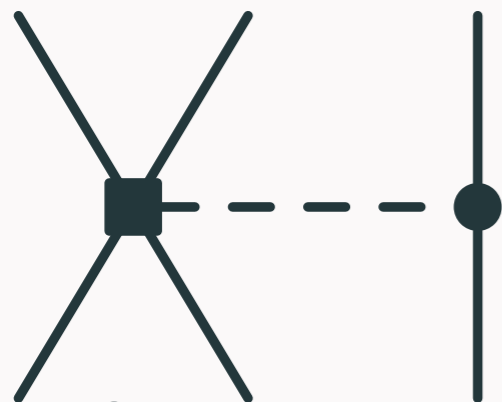
$C_E$

# Three-Nucleon Interaction



$C_1, C_3, C_4$

$$\mathcal{F} \left\{ \begin{array}{c} \text{---} \bullet \text{---} \blacksquare \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{---} \end{array} \right\} \rightarrow \sim \text{Tucson-Melbourne } a' \text{ Term}$$



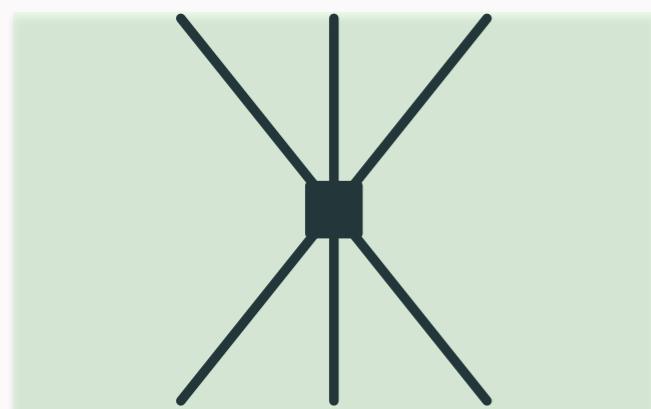
$C_D$

$$\mathcal{F} \left\{ \begin{array}{c} \text{---} \bullet \text{---} \blacksquare \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{---} \end{array} \right\} \rightarrow \sim \text{Fujita-Miyazawa}$$

$C_3, C_4$

$$\mathcal{F} \left\{ \begin{array}{c} \text{---} \bullet \text{---} \blacksquare \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{---} \end{array} \right\} \rightarrow 1\pi\text{-Exchange + Contact}$$

$C_D$



$C_E$

$$\mathcal{F} \left\{ \begin{array}{c} \text{---} \bullet \text{---} \blacksquare \text{---} \bullet \text{---} \\ | \quad | \quad | \\ \text{---} \end{array} \right\} \rightarrow \text{Contact}$$

$C_E$



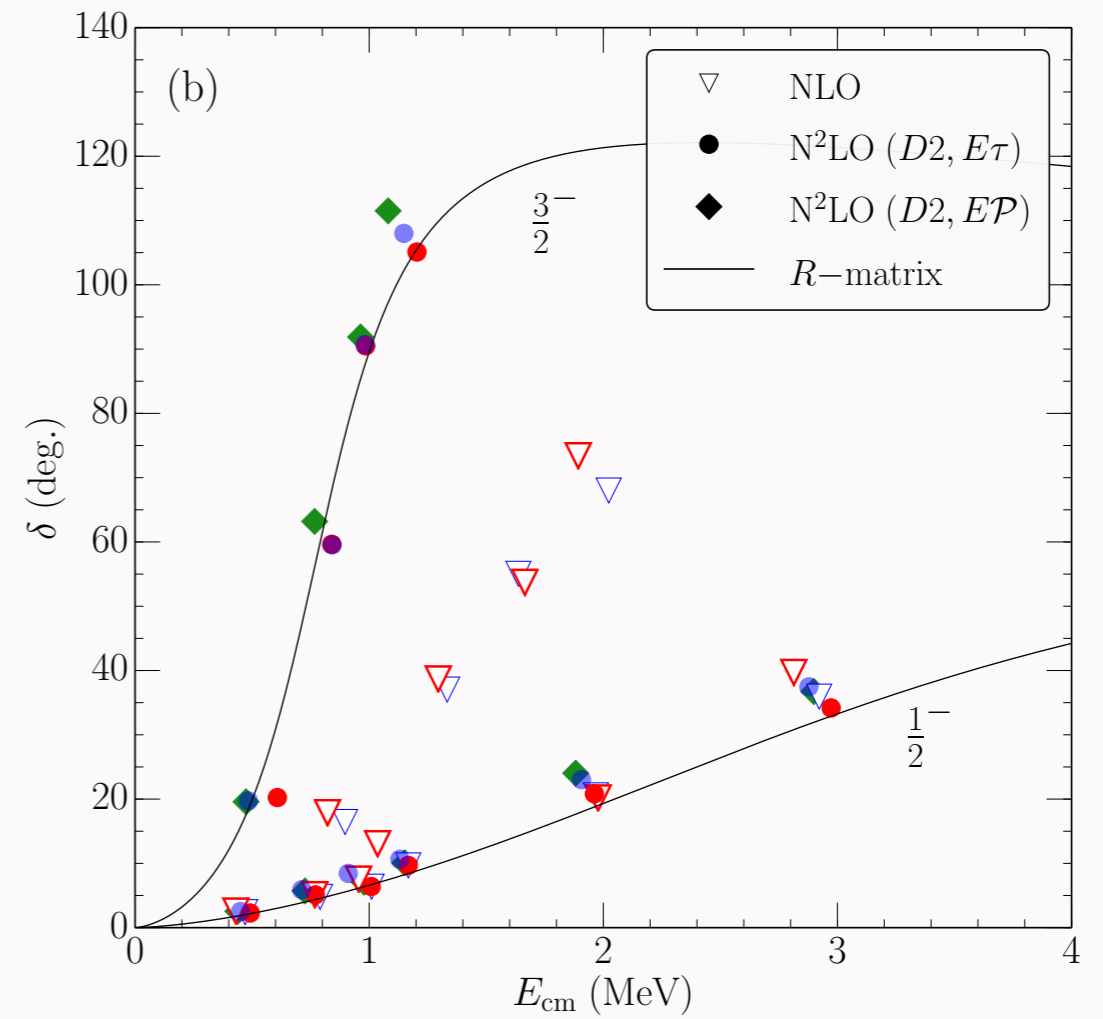
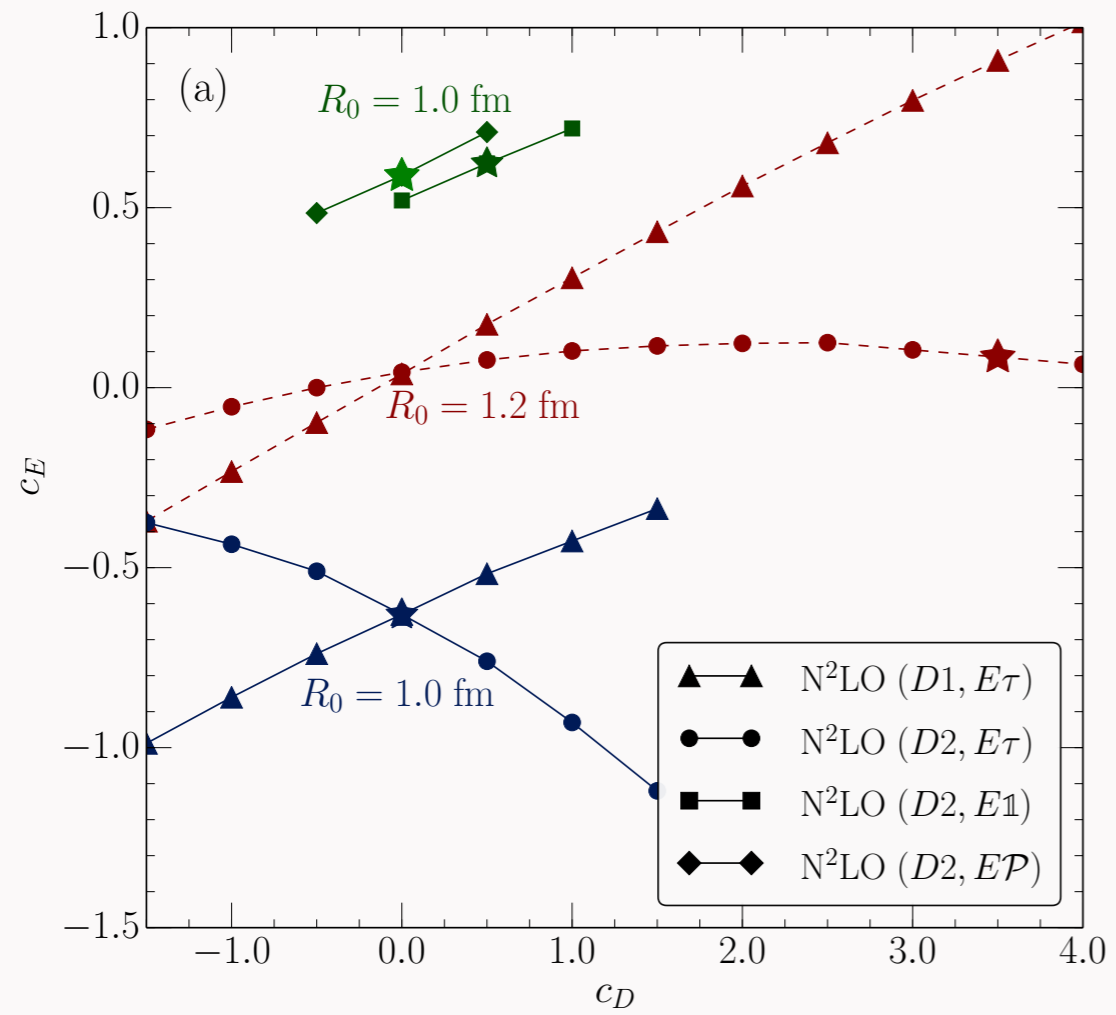
# Fitting $c_D$ And $c_E$

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# Choosing Observables

What to fit  $c_D$  and  $c_E$  to?

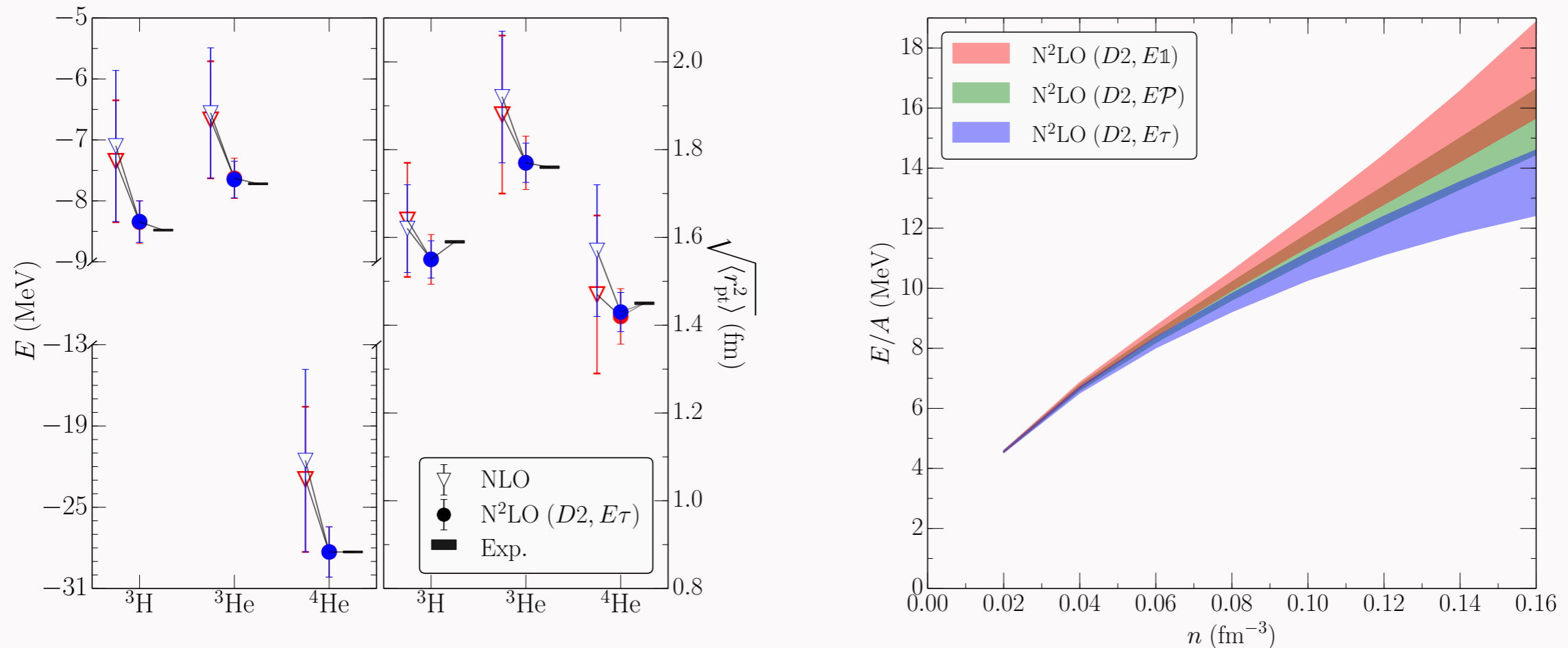
- Uncorrelated observables.
- Probe properties of light nuclei:  ${}^4\text{He}$   $E_B$ .
- Probe  $T = 3/2$  physics:  $n$ - $\alpha$  scattering phase shifts.



# Results

A simultaneous description of properties of light nuclei,  $n$ - $\alpha$  scattering and neutron matter is possible.

Uncertainty analysis as in  
E. Epelbaum et al, EPJ **A51**, 53 (2015).



# Few-Body Resonances

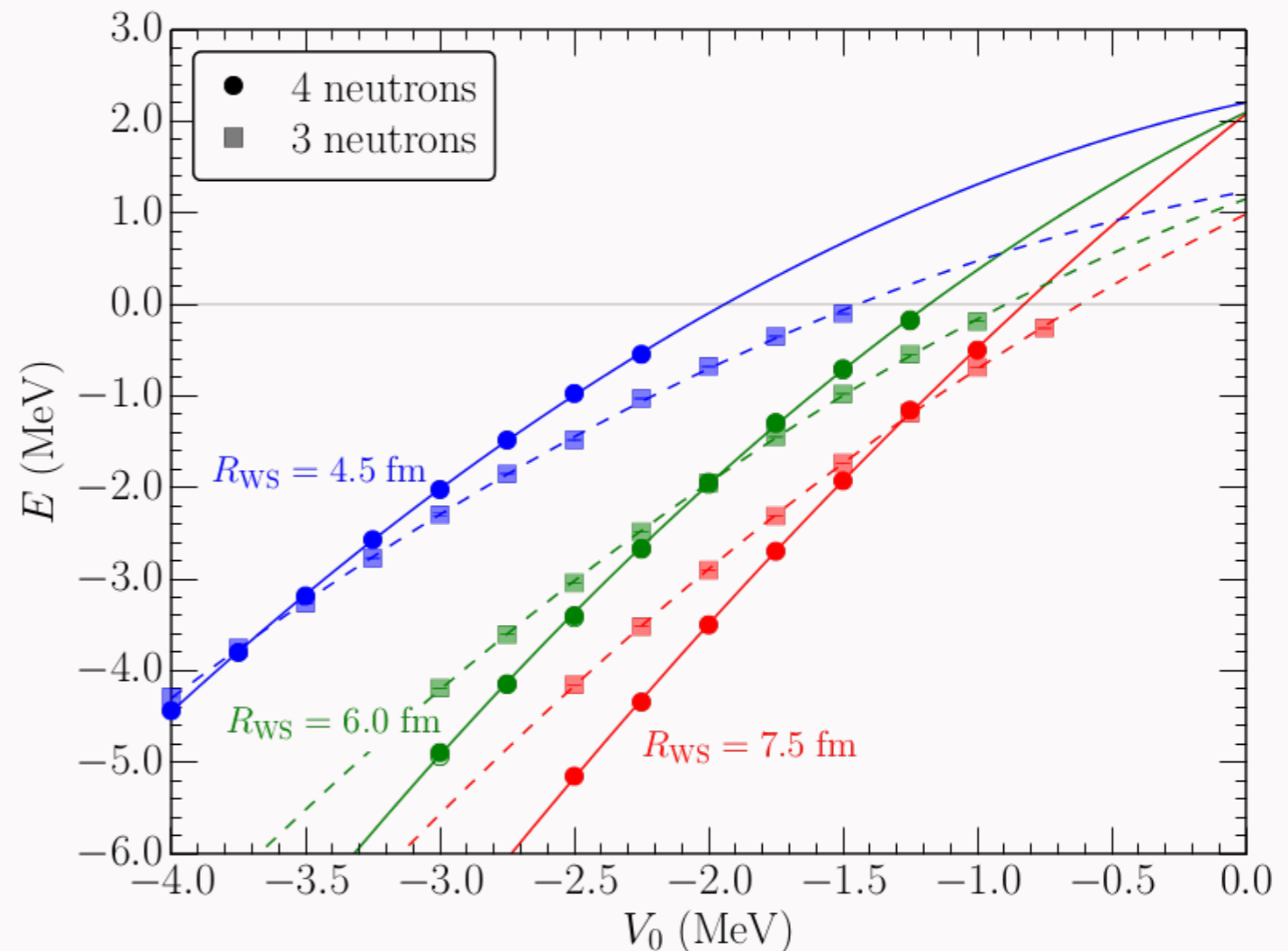
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# Neutrons In A Trap

We confine the neutrons in an external potential.

$$H = - \sum_i \frac{\hbar^2}{2m} \nabla_i^2 + \sum_i V_{WS}(r_i) + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk},$$

$V_{WS}(r) = V_0/[1 + e^{(r-R_{WS})/a}]$ , fixed diffuseness  $a = 0.65$  fm



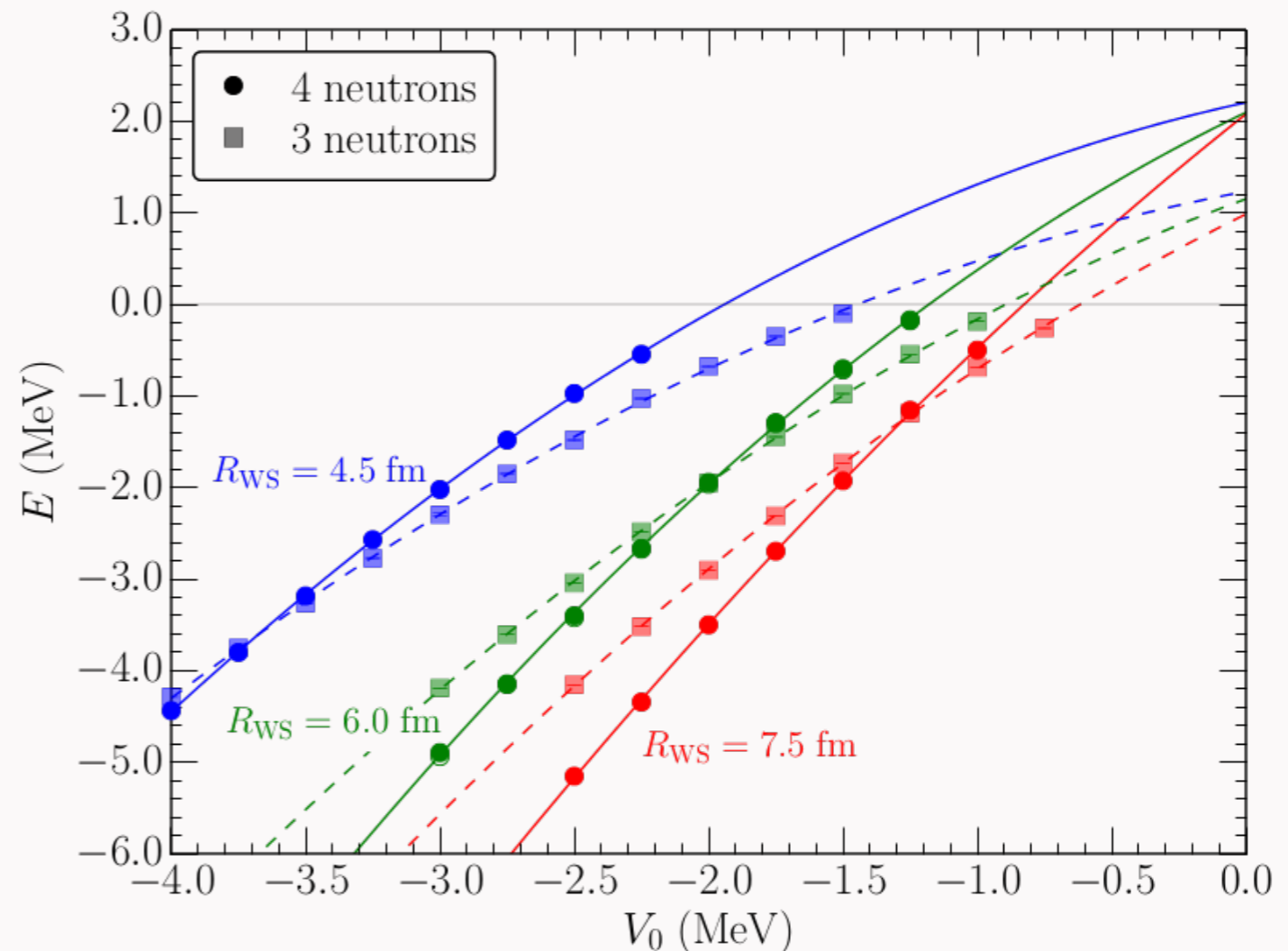
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$V_{WS}(r) = V_0/[1 + e^{(r-R_{WS})/a}]$ , fixed diffuseness  $a = 0.65$  fm

- Changing cutoff/  
removal of  $3N$   
interaction gives  
indistinguishable  
results.
- $E_{3n} = 1.1(2)$  MeV,  
 $E_{4n} = 2.1(2)$  MeV.
- ${}^3n$  resonance lower  
than  ${}^4n$  resonance.



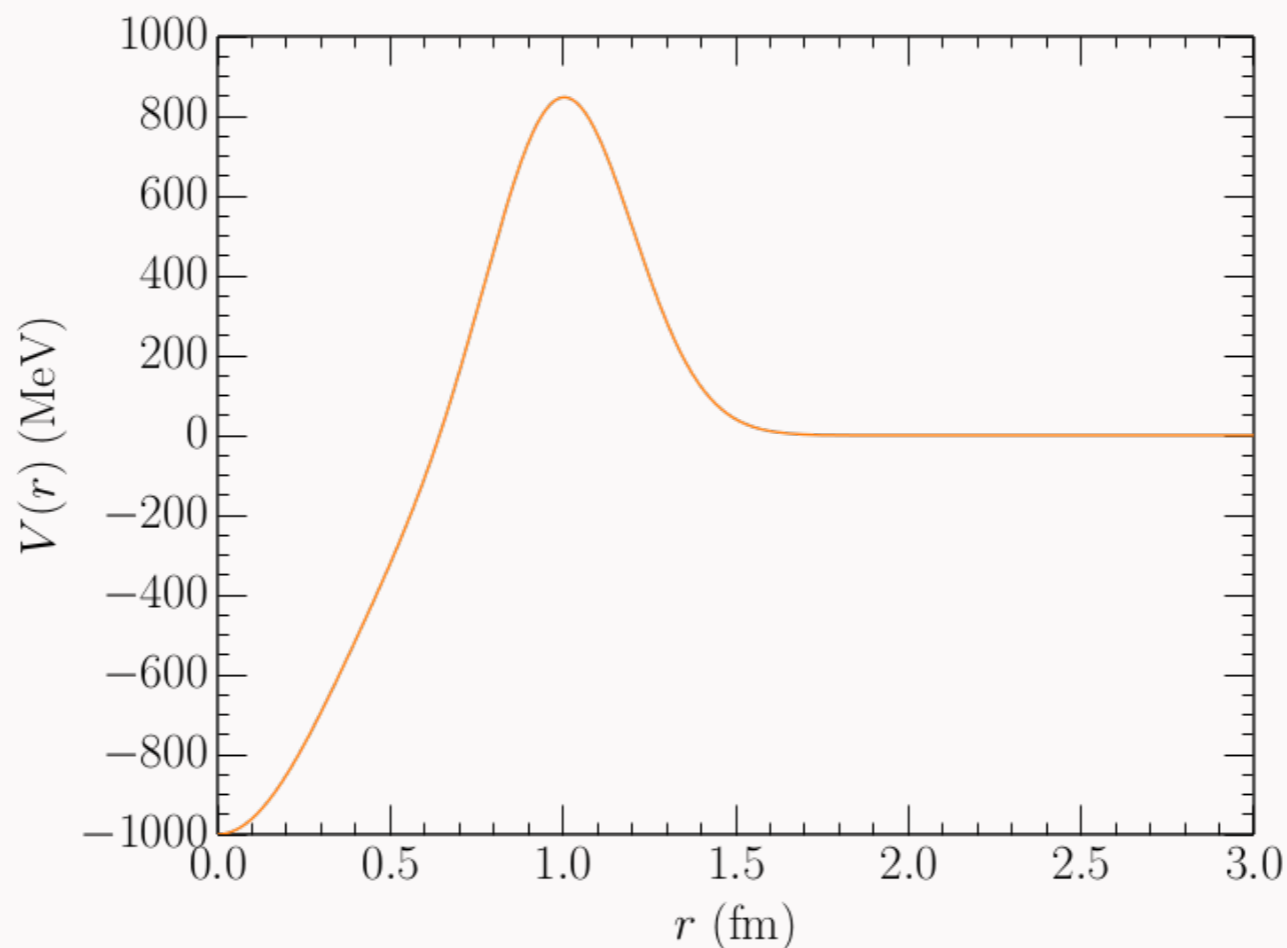
# A Two-Body Test

A simple S-wave potential:

$$V(r) = V_1 e^{-\left(\frac{r}{R_1}\right)^2} + V_2 e^{-\left(\frac{r-r_2}{R_2}\right)^2}$$

$$V_1 = -1000 \text{ MeV}, R_1 = 0.4981 \text{ fm},$$

$$V_2 = 865 \text{ MeV}, R_2 = 0.2877 \text{ fm}, r_2 = 0.9972 \text{ fm}$$



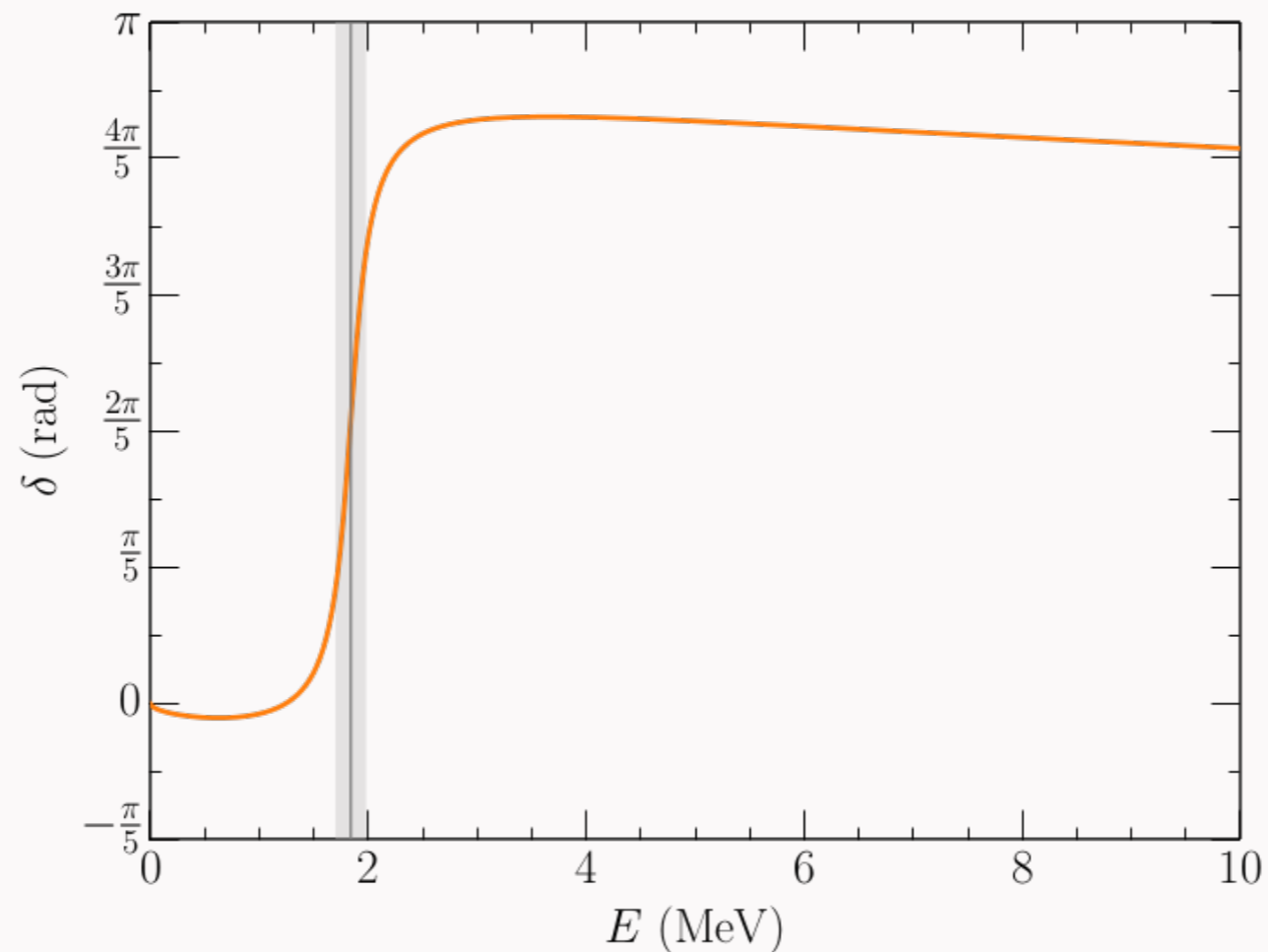


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$$V(r) = V_1 e^{-\left(\frac{r}{R_1}\right)^2} + V_2 e^{-\left(\frac{r-r_2}{R_2}\right)^2}$$

$$E_R = 1.84 \text{ MeV}, \quad \Gamma = 0.282 \text{ MeV}$$

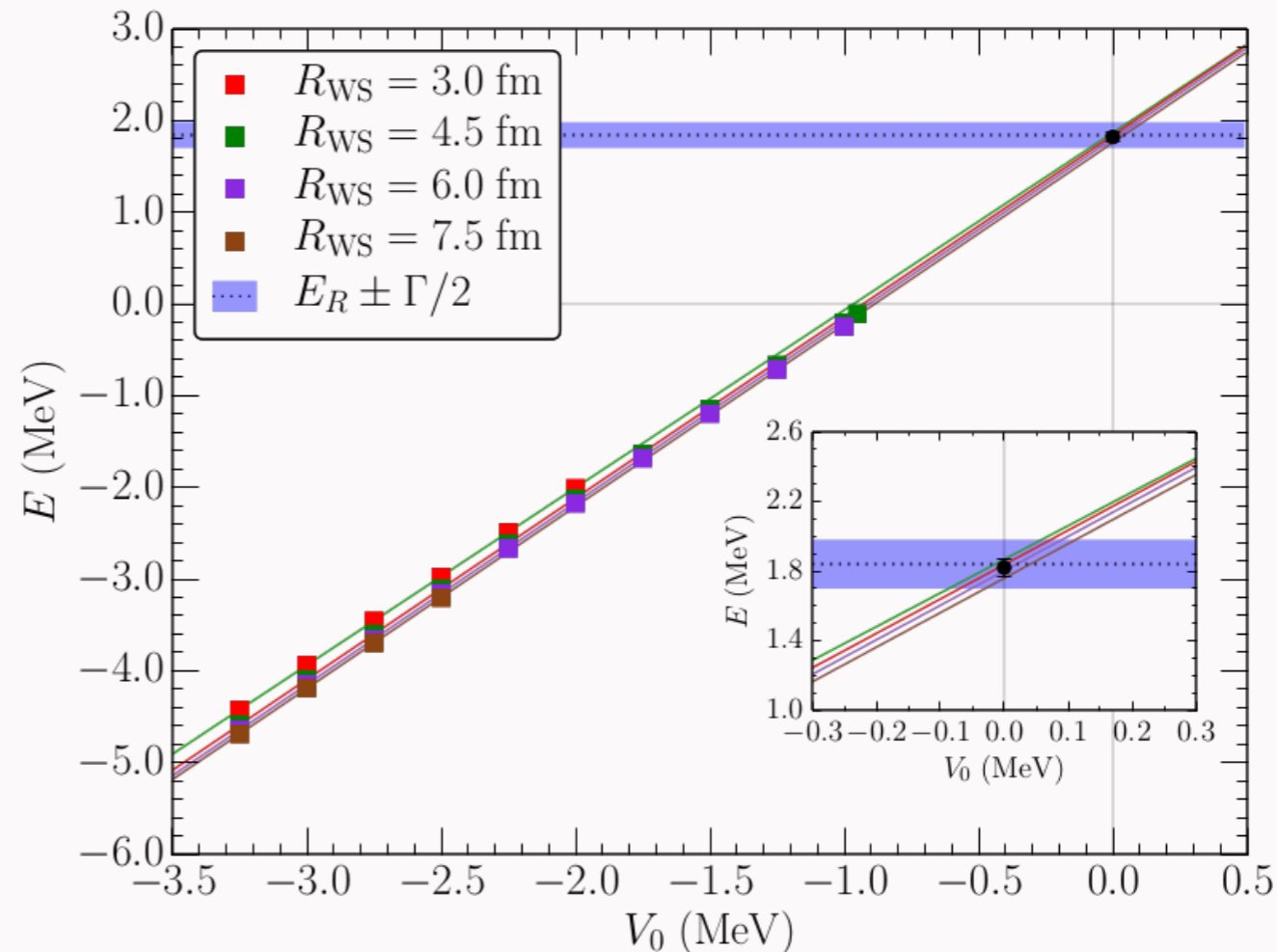


# A Two-Body Test

A simple  $S$ -wave potential + Woods-Saxon:

$$V(r) = V_1 e^{-\left(\frac{r}{R_1}\right)^2} + V_2 e^{-\left(\frac{r-r_2}{R_2}\right)^2}$$

$$V_{\text{WS}}(r) = V_0 / [1 + e^{(r-R_{\text{WS}})/a}], \text{ fixed diffuseness } a = 0.65 \text{ fm}$$



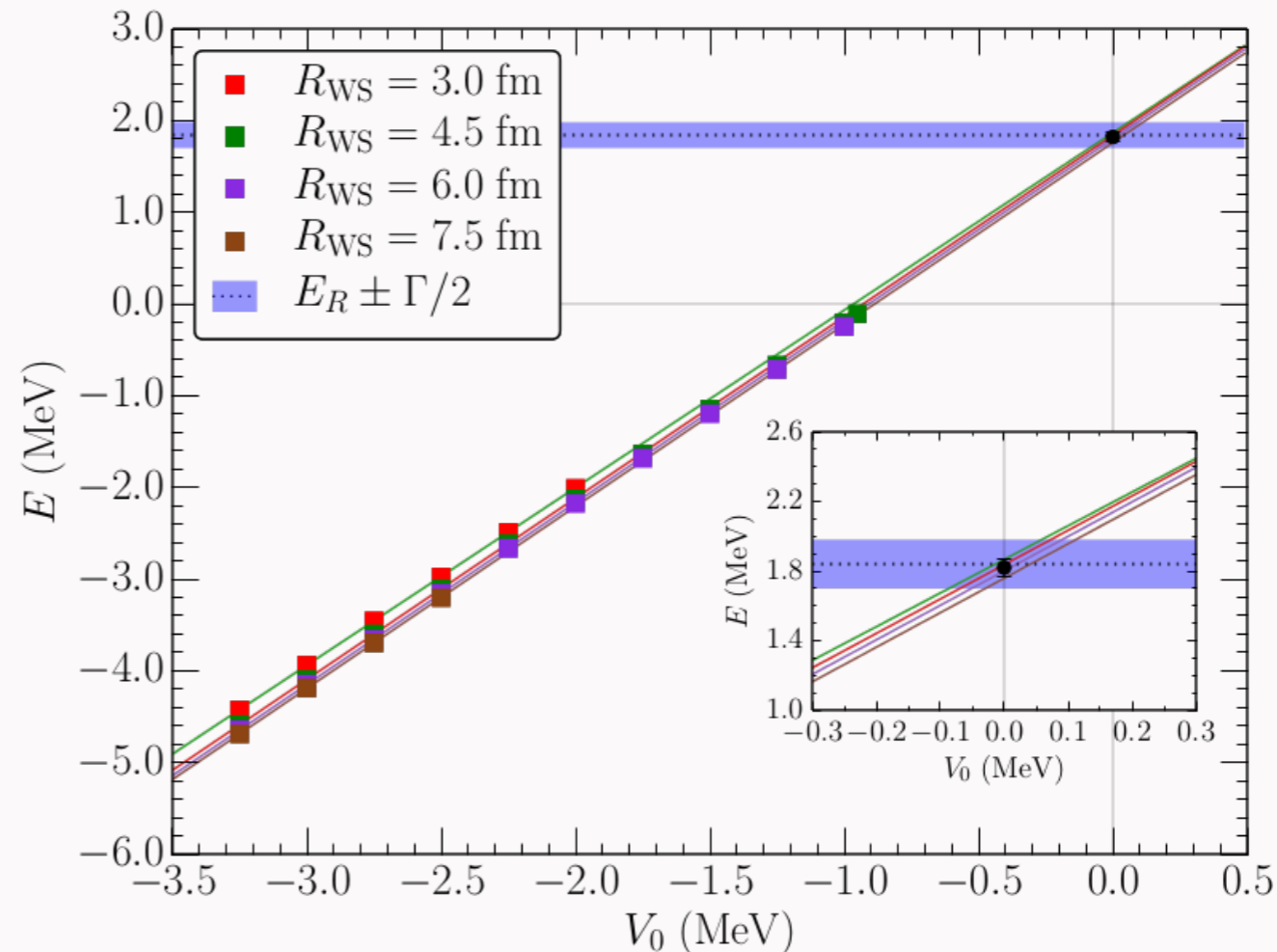
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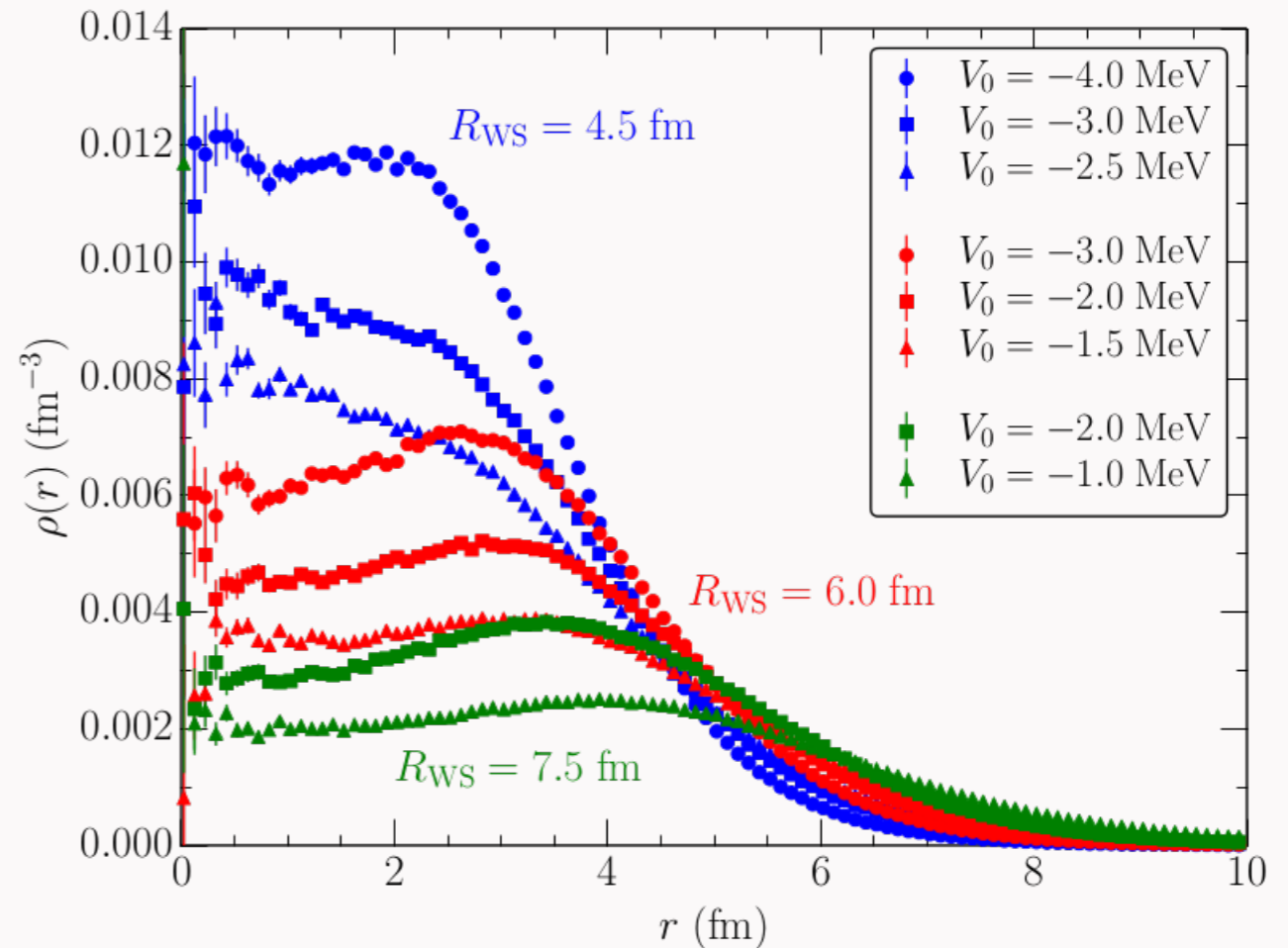
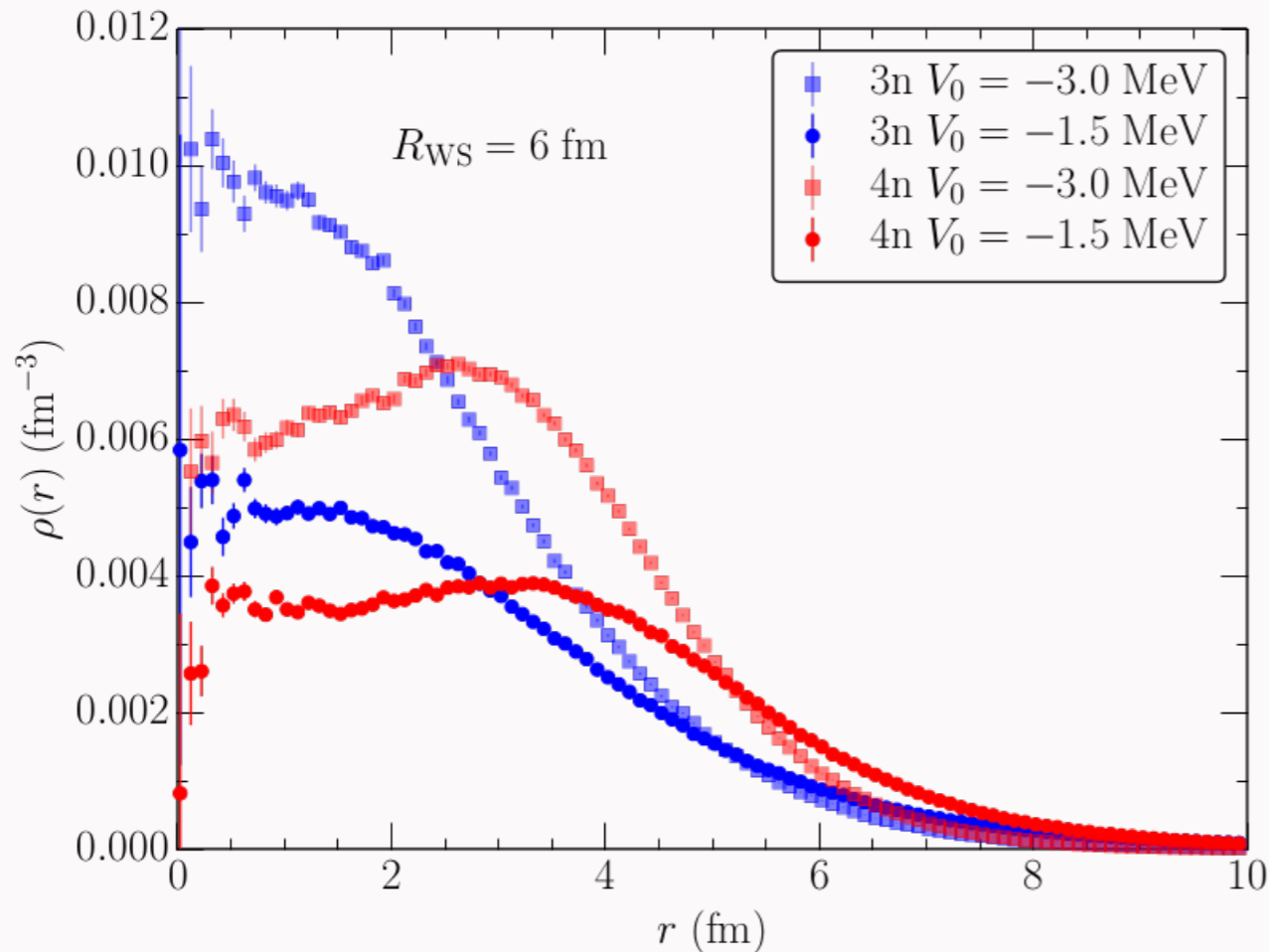
$$V_{\text{WS}}(r) = V_0 / [1 + e^{(r-R_{\text{WS}})/a}], \text{ fixed diffuseness } a = 0.65 \text{ fm}$$

- Different Woods-Saxon radii:  
Independence of trap geometry.
- Extrapolations give 1.83(5) MeV. (Compare to 1.84 MeV).



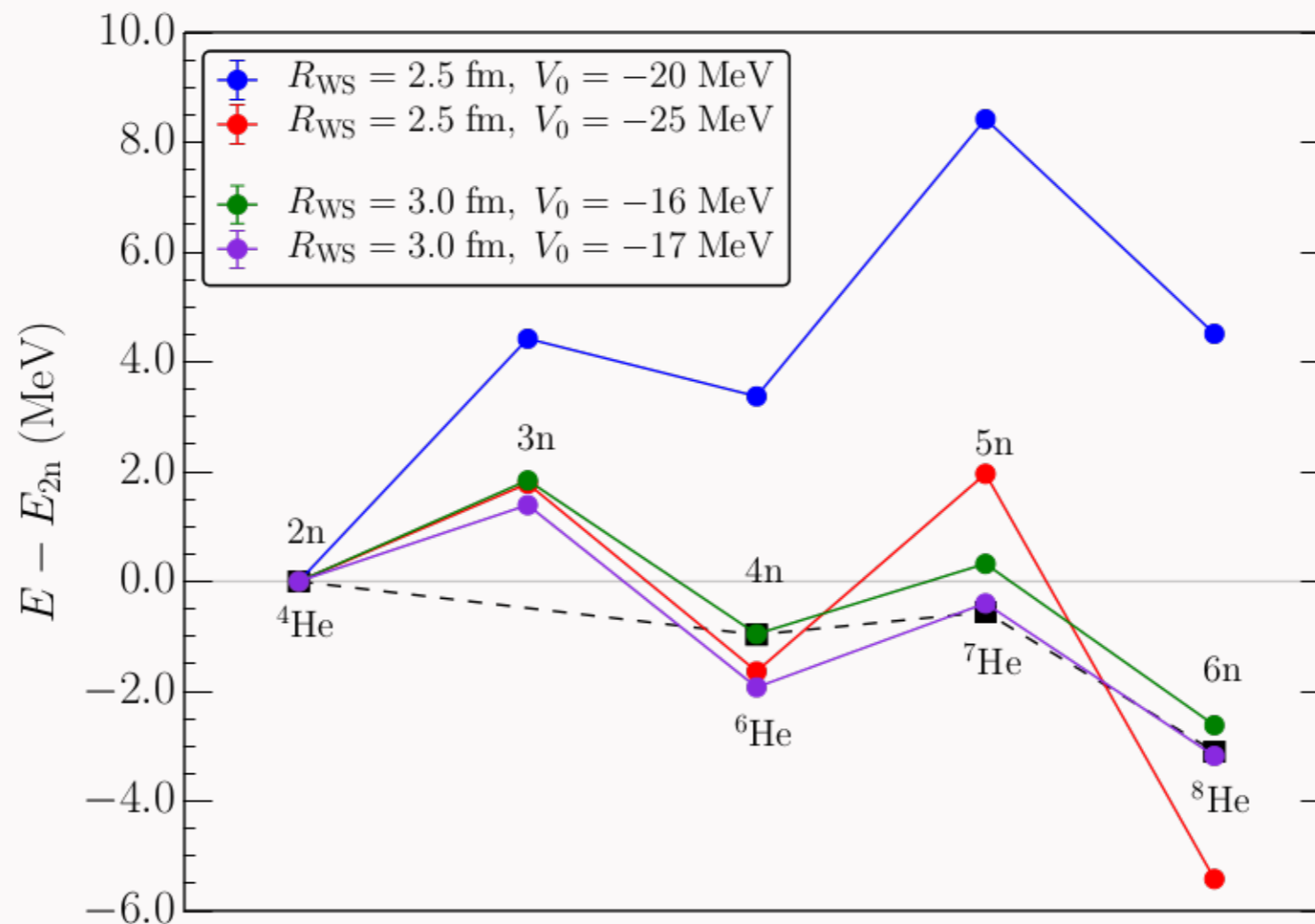
# One-Body Densities

- The  ${}^3n$  and  ${}^4n$  systems are very dilute.
- ${}^3n$  and  ${}^4n$  systems show different short-distance structure.



# Helium Chain

- That  ${}^3n$  is lower than  ${}^4n$  is not an artifact of the Woods-Saxon potential.
- In helium chain,  ${}^3n$  is always higher than  ${}^4n$ .



# Cold Atoms Connections

- Extrapolated energies for  ${}^3n$  and  ${}^4n$  are consistent with scaling like the number of pairs.  $E_{A_n} \sim \frac{A(A-1)}{2}$
- Mean-field interaction of dilute gas of spin-1/2 fermions:  $E_{MF}/A = \frac{k_F^2}{2m} \frac{2}{3\pi} (k_F a) \sim A \Rightarrow E_{MF} \sim A^2$
- Cold atomic gas experiments could determine if one-body density behavior is governed by large-scattering-length physics or details of nuclear interactions.

# Summary

- A recent experiment suggests the possibility of a low-lying tetra-neutron resonance.
- More experiments are needed (Coming soon!)
- Chiral two- and three-nucleon interactions at  $N^2LO$  support a tetra-neutron resonance at 2.1(2) MeV compatible with the experimental claim.
- A trineutron resonance might be lower in energy than a tetra-neutron resonance and therefore might be observable as well.
- Given the diluteness of the systems, connections to cold atomic gas experiments are possible.

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**Thank you for your attention!**