Impulssatz; Bernoulli'sche Gleichung

$$\frac{Dm}{Dt} = 0$$

$$\boxed{\frac{D\vec{I}}{Dt} = \vec{F}} \rightarrow \boxed{\text{Cauchy-Gleich.} \quad \rho\frac{D\vec{u}}{Dt} = \nabla\cdot\underset{\sim}{T} + \rho\vec{k}} \rightarrow \text{Bernoulli.}$$

Impuls- in integraler Form

Impulssatz in differentieller Form.

$$\underset{\sim}{T} = -p\underset{\sim}{I} \rightarrow \text{Euler-Gleich.}$$

$$\rho\frac{D\vec{u}}{Dt} = -\nabla p + \rho\vec{k}$$

Navier-Stokes'sche Gleich.

$$\underset{\sim}{T} = -p\underset{\sim}{I} + 2\eta\underset{\sim}{E}$$

$$\rho\frac{D\vec{u}}{Dt} = -\nabla p + \eta\Delta\vec{u} + \rho\vec{k}$$

Impulssatz in integraler Form

Die zeitliche Änderung des Impulses ist gleich der Kraft

$$\frac{D\vec{I}}{Dt} = \vec{F}$$

$$\vec{I} = \int_{V(t)} \rho \vec{u} \, dV$$

$$\vec{F} = \oint_{S} \vec{t} \, dS + \int_{V} \rho \vec{k} \, dV$$

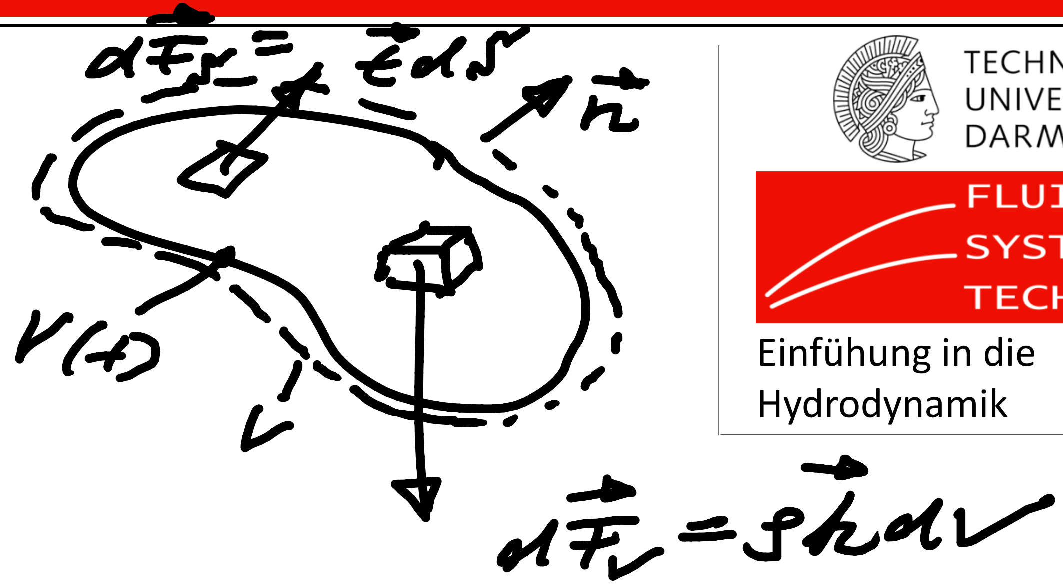
$\frac{D\vec{I}}{Dt} = 0$: Statik

$$\sum \vec{F}_i = 0$$

$$\Longleftrightarrow$$

$$\oint_{S} \vec{t} \, dS + \int_{V} \rho \vec{k} \, dV = 0$$

$$\frac{\partial}{\partial t}\int_V \rho \vec{u}\, dV + \oint_S \rho \vec{u}\, \vec{u}\cdot\vec{n}\, dS = \oint_S \vec{t}\, dS + \int_V \rho \vec{k}\, dV.$$



$$\vec{t} = \vec{n}\cdot\underline{\underline{T}}$$

⊕ Ziel: Kräfte auf benetzte Körper

$$\vec{F}_{Fl\to K} = \int_{S_K} \vec{t}\, dS$$

1. Möglichkeit:
Integration des Spannungsvektors
$\Rightarrow$ $\vec{t} = \vec{n} \cdot \underline{T}$ muß bekannt sein.

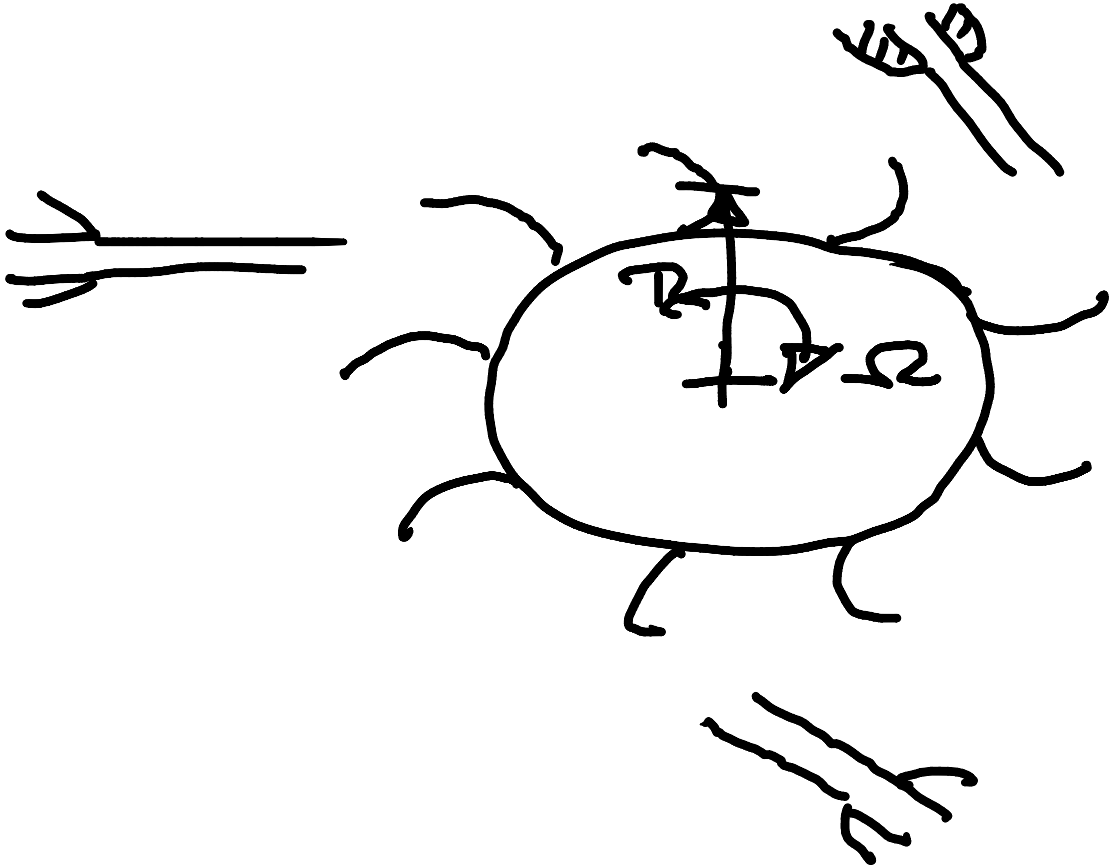
$$\vec{F}_{Fl \to K.} = \int_{S_K} \vec{t} \, dS$$

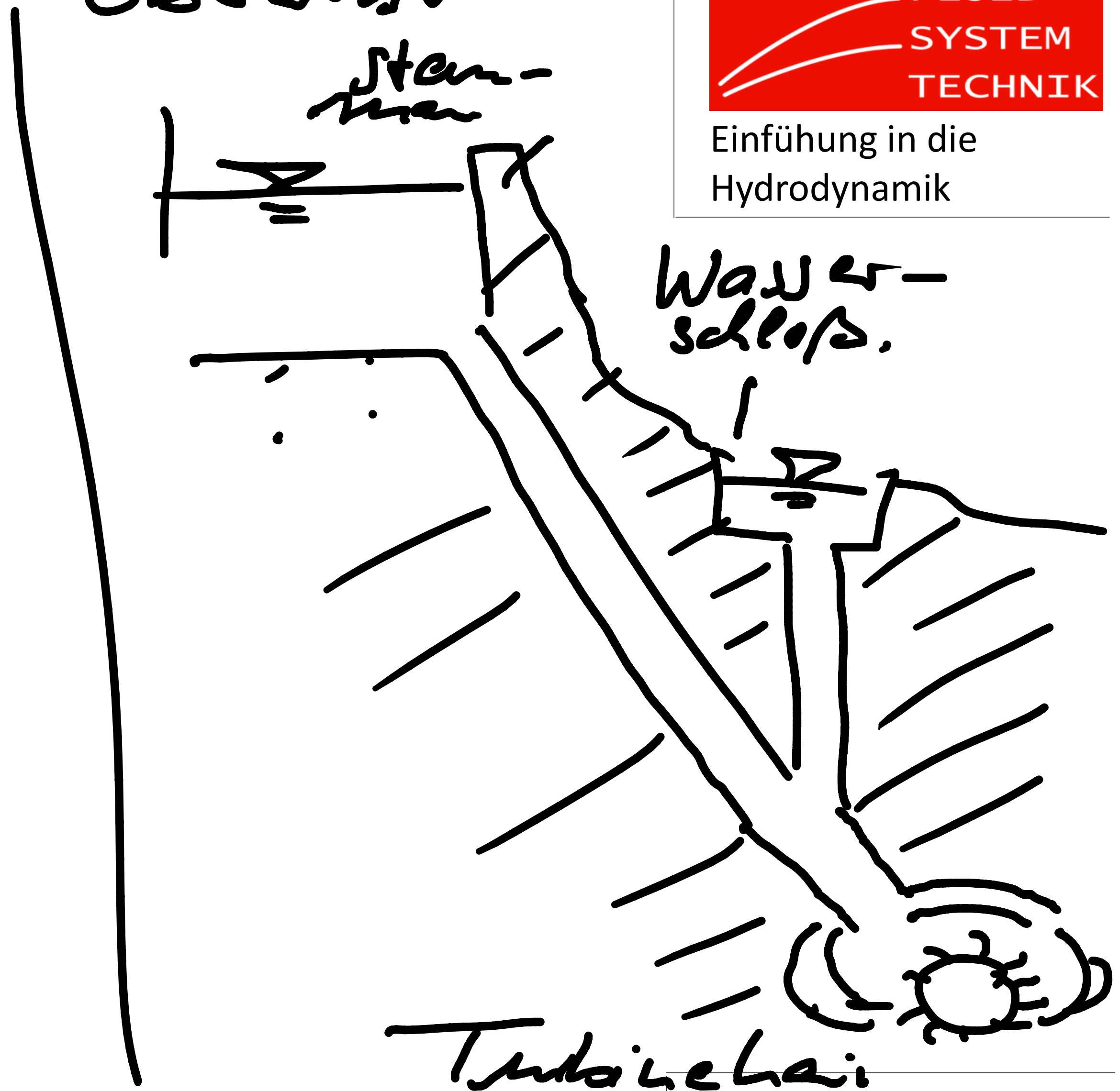
2. Möglichkeit: Impulssatz in integraler Form.

$$S = S_K + A_1 + A_2 \ldots\ldots$$

$$\int_{S_K} = ?$$

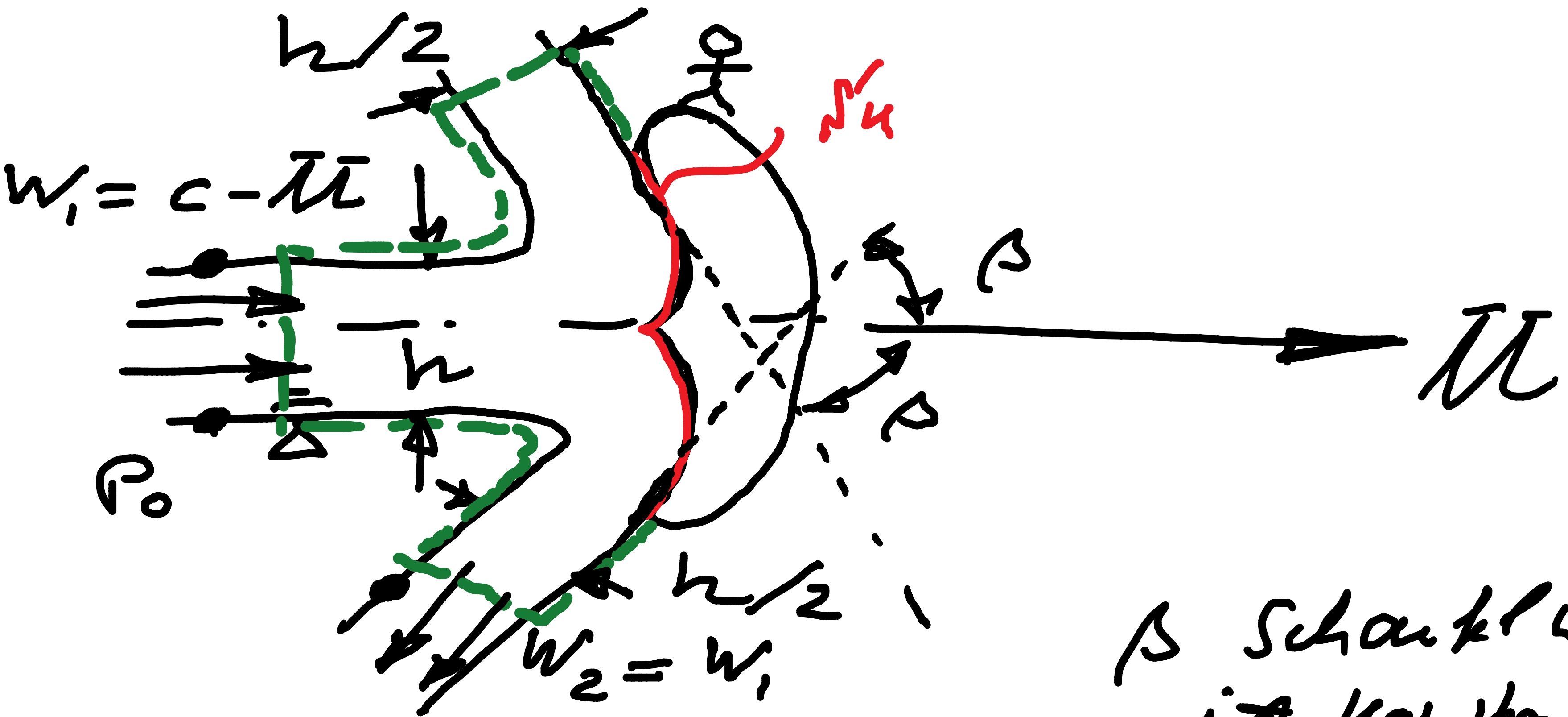
Anwendungsbeispiel Peltonmaschine.





TECHNISCHE UNIVERSITÄT DARMSTADT
FLUID SYSTEM TECHNIK

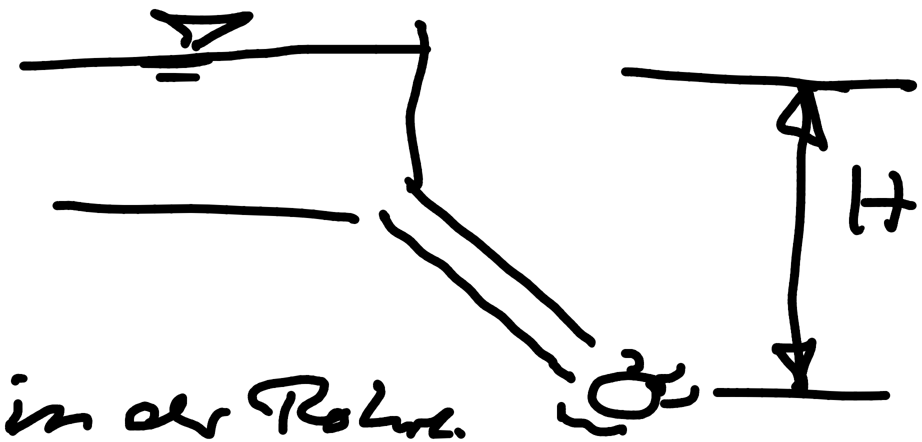
Einfühung in die Hydrodynamik



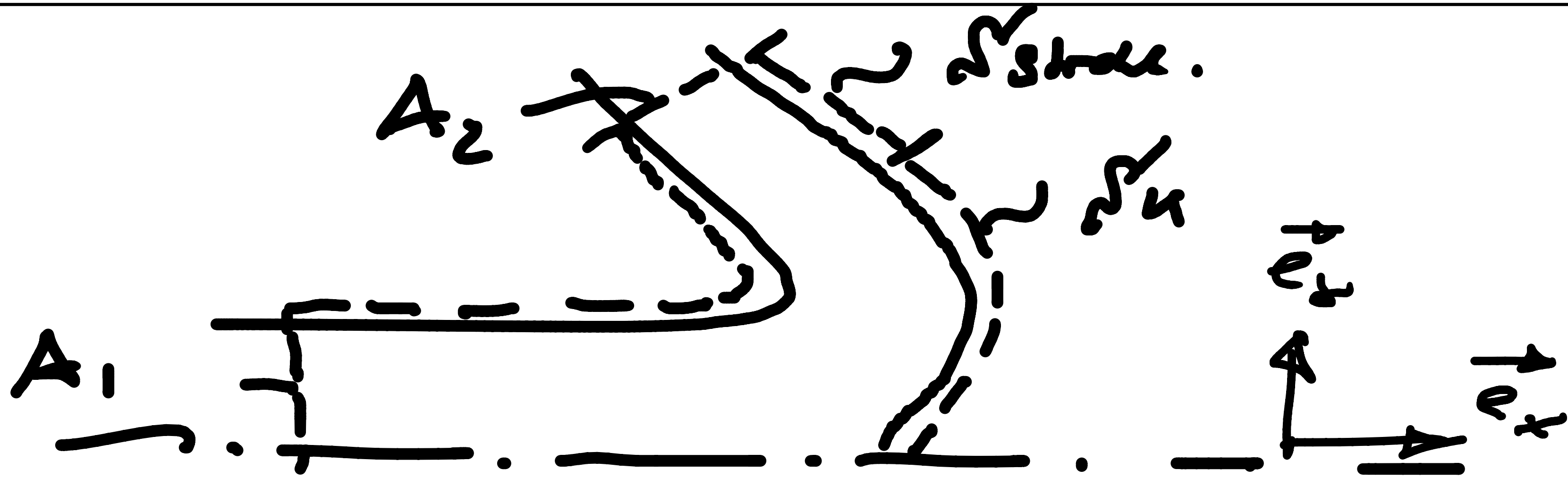
β Schaufelwinkel ist konstant vorgegeben.

Toricelli-Geschw.

$$c_{max} = \sqrt{2gH}$$



$c = c_{max}$, wenn keine Reibung in der Rohre.



$-\vec F_{Fl \to K}$

Stationärer Vorgang: $\sim \frac{\partial}{\partial t} \equiv 0$

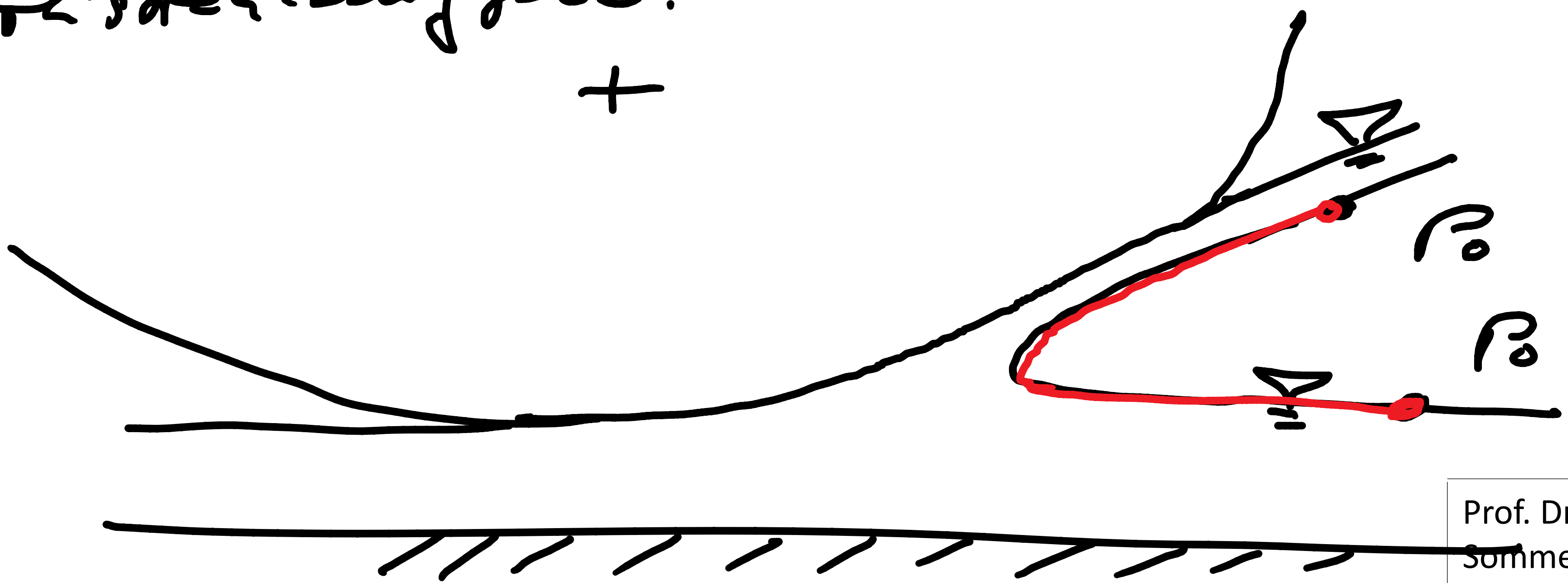
$$\vec e_x \cdot \Big| \int_{A_1+A_2+S_K} \rho \vec u \, \vec u \cdot \vec n \, dS = \underbrace{\int_{A_1+A_2+S_{Strahl}+S_K} -p_0 \vec n \, dS}_{=0} + \overbrace{\int_{S_K} \vec t \, dS}$$

nur Komponente in x-Richt.

$$\rho w_1^2 h_1 - \rho w_2^2 \frac{h}{2} \cos\beta \; 2 = F$$

$$w_1 = w_2 = c - U$$

Freistrahlaufgabe:



Impulssatz in differentieller Form

$$\frac{D}{Dt}\int_{V(t)} \vec{u}\,\rho\, dV = \int_S \vec{n}\cdot \underset{\sim}{T}\, dS + \int_V \rho \vec{k}\, dV$$

$$\int_S \vec{n}\cdot(\ )\, dS = \int_V \nabla\cdot(\ )\, dV$$

$$\int_V \frac{D\vec{u}}{Dt}\rho\, dV + \int \vec{u}\,\frac{D(\rho\, dV)}{Dt} = \int_V \nabla\cdot\underset{\sim}{T} + \rho\vec{k}\, dV$$

$$\frac{D\, dm}{Dt} \equiv 0$$

$$\int_V \rho\frac{D\vec{u}}{Dt} - \nabla\cdot\underset{\sim}{T} - \rho\vec{k}\, dV = 0$$

Die Integrationsgrenzen ✓ ist beliebig $\Longrightarrow$ der Integrand muss verschwinden.

$$\rho \frac{D\vec{u}}{Dt} = \nabla \cdot \underset{\sim}{T} + \rho \vec{k}$$ Cauchy-Gleich.

|| ||

Masse * Beschleunigung = Divergenz des Spannungstensors + Volumenkraft $\rho \vec{k} = \vec{f}$.

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho \vec{u} \cdot \nabla \vec{u} = \ldots\ldots$$

Turbulenz.

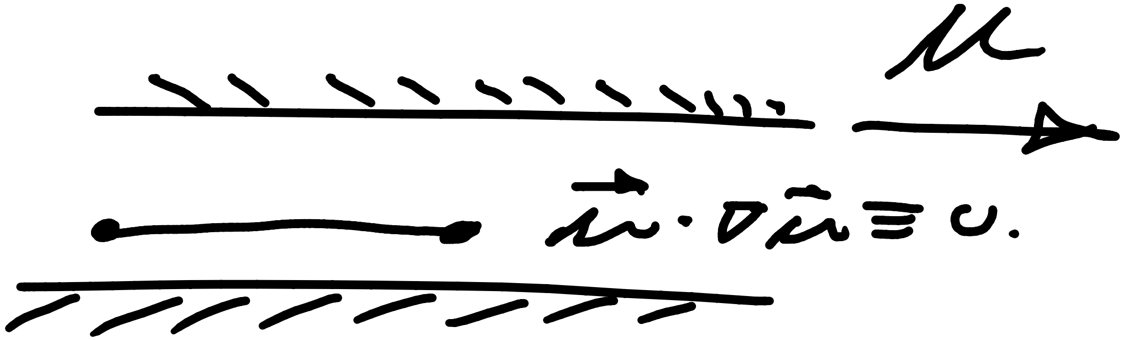
konvektive Beschl.

Nichtlinearität

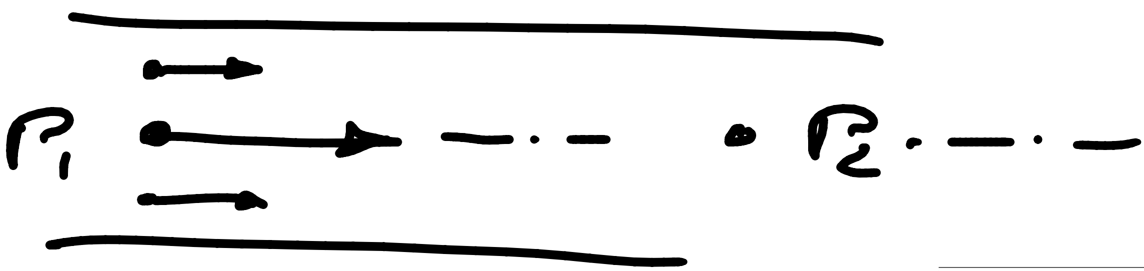
Spezielle kinematische Eigenschaft

$\vec{u}\cdot\nabla\vec{u} \equiv 0$ für laminare Schichtenströmung.

Couette-Strömung



Poiseuille-Strömung

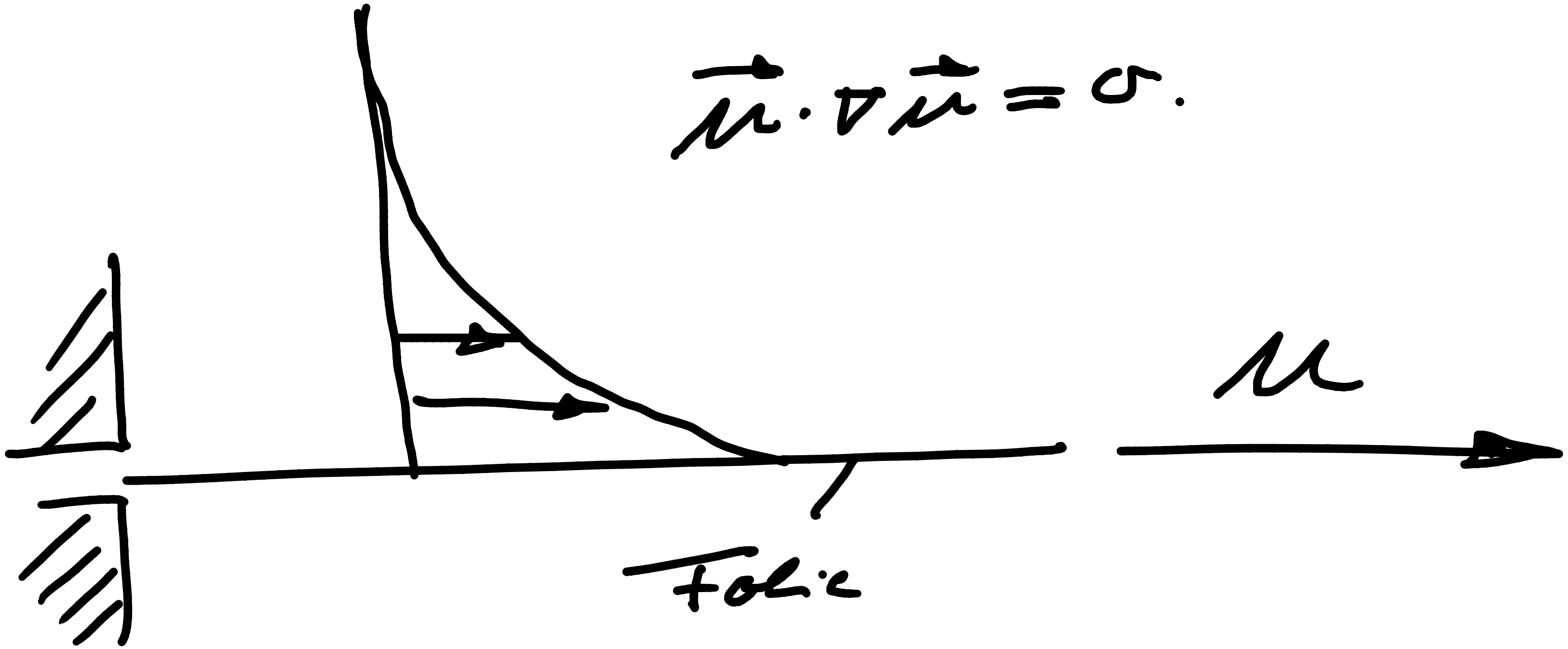


TECHNISCHE UNIVERSITÄT DARMSTADT

FLUID SYSTEM TECHNIK

Einfühung in die Hydrodynamik

$$\vec{u} \cdot \nabla \vec{u} = 0.$$



Navier Stokes-Gl.:

$$\varrho \frac{D\vec{u}}{Dt} = -\nabla p + \eta \Delta \vec{u} + \varrho \vec{k}$$

$$\mathrm{rot}\,\vec{u} = \nabla \times \vec{u} = 0$$

$$\hookrightarrow \eta \Delta \vec{u} \equiv 0$$

$$\vec{k} = -\nabla \psi$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{\nabla p}{\varrho} - \nabla \psi \quad \Big| \cdot \vec{u}$$

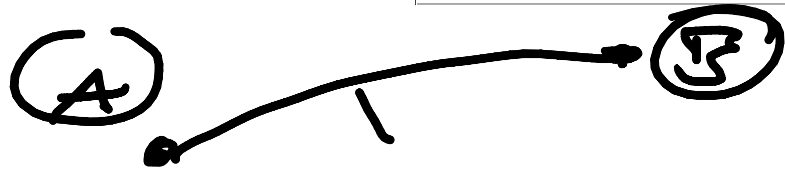
Integration längs einer Stromlinie

$$u \, d\vec{x} = \vec{u} \, ds \qquad u = |\vec{u}|$$

Differentialgleichung längs der Stromlinie..

$$\vec u \cdot \frac{\partial \vec u}{\partial t} + \underbrace{\vec u \cdot (\vec u \cdot \nabla \vec u)}_{2} =$$

$$= -\vec u \cdot \frac{\nabla p}{\rho} - \vec u \cdot \nabla \psi$$



Stromlinie

② $$\vec u \cdot (\vec u \cdot \nabla \vec u) = \vec u \cdot \nabla \left(\frac{u^2}{2} \right)$$

$$\vec u \cdot \nabla = u \frac{d}{ds}$$ längs der Stromlinie

$$\frac{\partial u}{\partial t} + \frac{d}{ds}\left(\frac{u^2}{2} \right) = -\frac{1}{\rho}\frac{dp}{ds} - \frac{d\psi}{ds}$$

Integration längs der Stromlinie

$$\boxed{\int \frac{\partial u}{\partial t}\, ds + \frac{u^2}{2} + \int \frac{dp}{\rho} + \psi = C}$$

C Bernoullische Konstante.

$$P = \int \frac{dp}{\rho} \quad \text{Druckfunktion}$$

$= \frac{p}{\rho}$ für $\rho = const$

$= \frac{\gamma}{\gamma - 1} \frac{p}{\rho}$ für $p = C \rho^{\gamma}$ Isentrope Fl.

$$\frac{Dm}{Dt} = 0 \iff \frac{D\rho}{Dt} + \rho\, \nabla \cdot \vec{u} = 0$$

$$\frac{\partial}{\partial t}\int_V \rho \, dV + \oint_S \rho\, \vec{u} \cdot \vec{n}\, dS = 0$$

$\vec{u}$ Absolutgeschw.

$$\left[\frac{\partial}{\partial t}\int_V \rho \, dV\right]_B + \oint_S \rho\, \vec{w} \cdot \vec{n}\, dS = 0$$

$\vec{u}$ Umfangsgeschw.

$\vec{w}$ Relativgeschw.

$\hat{\vec{u}} \mathrel{\hat{=}} \vec{c}$ Absolutgeschw.

$\vec{v}$ Führungsgeschw.