A02: Effective field theories and *ab initio* calculations of light nuclei

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Goals of Project A02

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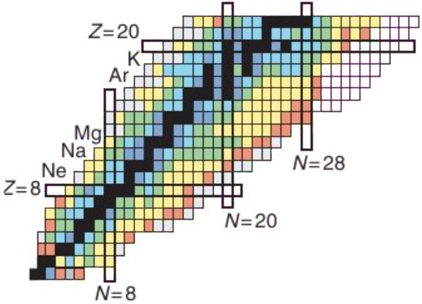
- develop and apply the theoretical framework for **precision *ab initio* calculations** of nuclear-structure observables based on χEFT
- combine *ab initio* calculations with EFT-based description to **identify correlations** and perform extrapolations
- rigorous **quantification of theory uncertainties** at all stages of the calculation
- focus on radii and **electromagnetic observables**
- include electromagnetic currents and pion production operators



Outline

1. project overview and motivation
   - universal correlations
   - *ab initio* and Halo EFT approaches
2. Halo EFT for ^{15}C
   - extension to d-wave states
   - electric form factor and B(E2) results
3. correlation between E2 observables
   - combine Halo EFT with IT-NCSM data
4. summary and outlook

Universal Correlations for Shallow Bound States

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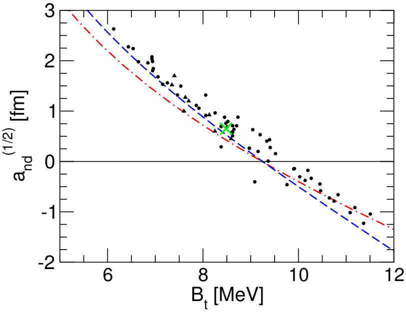
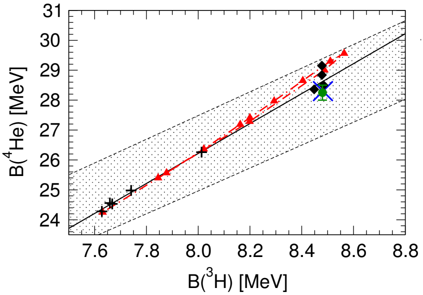
- **universal correlations** between observables for loosely bound few-body systems
  ⇒ **Phillips** (Phillips, 1968) and **Tjon Line** (Tjon, 1975) in few nucleon systems





- correlation universal: nucleons, ^{4}He atoms, ...

Ab Initio and Halo EFT Approaches

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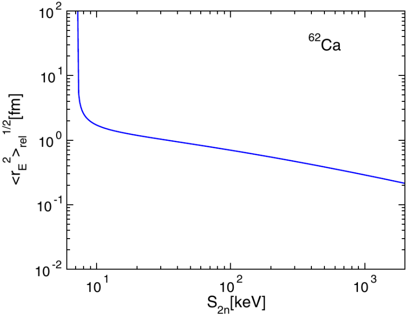
- *ab initio* approaches **successful tool** to calculate nuclear observables
  ⇒ **limited** by the computational complexity of the nuclear many-body problem
- exotic isotopes as **halo nuclei** important for our understanding of nuclear structure
- Halo EFT as a **complementary approach** to *ab initio* methods
  ⇒ useful tool to identify **universal correlations** between observables
  ⇒ combine with *ab initio* results (or experimental) for predictions
- correlations for ^{60}Ca-n-n system [Hagen et al., 2013]



One-Neutron Halo Nuclei

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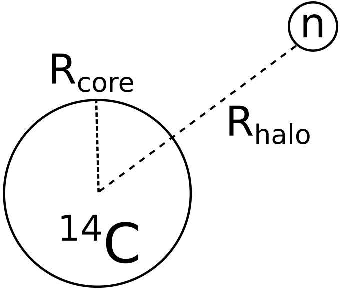
- study of electric properties of one-neutron Halo nuclei provide insights in **universal properties** $\rightarrow$ ^{15}C as example
- one-neutron Halo EFT already for *s*- & *p*-waves $\rightarrow$ extension to *d*-waves
- neutron separation energy of $\frac{1}{2}^+$ $\left[\frac{5}{2}^+\right]$ state of ^{15}C is 1218 [478] keV
- first excitation of ^{14}C is 6.1 MeV above 0^+ ground state



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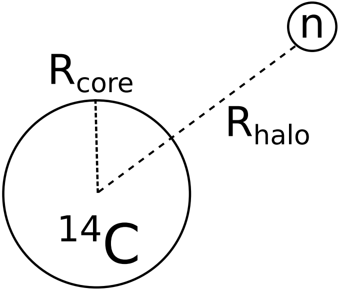
- first excitation of ^{14}C is 6.1 MeV above 0^+ ground state
- exploit **separation of scales** in weakly-bound nuclei $\Rightarrow R_{core} \ll R_{halo}$
- compute observables in a **Halo EFT** in powers of $\mathbf{R_{core}/R_{halo} \approx 0.3}$
- relevant degrees of freedom: **core** and **halo neutron**



Halo EFT Formalism

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- follow the approach of [Hammer and Phillips, 2011] for ^{11}Be (*s*- & *p*-waves) and [Rupak et al., 2012, Fernando et al., 2015] for ^{15}C (*s*-waves)
- include strong *s*-wave and *d*-wave interaction through **auxiliary spinor fields** σ and *d*, respectively

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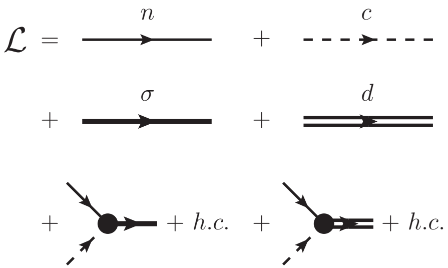
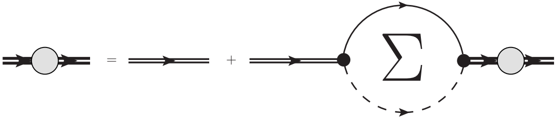


Dressing the D-Wave State



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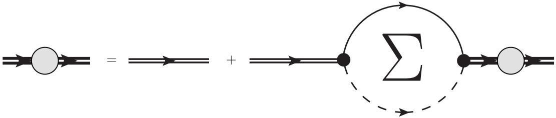

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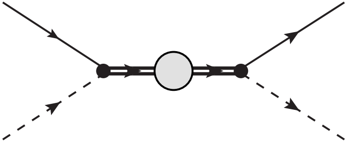
- match scattering amplitude to the **effective-range expansion** (ERE)

$$t_2(k, \cos\theta) \sim \frac{k^4 P_2(\cos\theta)}{1/a_2 - \frac{1}{2} r_2 k^2 + \frac{1}{4}\mathcal{P}_2 k^4 + i k^5}$$



Power-Counting Scheme for Shallow Bound States

- power-counting scheme for arbitrary l-th partial wave shallow bound states
- **(l + 1) ERE parameters** needed at LO for matching due to higher divergences
- **minimal number** of fine tunings $\rightarrow$ **l fine tunings** for $l \geq 1$
- every additional fine tuning less likely in nature $\rightarrow$ proof that shallow bound states for higher partial waves **less likely to occur** in nature

$$\gamma_l \sim 1/R_{halo} \qquad a_l \sim \begin{cases} R_{halo}, & l = 0 \\ R_{halo}^{2l}\, R_{core}, & l > 0 \end{cases}$$

$$r_l \sim \begin{cases} R_{core}^{1-2l}, & l = 0 \\ 1/\left(R_{halo}^{2l-2}\, R_{core}\right), & l > 0 \end{cases} \qquad \mathcal{P}_l \sim \begin{cases} R_{core}^{3-2l}, & l \leq 1 \\ 1/\left(R_{halo}^{2l-4}\, R_{core}\right), & l > 1 \end{cases}$$

Electromagnetic Interactions

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- include electromagnetic interactions via **minimal substitution** in the Lagrangian $\partial_\mu \rightarrow D_\mu = \partial_\mu + ie\hat{Q}A_\mu$
- add **local gauge-invariant** operators involving the electric field $\mathbf{E} = \nabla A_0 - \partial_0 \mathbf{A}$

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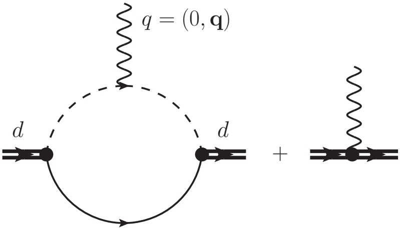
- add **local gauge-invariant** operators involving the electric field $\mathbf{E} = \nabla A_0 - \partial_0 \mathbf{A}$
- compute electric form factors in the **Breit frame** $q = (0, \mathbf{q})$
- electric form factors get contributions only from irreducible **Γ_0 vertex** up to NLO
- for d-wave obtain $G_E(|\mathbf{q}|)$, $G_Q(|\mathbf{q}|)$ & $G_H(|\mathbf{q}|)$ form factors



Electric Form Factors

- need additional local gauge-invariant operators for $r_E \sim L^{(d)}_{C0E}$ and $\mu_Q \sim L^{(d)}_{C0Q}$ at LO to treat arising divergences for $G_E(|\mathbf{q}|)$ and $G_Q(|\mathbf{q}|)$

$$G_E(|\mathbf{q}|) \approx 1 - \frac{1}{6}\left\langle r_E^2 \right\rangle |\mathbf{q}|^2 + \ldots \quad \xrightarrow{\text{LO}} \quad \left\langle r_E^2 \right\rangle^{(d)} = -\frac{6\tilde{L}^{(d)\,\text{LO}}_{C0E}}{r_2 + \gamma_2^2 \mathcal{P}_2}$$

$$G_Q(|\mathbf{q}|) \approx \mu_Q \left(1 - \frac{1}{6}\left\langle r_Q^2 \right\rangle |\mathbf{q}|^2 + \ldots\right) \quad \xrightarrow{\text{LO}} \quad \mu_Q^{(d)} = \frac{40\tilde{L}^{(d)\,\text{LO}}_{C0Q}}{3\left(r_2 + \mathcal{P}_2 \gamma_2^2\right)}$$

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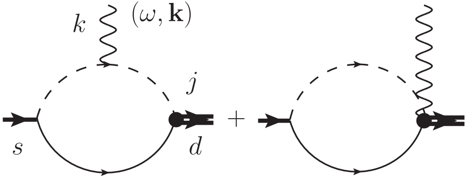
$$\langle r_Q^2\rangle^{(d)} = \frac{90}{7}\frac{f^4}{\gamma_2\left(r_2 + \mathcal{P}_2\gamma_2^2\right)}$$

$$G_H(|\mathbf{q}|) \approx \mu_H\left(1 - \frac{1}{6}\langle r_H^2\rangle |\mathbf{q}|^2 + \ldots\right) \quad \xrightarrow{\text{LO}} \quad \mu_H^{(d)} = -\frac{2}{3}\frac{f^4}{\gamma_2\left(r_2 + \mathcal{P}_2\gamma_2^2\right)}$$

- **obtain correlations** between different electric observables: $\langle r_Q^2\rangle^{(d)} \sim \mu_H^{(d)}$

E2 Transition

- calculate irreducible $\mathbf{\Gamma_{j\mu}}$ **vertex** for E2 transition from the $1/2^+$ to the $5/2^+$ state at LO
- neutron spin unaffected
- **divergences cancel**



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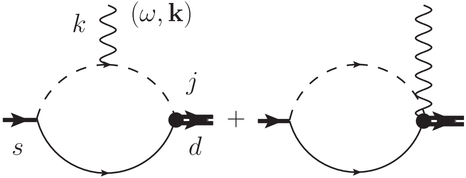


$$B(E2) = \frac{6}{\pi}\left(\frac{\bar{\Gamma}_E}{\omega}\right)^2 = \frac{6}{15\pi} Z_{eff}^2 e^2 \frac{\gamma_0}{-r_2 - \mathcal{P}_2\gamma_2^2}\left[\frac{3\gamma_0^2 + 9\gamma_0\gamma_2 + 8\gamma_2^2}{(\gamma_0+\gamma_2)^3}\right]^2$$

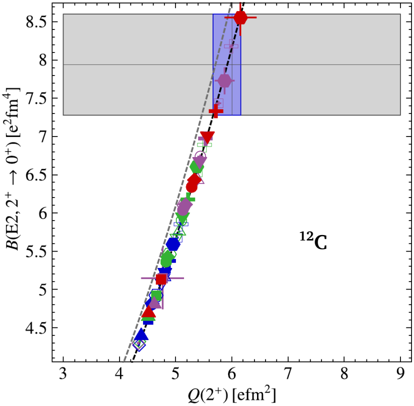
- **experimental result** for B(E2) = 0.967(22) e^2 fm^4
  $\rightarrow$ extract $1/\left(r_2 + \mathcal{P}_2\gamma_2^2\right)$
- **numerical predictions** for $\langle r_Q^2\rangle^{(d)} = -0.578$ fm^4 and $\mu_H^{(d)} = 0.030$ fm^4

Correlation between E2 observables

- robust **correlation** between μ_Q and **B(E2)** in *ab initio* calculations

- interpreted by **rigid rotor** model (Bohr and Mottelson, 1975)

$$B(E2, J_i \rightarrow J_f) = f(J_i, J_f, J) \left(\frac{Q_{0,t}}{Q_{0,s}}\right)^2 \mu_Q(J)^2$$



[Calci and Roth, 2016]

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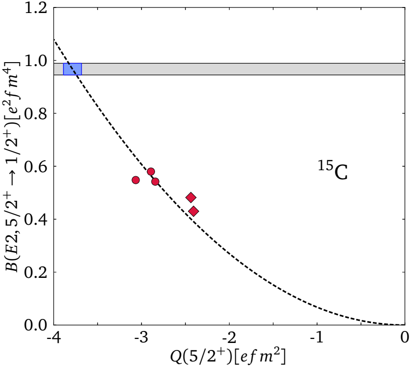
- use correlation for **prediction of** μ_Q for $^{15}C \to$ fit $Q_{0,t}/Q_{0,s} \approx 0.5$

- approximation for $\tilde{L}^{(d)\ \mathrm{LO}}_{C0Q}$



ab initio data from R. Roth

Correlation between E2 observables

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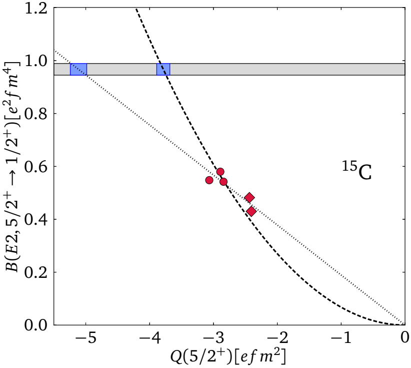
- **LO correlation** between μ_Q and **B(E2)** from Halo EFT

$$\mathrm{B(E2)} = \frac{6}{15\pi} Z_{eff}^2 e^2 \frac{3\gamma_0}{40\, \tilde{L}_{C0Q}^{(d)\,\mathrm{LO}}} \times \left[\frac{3\gamma_0^2 + 9\gamma_0\gamma_2 + 8\gamma_2^2}{(\gamma_0 + \gamma_2)^3}\right]^2 \mu_Q$$

- **linear** dependence $\rightarrow$ fit $\tilde{L}_{C0Q}^{(d)\,\mathrm{LO}}$
- **combine Halo EFT with *ab initio*** and experimental results for approximation of μ_Q and $\tilde{L}_{C0Q}^{(d)\,\mathrm{LO}}$
- more data points useful



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Summary & Outlook

Summary

- Halo EFT formalism to calculate electric observables of weakly-bound *s*, *p*, & **d-wave** states established
- number of matching parameters increases
- shallow bound states in lower partial waves more likely
- correlations between electric observables
- combination with *ab initio* calculations
- states with smaller / more universal

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Outlook

- compare results with more data from *ab initio* calculations
- focus on pion production (χEFT)

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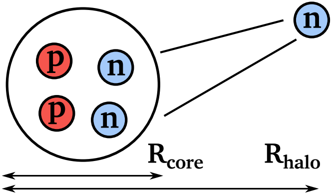
Scales and Antisymmetrization

- scales: $R_{halo} \gg R_{core}$
- **antisymmetrization** w.r.t. neutrons in core?
- core neutrons not active dof in Halo EFT



- only contribution if **significant spatial overlap** between wave functions of core and halo nucleon

  $\Rightarrow$ **small** for $R_{halo} \gg R_{core}$
- effects **subsumed in low-energy constants** and included perturbatively in expansion R_{core}/R_{halo}