$$\left[\frac{D\vec{I}}{Dt}\right]_I = \left[\frac{D\vec{I}}{Dt}\right]_B + \vec{\Omega} \times \vec{I}.$$

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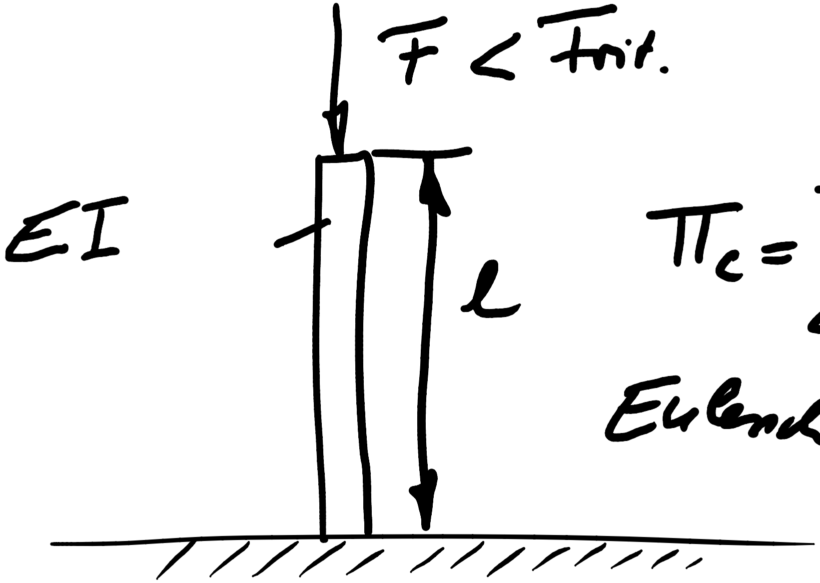
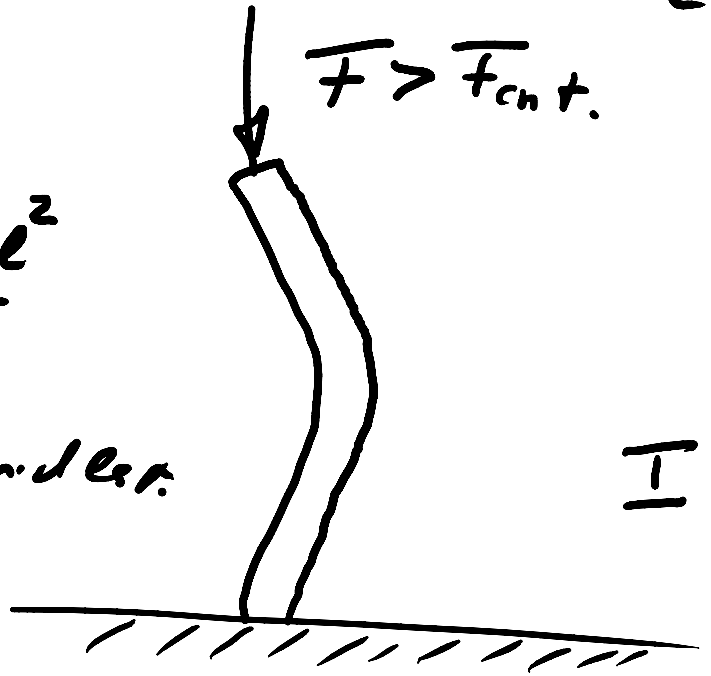
stabil $\Pi < \Pi_c$ instabil. $\Pi > \Pi_c$



$$\Pi_c = \frac{F_c l^2}{EI}$$

Eulersche Kennzahl



$$I = \int_S y^2 \, dS$$

$$\underbrace{F_{krit.}}_{} = fn(l, EI)$$

$\{\ \}$ Einheit

$[\ ]$ Größenart.

Eigenart.

$\{EI\} = Nm^2$ $[EI] = FL^2$

$\{F_{krit}\} = N$; $[F_{krit}] = F$

$\{l\} = m$; $[l] = L$

Ueberlegung:

$$F_{krit} = fn(\ell, EI)$$

F_{krit}: N $\qquad$ ℓ: 10 m $= 10^3$ cm $\qquad$ EI: Nm^2

1. dimensionshomogener Zusammenhang.

⇓ Produktbildung

2. Gerechnet wird nur mit Maßzahlen!?

$$F_{krit} = fn\left(\frac{EI}{\ell^2}, \cancel{EI}\right)$$

F_{krit}: N $\qquad$ $\frac{EI}{\ell^2}$: N $\qquad$ $\cancel{Nm^2}$

⇓

3. Physikalisch-technische Zusammenhänge sind invariant gegenüber Änderung der Basiseinheiten. } Bridgman Postulat.

Bridgman-Postulat

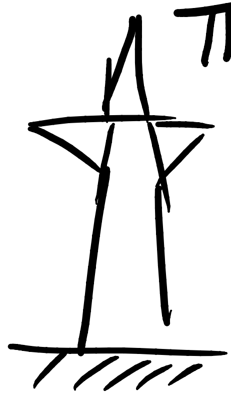
$$\underbrace{\text{„Absolute Bedeutung}}\ \underbrace{\text{relativer Größen"}}_{\text{dimensionslosen Produkte.}}$$

$$F_{crit} = fn(l, EI)$$

$$F_{crit} = fn\left(\frac{EI}{l^2}, \cancel{EI}\right)$$

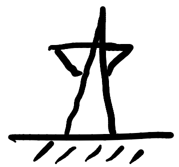
$$\Pi_c = \frac{F_{crit}\, l^2}{EI} = fn\left(\cancel{\frac{EI}{l^2}}\right) = const$$

dimensionslose Konstante

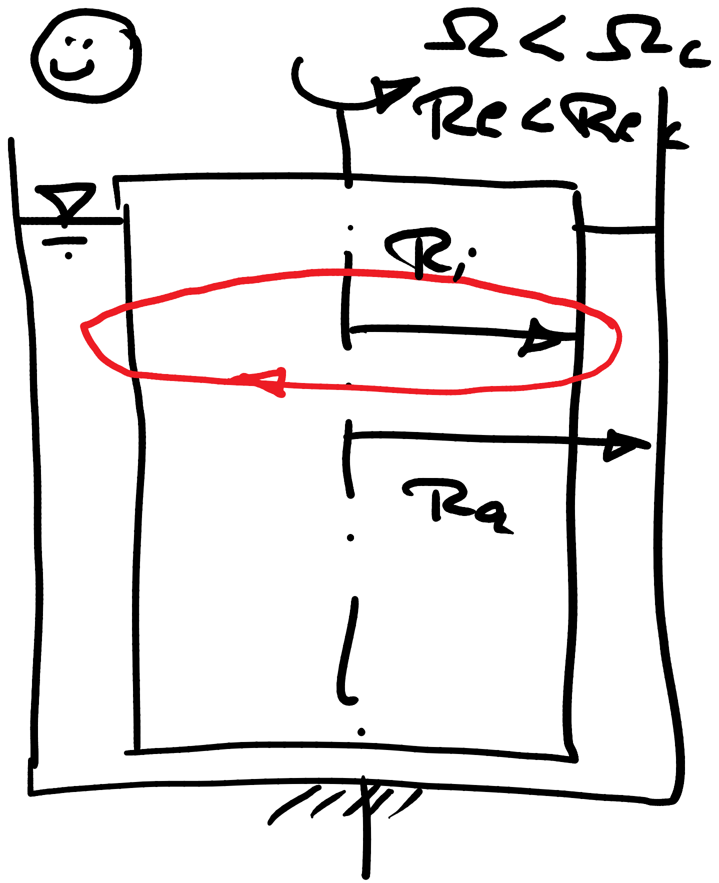


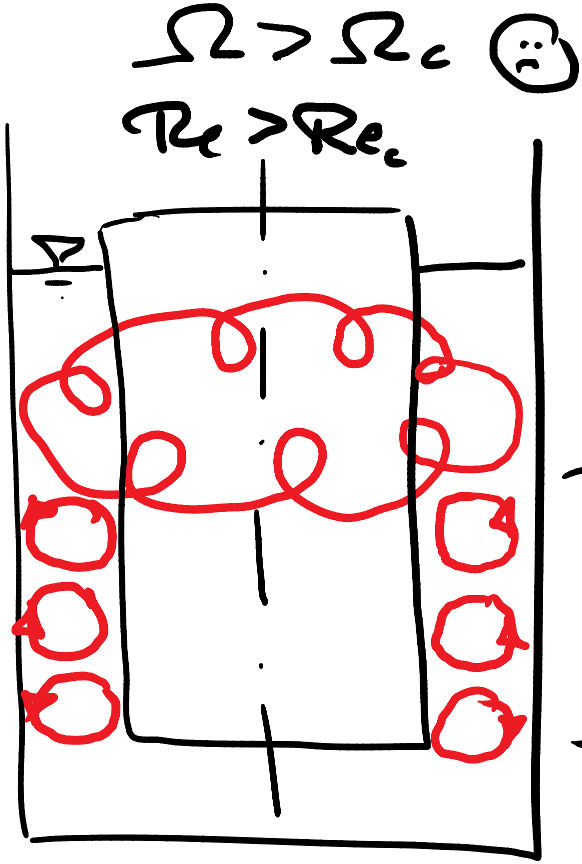
$$\Pi_c \overset{!}{=} \Pi_c'$$

geometrische Ähnlichkeit.
⇔ gleiche Gestalt.



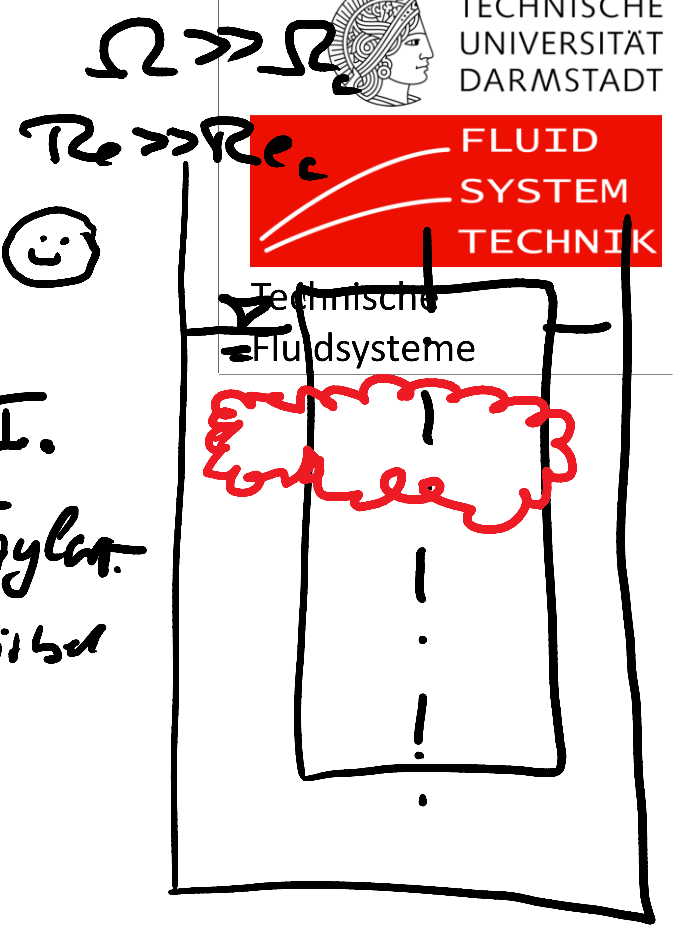
physikalische Ähnlichkeit.
⇔

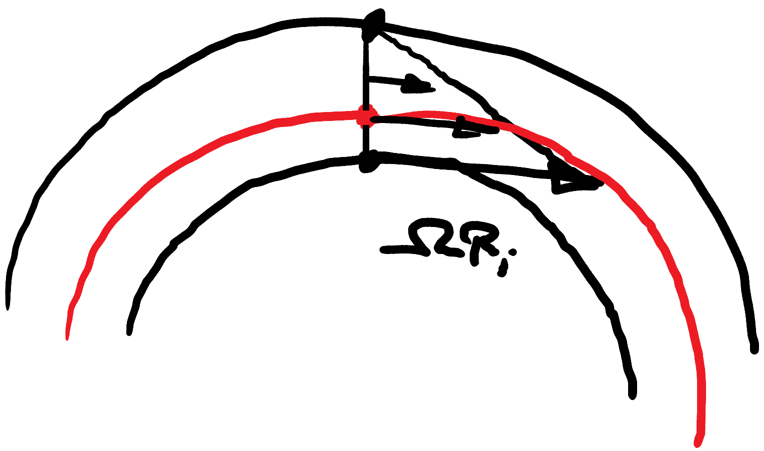


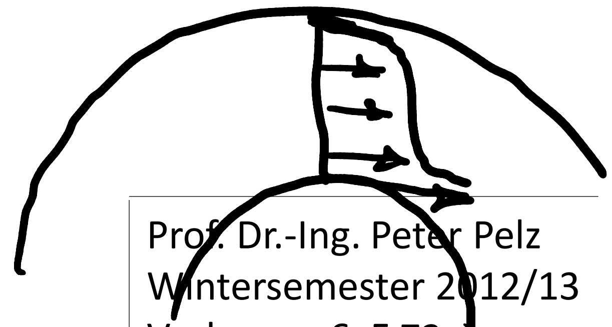


G.I. Taylor Wirbel

räumliche und
zeitliche Strukturen
bilden sich ab







Der Weg in die Turbulenz geschieht über Verzweigungen (Instabilität).

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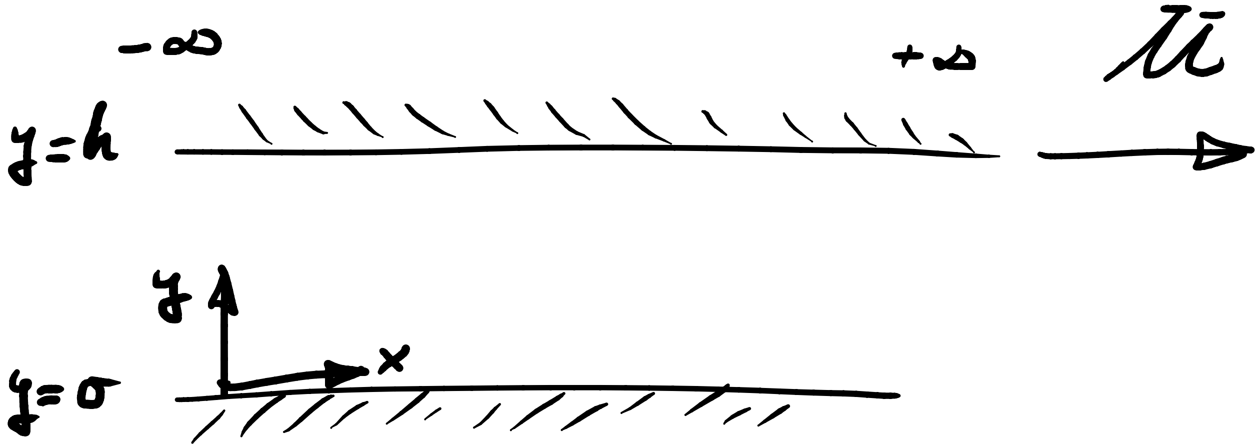
Couette-Strömung



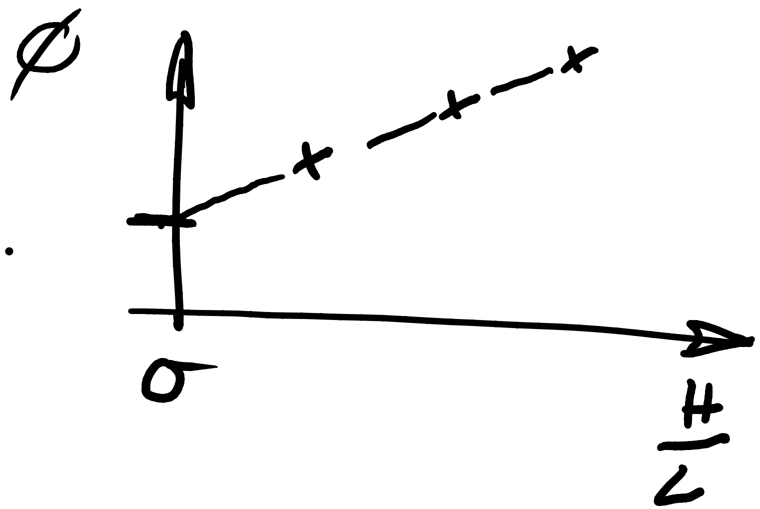
- im zeitlichen Mittel quasistationäre Strömung

$$\overline{\frac{\partial}{\partial t}} \equiv 0$$

- unendlich ausgedehnte Platte ⇝ keine Randeffekte.

Hinweis: Wenn Randeffekte zu erwarten sind, sollte der Schlankheitsgrad $\frac{L}{H}$

Extrapol.:

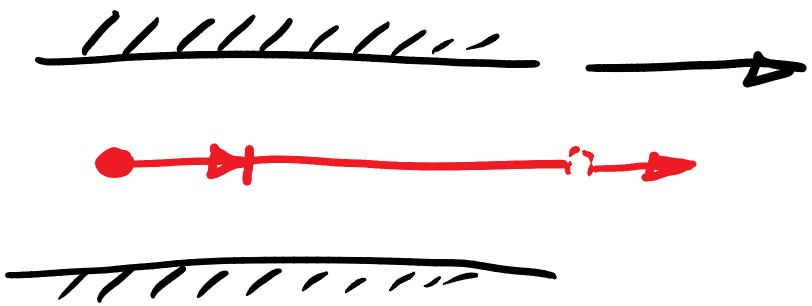


$$\rho \frac{D\vec{u}}{Dt} = \nabla \cdot \underset{\sim}{T} + \vec{f} \quad \Big| \; \cdot \vec{e}_x \; ; \; \cdot \vec{e}_y$$

$$0 = \rho \frac{Du}{Dt} = \left(\nabla \cdot \underset{\sim}{T}\right) \cdot \vec{e}_x + f_x$$

$$0 = \rho \frac{Dv}{Dt} = \left(\nabla \cdot \underset{\sim}{T}\right) \cdot \vec{e}_y + f_y$$

- laminare Strömung.
- keine Volumenkraft $\vec{f} \equiv 0$.
- $\vec{v} = u \vec{e}_x$



$$\nabla \cdot \underset{\sim}{T} = 0 ; \quad v \equiv 0$$

Bewegungsgl.

$$\underset{\sim}{\tau} = \tau_{xx}\vec{e}_x\vec{e}_x + \tau_{xy}\vec{e}_x\vec{e}_y + \tau_{yx}\vec{e}_y\vec{e}_x + \tau_{yy}\vec{e}_y\vec{e}_y$$

$$\tau_{xx} = -\underbrace{\overset{0 \,=}{p}}_{} + \overbrace{\underbrace{2\eta\frac{\partial u}{\partial x}}_{}}^{\equiv 0}$$

Viskose Normalspannung.

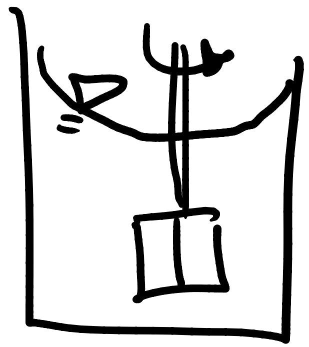
$$\tau_{xy} = \eta\frac{\partial u}{\partial y} = \tau_{yx}$$

$$\tau_{yy} = -\overset{0 \,=}{p} + \underbrace{2\eta\frac{\partial v}{\partial y}}_{\equiv 0.}$$

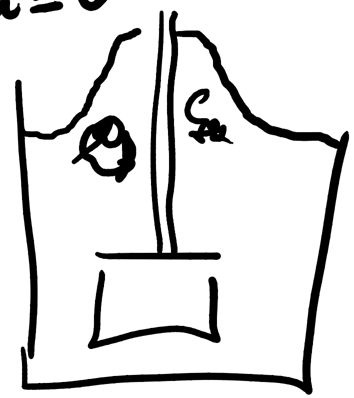
$$v \equiv 0$$

Konti: $\frac{D\rho}{Dt} + \rho\nabla\cdot\vec{u} = 0.$

$\circ\ \frac{D\rho}{Dt} \equiv 0$ Inkompressibel. $\rightarrow \nabla\cdot\vec{u} = 0$



Newton.



Nicht-Newton.

$$\frac{\partial u}{\partial x} + \underbrace{\frac{\partial v}{\partial y}}_{\equiv 0} = 0$$

Normalspannungseffekt

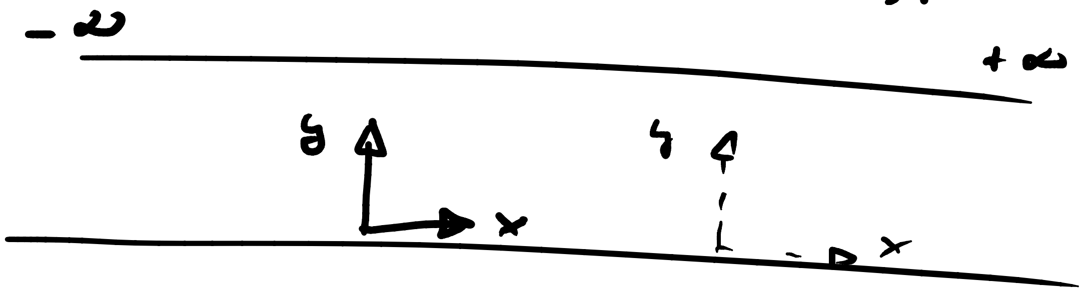
Weissenbergeffekt.

$$\sigma = \frac{d}{dy}\left(\tau_{xy}\right)$$

$$v \equiv 0$$

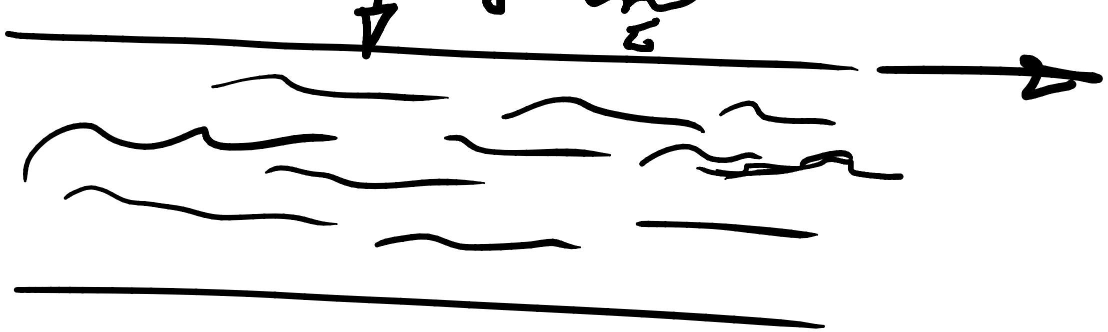
$$\left.\begin{aligned} v(y=0) &= 0 \\ v(y=h) &= 0 \end{aligned}\right\} \text{kinematische Randbed.}$$

$$\cancel{\frac{\partial u}{\partial x}} + \frac{\partial v}{\partial y} = 0 \quad \rightsquigarrow \quad v = \text{const} + \underbrace{f(x)}_{=\text{const.}}$$



$$\frac{\partial u}{\partial x} = 0 \quad \text{aus Symmetriegrün.}$$

$$F_y = \underbrace{\nu_{10}}_{\varepsilon} \dot{\gamma}^2 A$$



$$F_x = \underbrace{\eta}_{\tau} \dot{\gamma} A$$

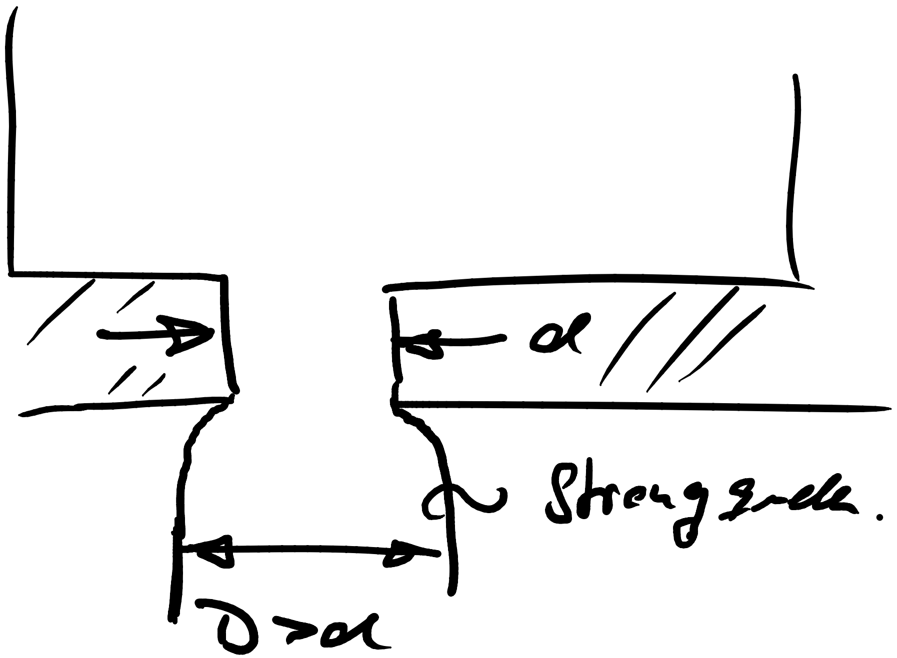
ν_{10} Materialparam.

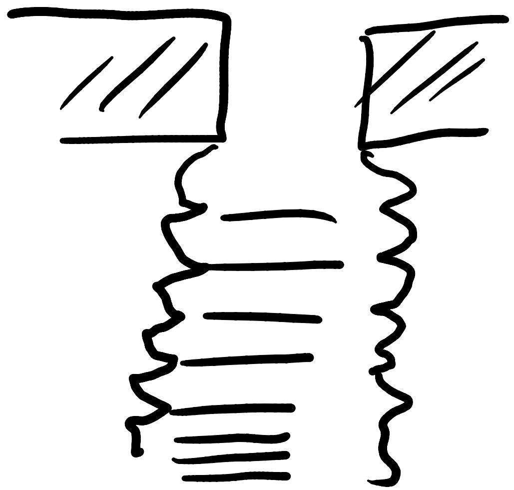
$$\uparrow We = \frac{\sigma_E}{\tau_v} = \frac{\nu_{10} \dot{\gamma}}{\eta} \uparrow$$

$We > We_c$ Viskoelastisch Teilbruch. Schmelzebruch.

$$Re = \frac{\sigma_T}{\tau_v} = \frac{\rho u^2}{\eta \dot{\gamma}}$$

$Re > Re_c$ turbulent

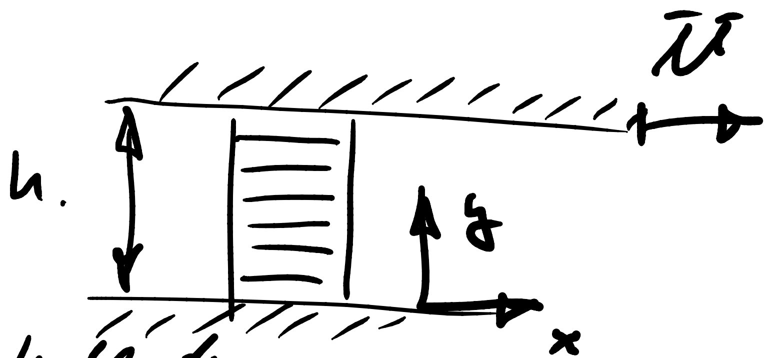

d
Strahlgrenze.
D > d



Bewegungsgleichung:

$$0 = \frac{d}{dy}\left(\tau_{xy}\right)$$

$\tau_{xy} = \text{const.} = \tau_w$ Wandschubspannung.



$$u(y) = \frac{\tau_w}{\eta} y = \bar{U}\,\frac{y}{h}$$

Newtonsches Gesetz.

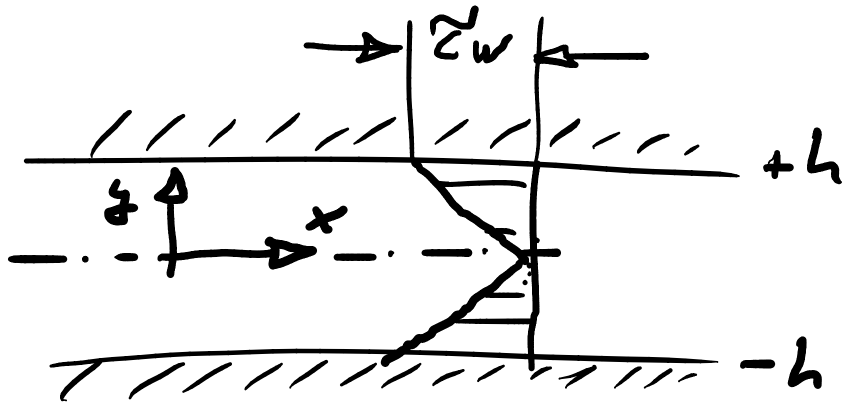
$$\tau_{xy} = \eta \frac{du}{dy}$$

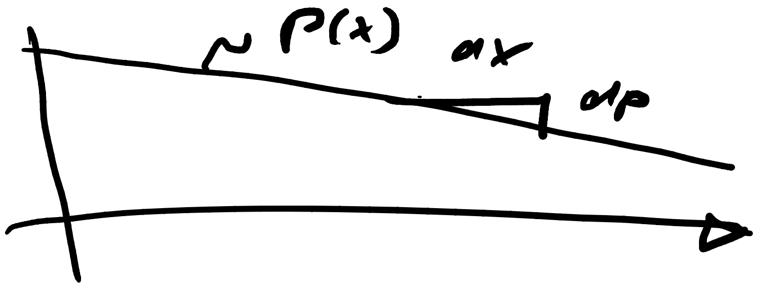
$$\tau_w = \eta \frac{du}{dy} \rightsquigarrow \frac{\tau_w}{\eta} y + \underbrace{\text{const}}_{=0} = u(y)$$

Haftbed.

Druckgetriebene Strömg. $\overline{\frac{\partial}{\partial t}} \equiv 0$

$$\sigma = -\frac{dp}{dx} + \frac{d}{dy}(\tau)$$





$$\frac{dp}{dx} y + const = \tau(y)$$

$\|$

0 aus Symmetriegrün.

Schubspannung an der Wand

$$-\frac{dp}{dx} h = \tau_w$$

$$\frac{\tau}{\tau_w} = -\frac{y}{h}.$$

für Newtonsche Rel.:

$$\tau = \eta \frac{du}{dy}$$

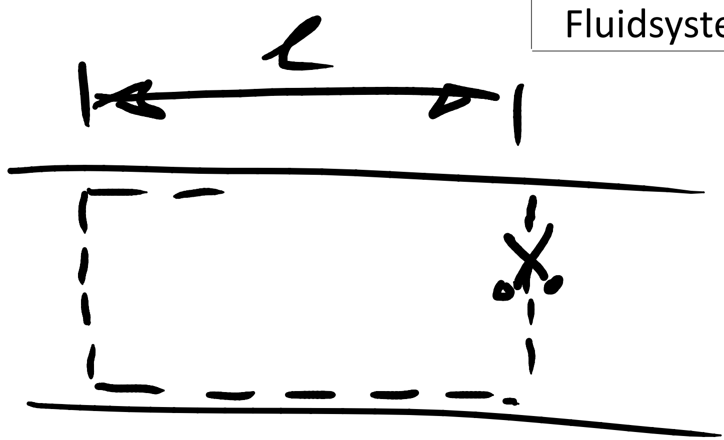
$$\frac{du}{dy} = -\frac{y}{h}\frac{\tau_w}{\eta}$$

$$u = const - \frac{1}{2}\frac{y^2}{h}\frac{\tau_w}{\eta}$$

$u(h) = 0$ Haftbedingung.

↳ $\text{const} = \frac{1}{2} h \frac{\tau_w}{\eta}$

$$u(y) = \underbrace{\frac{1}{2}\frac{\tau_w h}{\eta}}_{u_{max}} \left(1 - \left(\frac{y}{h}\right)^2\right).$$



$$\cancel{p 2h} - (\cancel{p} + \underbrace{\frac{dp}{dx}\cancel{l}}_{\Delta p})\cancel{2}h = \cancel{2}\cancel{l}\tau_w$$

$$\boxed{\tau_w = -\frac{dp}{dx} h}$$

mittlere Strömungsgeschw.

$$\boxed{\bar{u} = \frac{1}{h}\int_0^h u(y)\,dy = \frac{1}{\cancel{2}}\frac{\tau_w h}{\eta}\frac{\cancel{2}}{3} = \frac{1}{3}\frac{\tau_w h}{\eta}}$$

<u>Widerstandsgesetz</u>