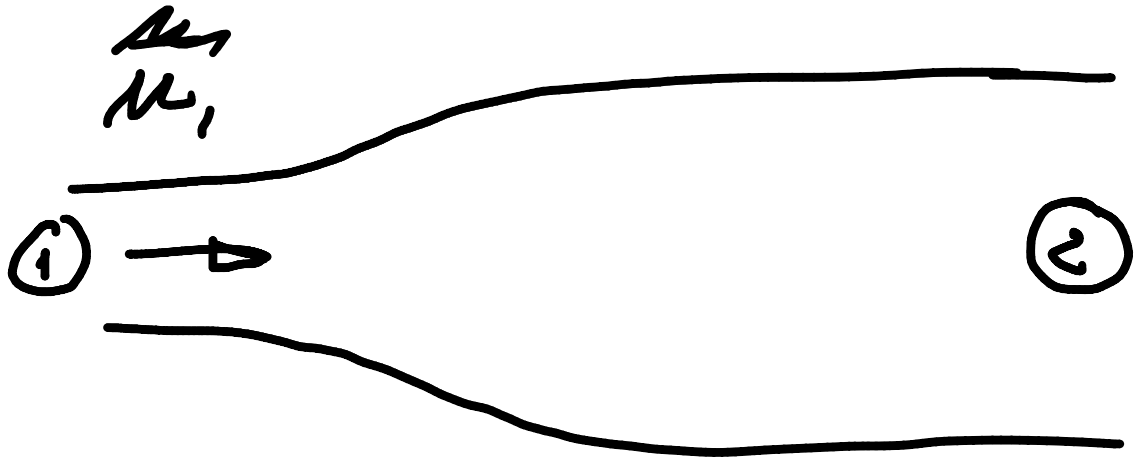
Schichtenströmung; Turbulenz

$$\boxed{p_1 + \frac{\rho}{2}u_1^2 + \rho g z_1 = p_2 + \frac{\rho}{2}u_2^2 + \rho g z_2} + \Delta p_v$$



Δp_v – Trägheits Druckverlust
Δp_v – viskoser Druckverlust

$$\zeta = \frac{\Delta p_v}{\frac{\rho}{2}u_1^2}$$ dimensionsloser Druckverlust

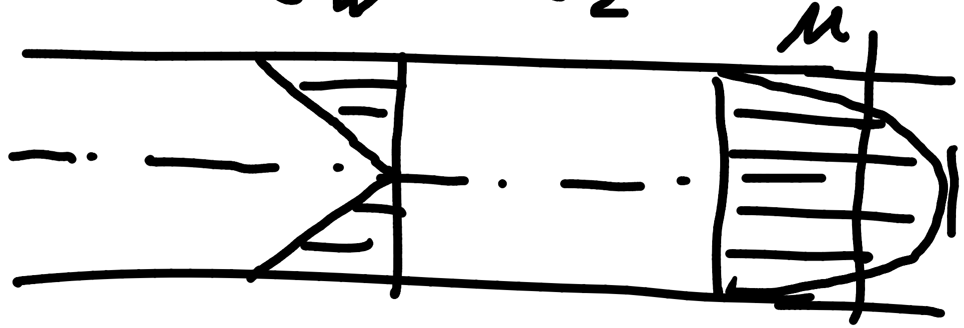
Bei Verlusten, die proportional zur Länge sind, ist es sinnvoll einen Verlust pro Längeneinheit zu definieren

laminare Rohrströmung.

$$\zeta := \frac{L}{d}\lambda$$

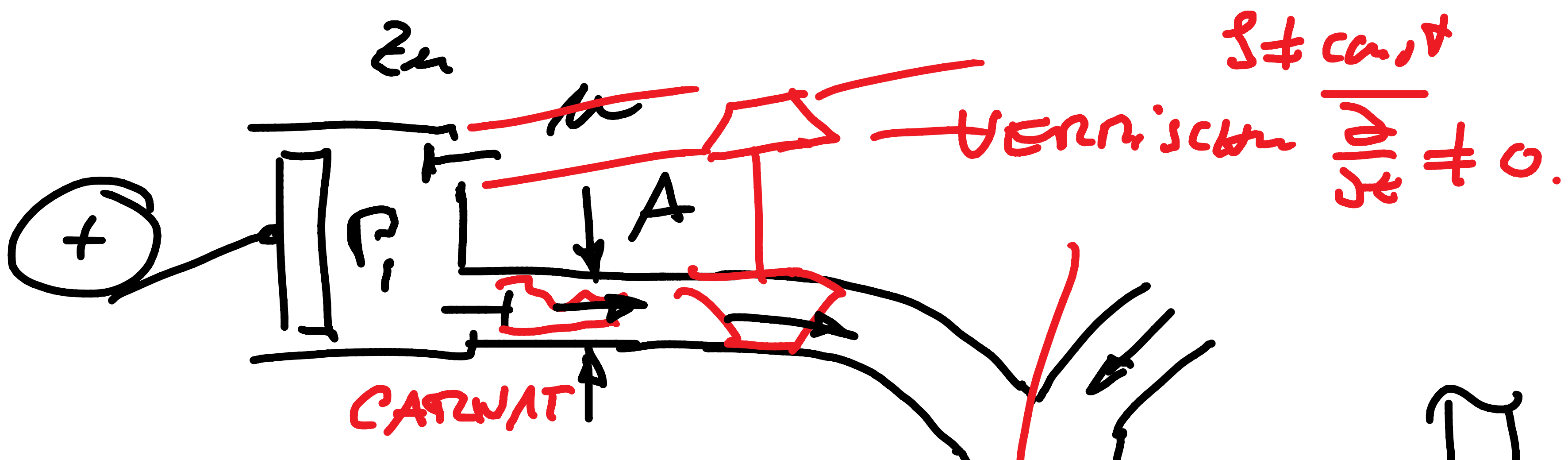
$$\tau_w = \lambda \frac{\rho}{2}\bar{u}^2$$

λ Widerstandszahl.



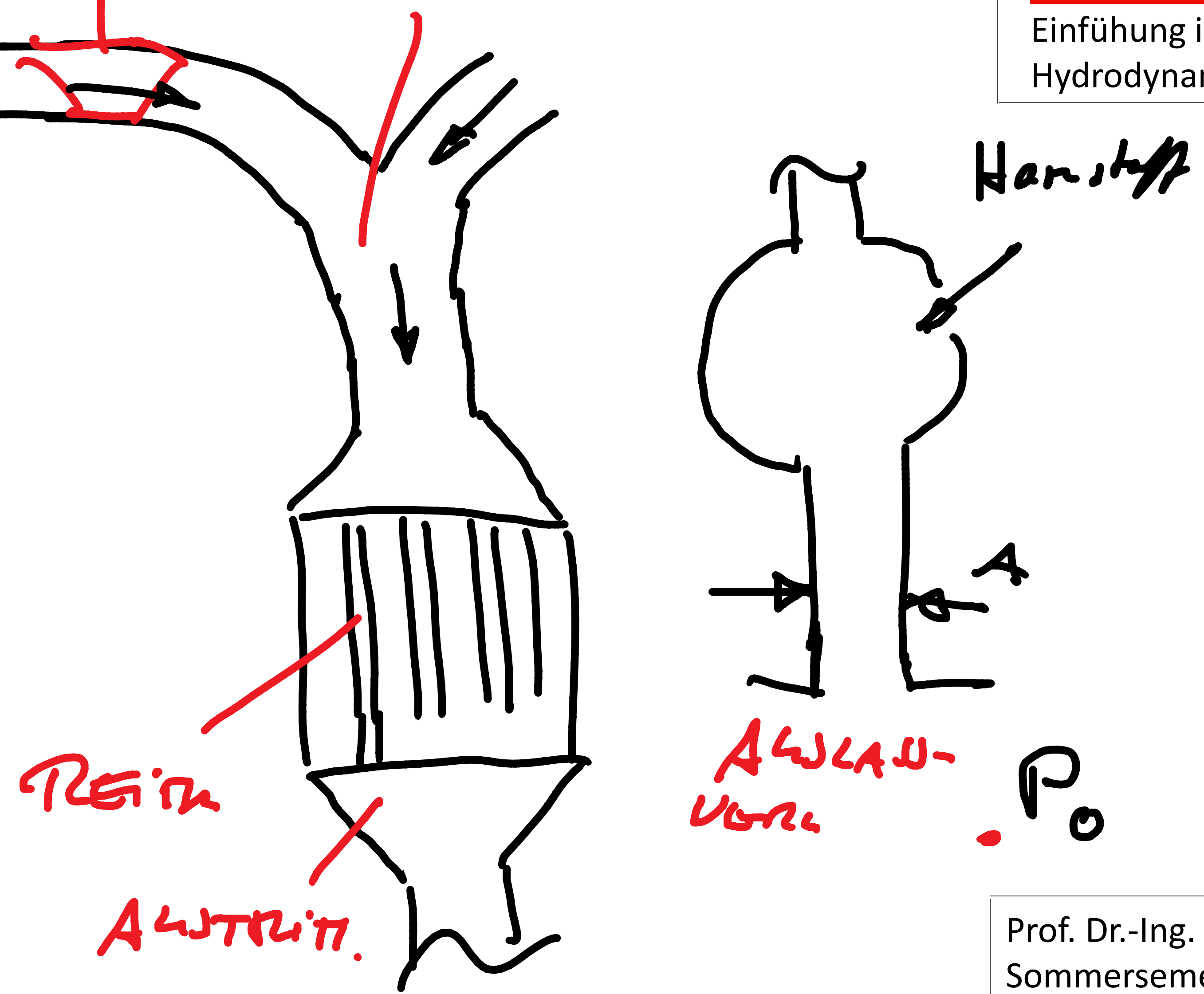
$$\tau = \tau_w \frac{r}{R} \qquad u_{max} = 2u$$

$$\tau = \eta \frac{du}{dr}$$



$$\Delta p_v = \sum_{i=1}^{N} \Delta p_{vi}$$

$$= \frac{\rho}{2} u^2 \sum_{i=1}^{N} \zeta_i$$

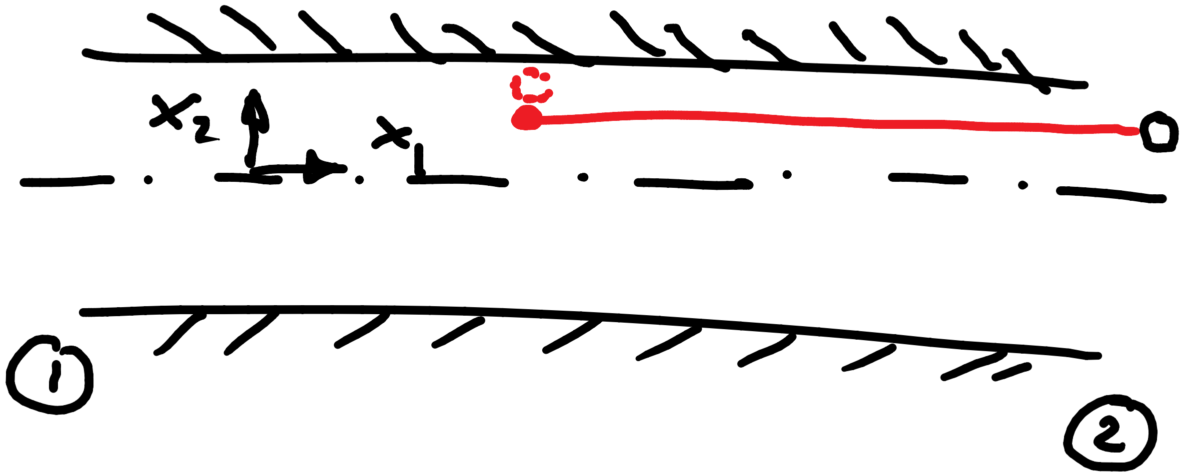


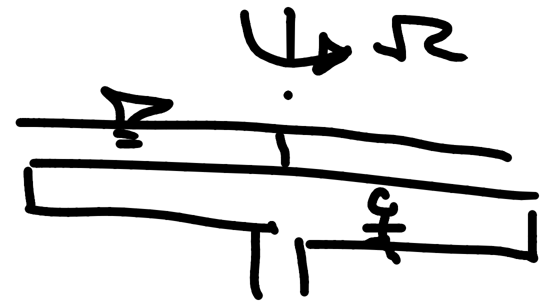
TECHNISCHE UNIVERSITÄT DARMSTADT

FLUID SYSTEM TECHNIK

Einfühung in die Hydrodynamik

laminare Strömung für den ebenen Kanal





$$\rho \frac{Du_i}{Dt} = f_i + \frac{\partial}{\partial x_j} \tau_{ij}$$

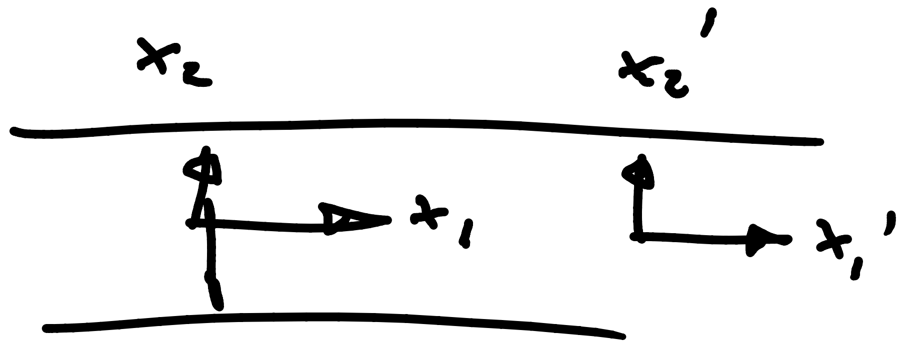
$$0 = f_i + \frac{\partial}{\partial x_j} \tau_{ij} \quad \text{für stationäre Schichtstr.}$$

$$f_1 = i B = \text{const} \qquad f_2 = 0$$

Tip: Wie sieht τ für $f_i \neq 0$ aus?

$$f_i \equiv 0$$

$$0 = \frac{\partial}{\partial x_j} \tau_{ij}$$



$$\frac{\partial}{\partial x_1} \equiv 0.$$

$i = 1$:

$$0 = \frac{\partial}{\partial x_1} \tau_{11} + \frac{\partial}{\partial x_2} \tau_{12}$$

$$\tau_{ij} = -p\,\delta_{ij} + P_{ij}$$

$$0 = \underbrace{-\frac{\partial p}{\partial x_1}}_{\text{Druckgrad.}} + \frac{\partial}{\partial x_2}\underbrace{P_{12}}_{\tau}$$

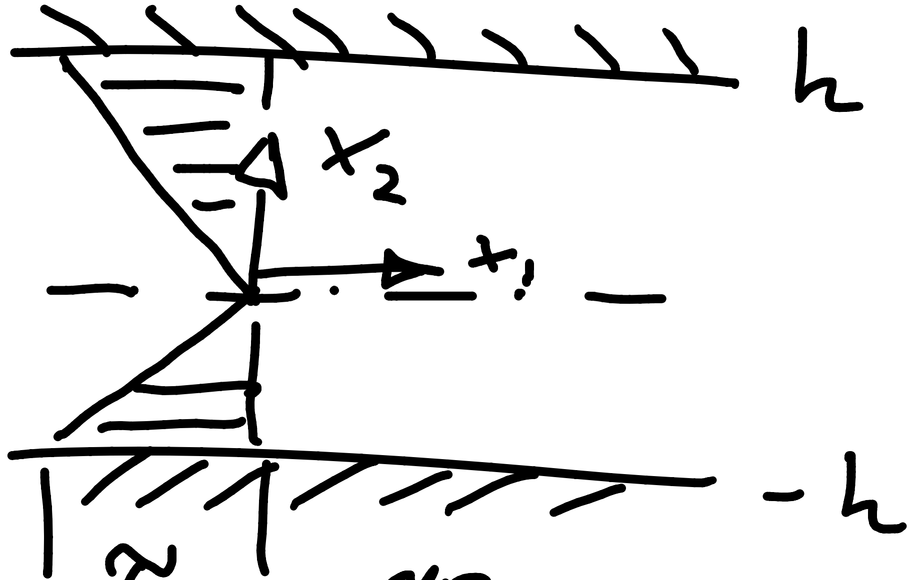
$i = 1$

$$C' = -\frac{\partial p}{\partial x_1} x_2 + \tau$$

$C' \equiv 0$ aus Symmetrie.

$$\tau(x_2 = 0) = 0$$

$$\tau(x_2) = \frac{\partial p}{\partial x_1} x_2$$



$$\tau_w = \frac{dp}{dx} h$$

Bei <u>laminaren Schichtenströmungen</u> können die Geschwindigkeitsfelder superponiert werden?

Grund: Die Bewegungsgleichung ist linear

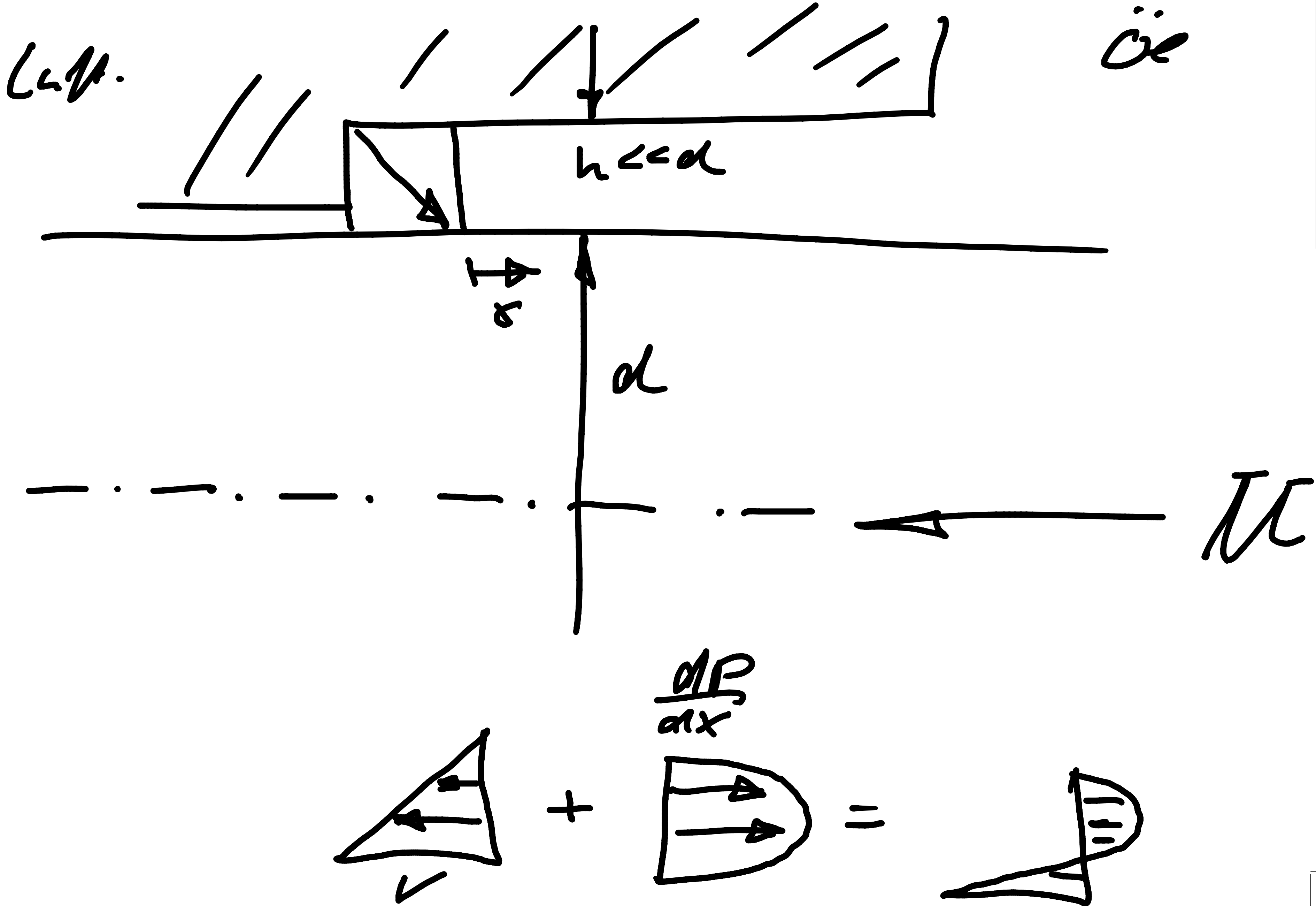
$u_1 \frac{\partial u_i}{\partial x_1} = 0$ bei laminarer Schicht

$$\underbrace{\rho \frac{Du_i}{Dt}}_{\text{lin}} = -\frac{\partial p}{\partial x_i} + f_i + \underbrace{\eta \frac{\partial^2 u_i}{\partial x_j \partial x_j}}_{\text{lin}}$$

$$\underbrace{\rho \frac{\partial u_i}{\partial t}}_{\text{lin}} + \cancel{u_j \frac{\partial u_i}{\partial x_j}} = \dots$$

Konvektiven Term.

nichtlinear.



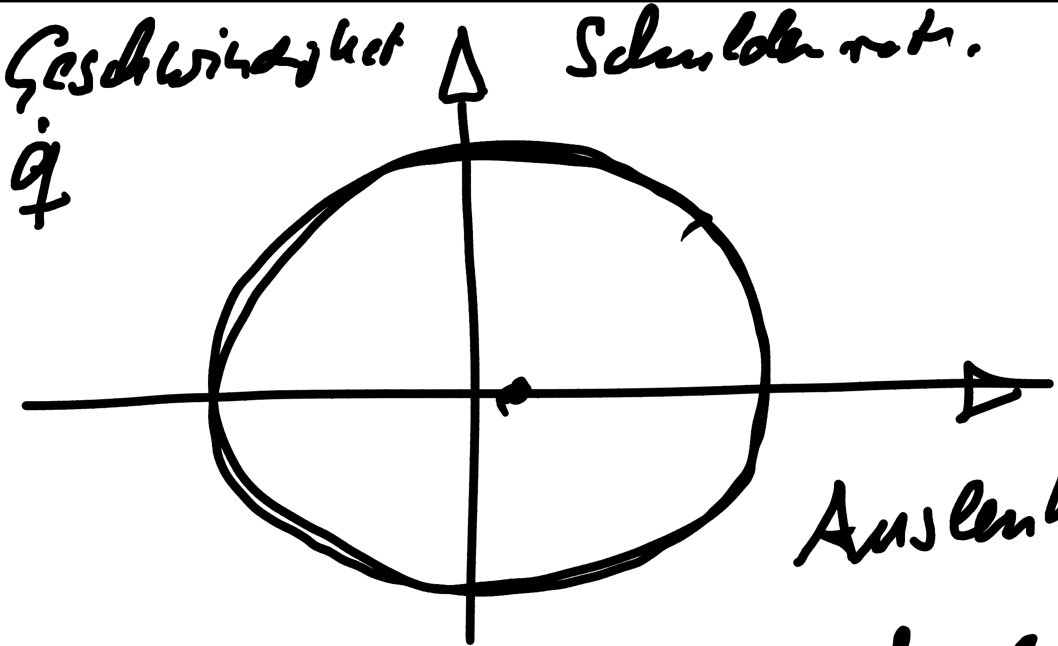
TECHNISCHE UNIVERSITÄT DARMSTADT

FLUID SYSTEM TECHNIK

Einfühung in die Hydrodynamik

Geschwindigkeit $\dot{q}$

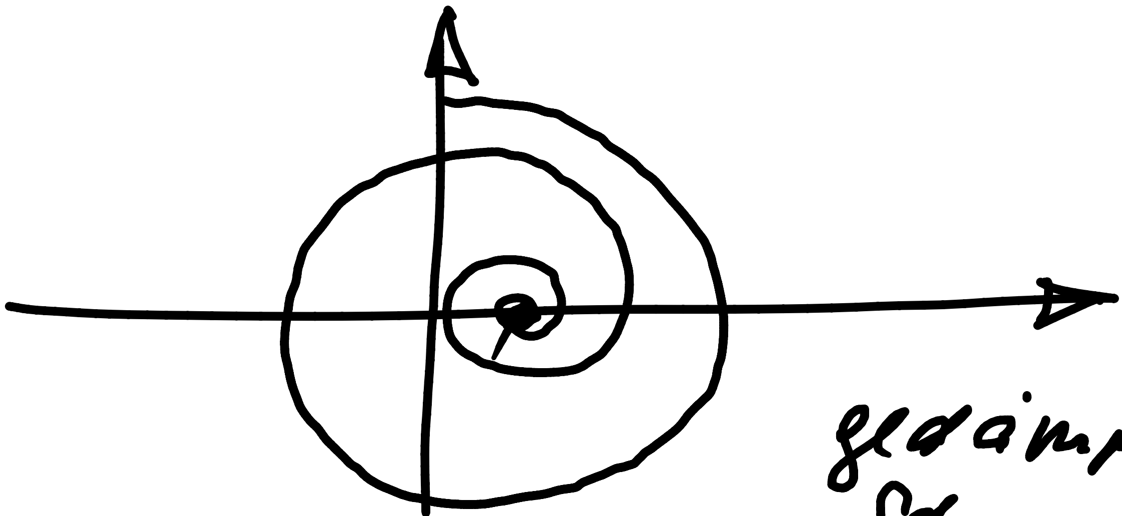
Schwebel rot.



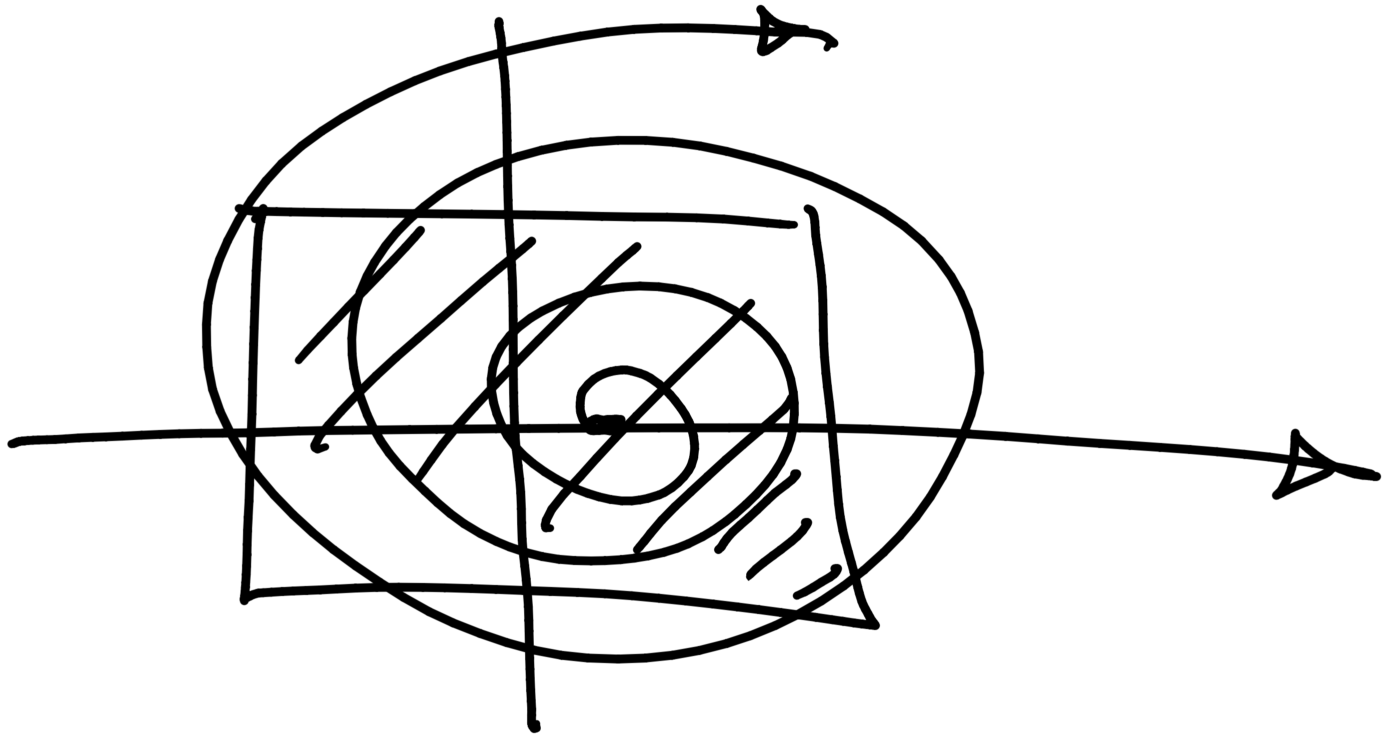
Auslenkung q

Schwebelstand

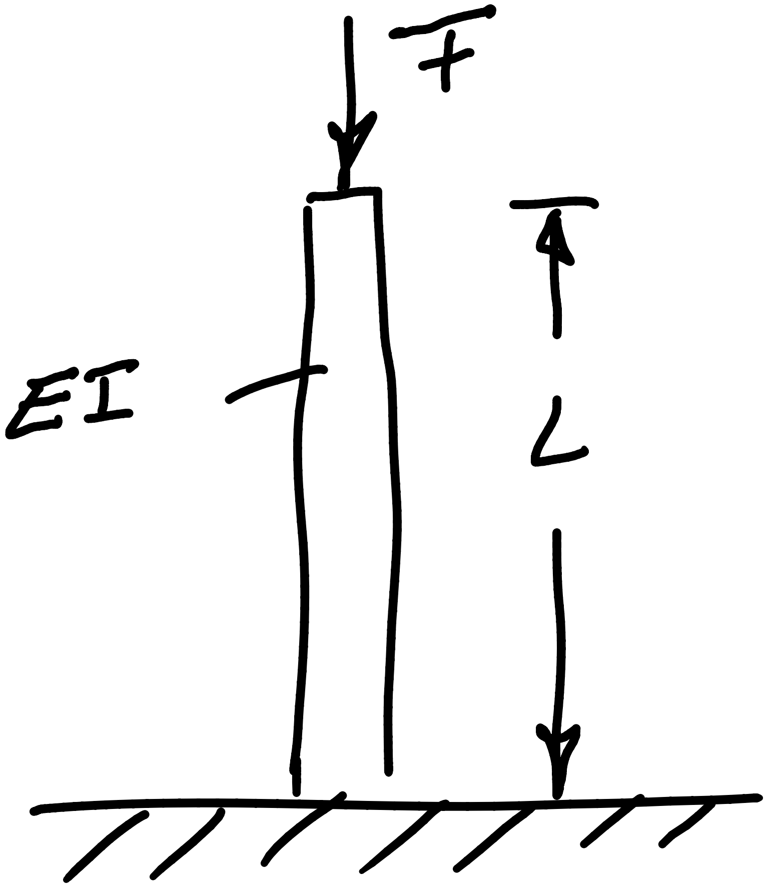
erregte Schwing. od. ungedämpfte frei Sch.

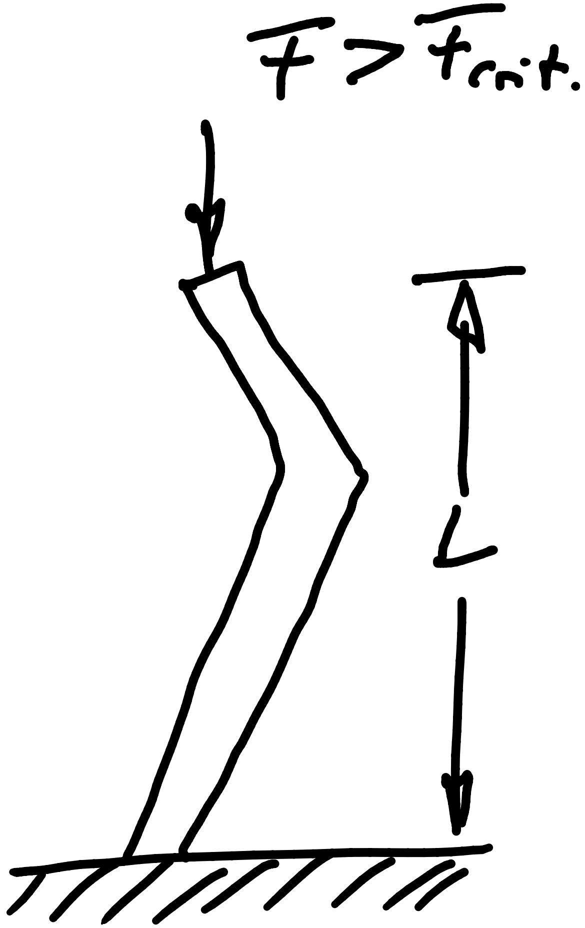


gedämpfte frei. Sch.



angefachte Schwing





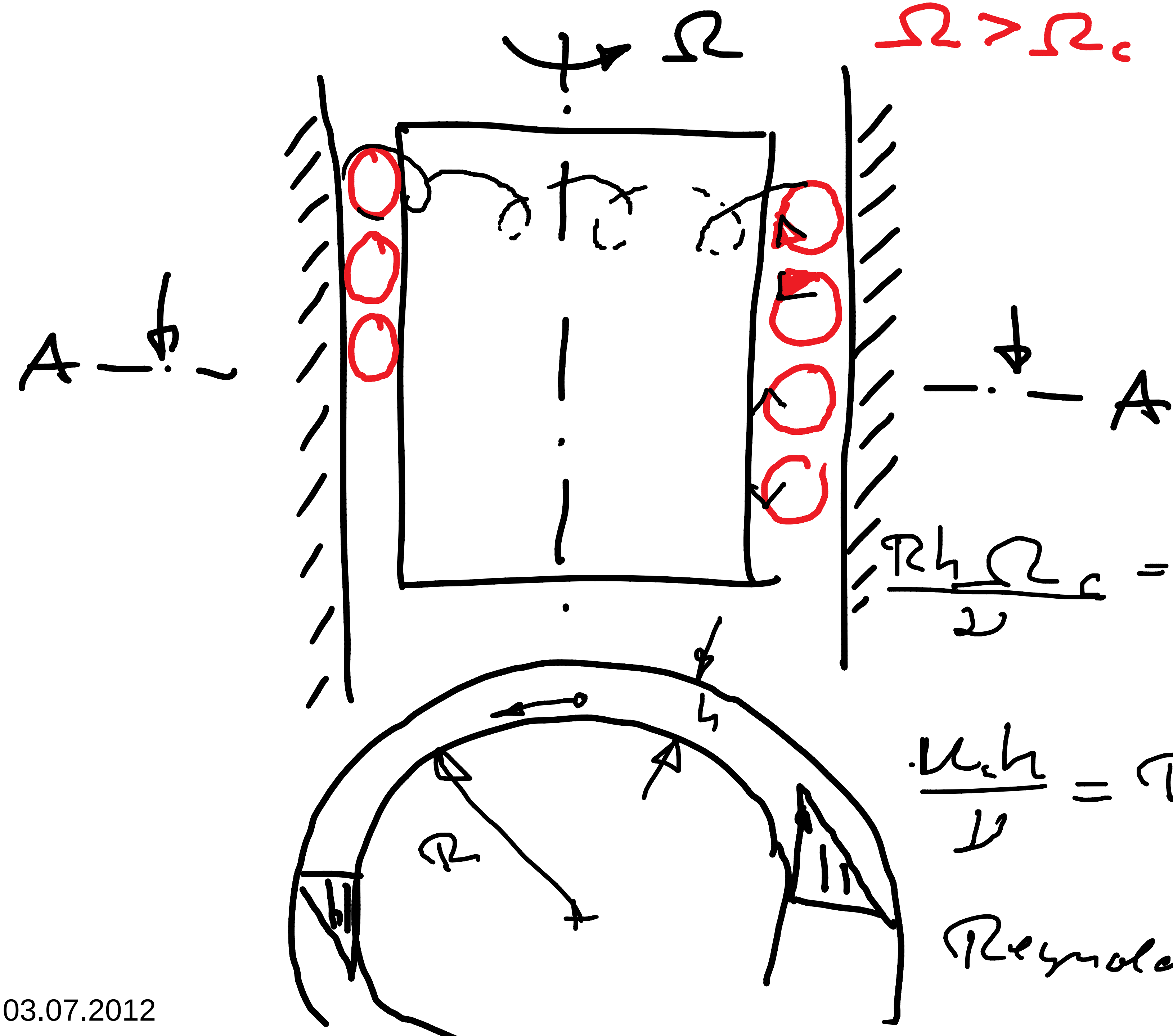
$$F_c = \frac{EI}{L^2} \frac{1}{12}$$ Eulerknickfall

$F < F_c$ statische Problem

$F > F_c$ dynamisches Problem.

Verzweigungsproblem $\longrightarrow$ Bifurcation.

$\Omega > \Omega_c$



$$\frac{R h \Omega_c}{\nu} = 41.3 \sqrt{\frac{R}{h}}$$

$$\frac{u_c h}{\nu} = Re \qquad u_c = R\,\Omega_c$$

Reynoldszahl

$$\text{Reynoldszahl} = \frac{\text{Trägheitsspannung}}{\text{viskose Spannung}} = \frac{\rho u^2}{\eta \frac{u}{h}}$$

$$= \frac{uh}{\eta/\rho}$$

$$= \frac{uh}{\nu}$$

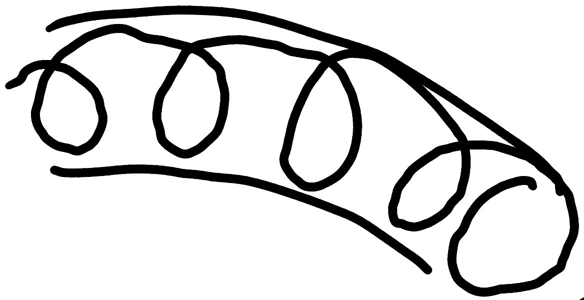
$Re > Re_c$, dann bilden sich räumliche Strukturen aus.

$Re \gg Re_c$, dann bilden sich chaotisch zeitlich & räumlich Strukturen.

$Re < Re_c$, dann ist die Strömung laminar.

$Re < Re_c$

$Re > Re_c$



$Re \gg Re_c$

Ludwig Prandtl.
G.I. Taylor.
von Karman
Kolmogorov

Turbulenz ballen.

vollkommen turbulente Strömung

dreidimensional, instationär, chaotisch.