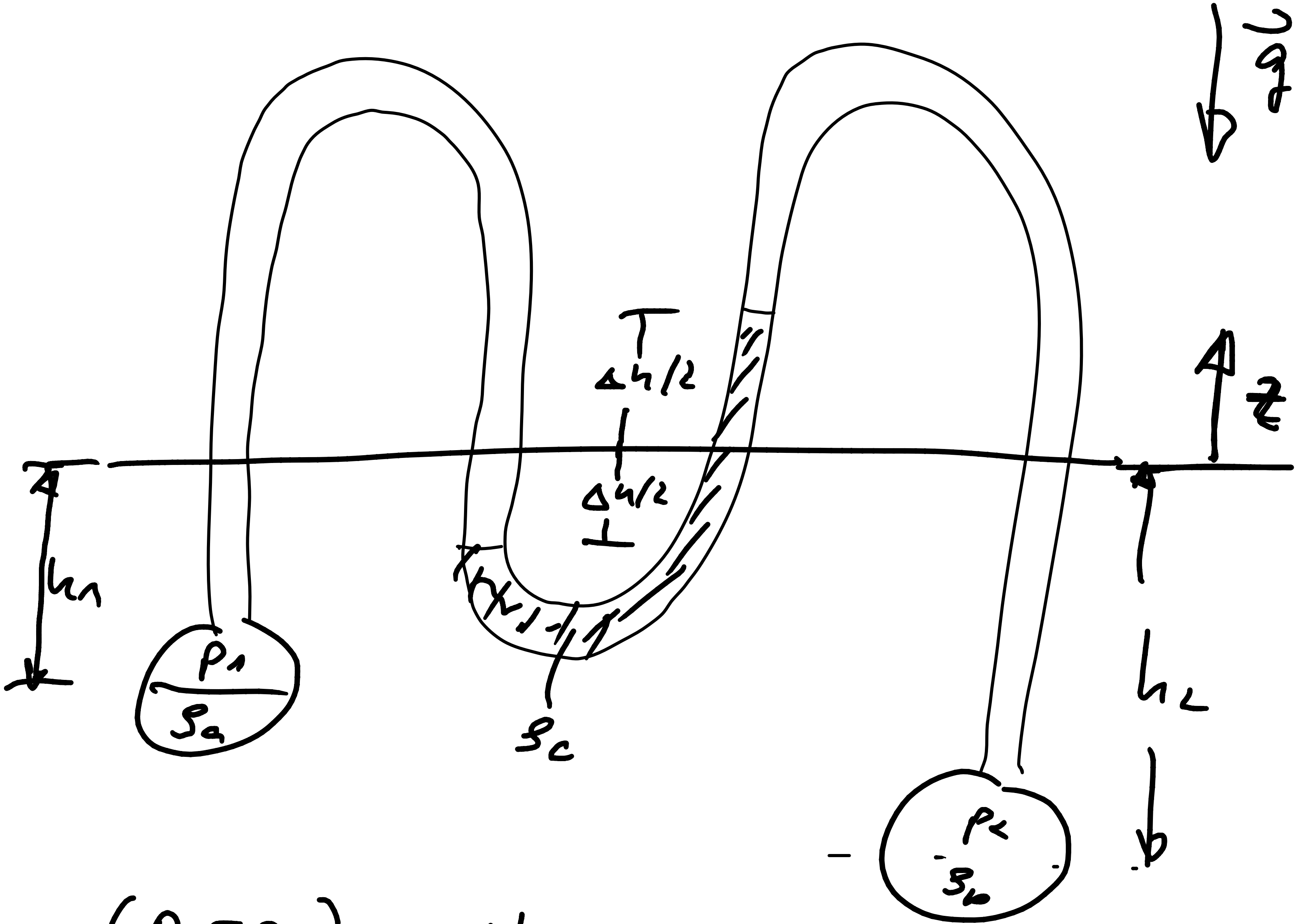

$$(p_1 - p_2) = \Delta h$$

$$p + \rho g z = \text{const}$$

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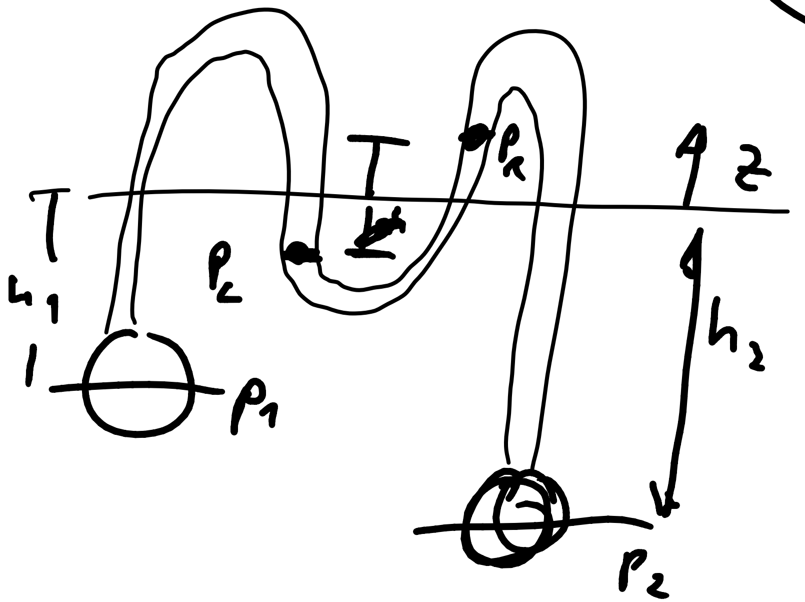
$$p_1 + \cancel{\frac{\rho}{2}u_1^2}^{\,0} + \rho g z_1 = \text{const}$$

$$p + \rho g z = \text{const}$$



$$p_1 + \rho_a g(-h_1) = p_L + \rho_a g\left(-\frac{\Delta h}{2}\right)$$

$$p_R + \rho_b g\left(\frac{\Delta h}{2}\right) = p_2 + \rho_b g(-h_2)$$

$$p_L + \rho_c g\left(-\frac{\Delta h}{2}\right) = p_R + \rho_c g \frac{\Delta h}{2}$$

3 Gl

3 Unbekannte $p_L, p_R, p_1 - p_2$

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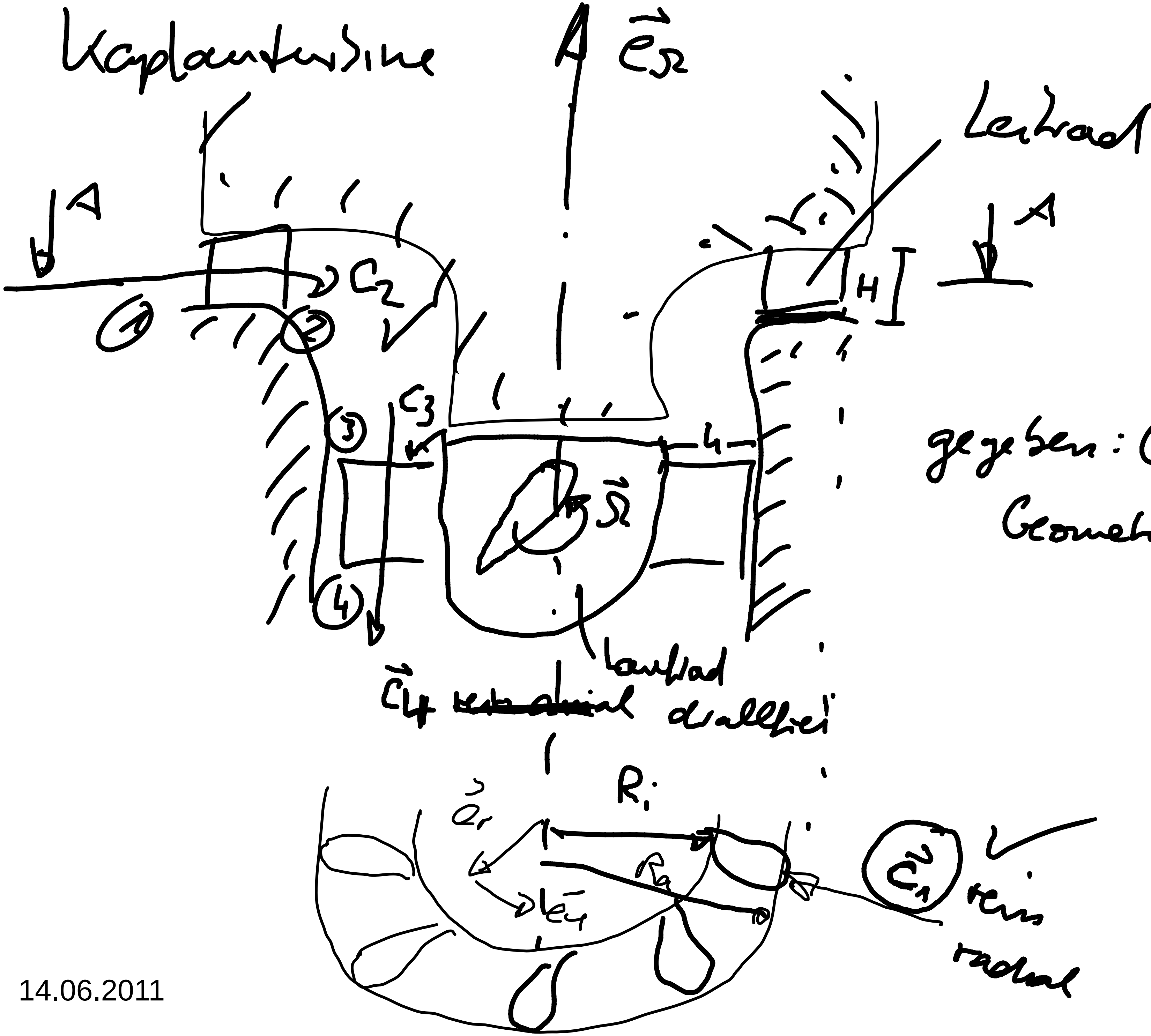
Kaplanturbine



gegeben: Q, ρ

Geometrie

$\vec{c}_4$ ~~rein axial~~ drallfrei

$\vec{c}_1$ rein radial

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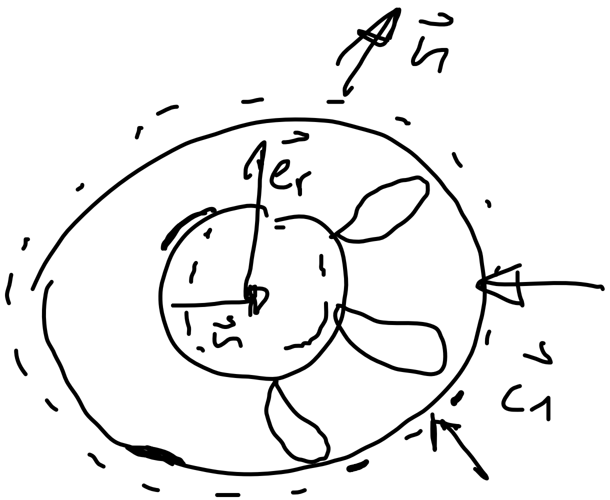
1) $\vec{c}_1 = ?$

$$Q = -\int_{S_e} \vec{c} \cdot \vec{n} \, dS$$

$\vec{c} = c_{r1} \vec{e}_r$



$$\boxed{\begin{aligned} \vec{n} &= \vec{e}_r \\ \vec{c}_1 &= c_{1r} \cdot \vec{e}_r \\ dS &= R_a \, d\varphi \, dz \end{aligned}}$$

$$Q = -\int_{S_e} \underbrace{c_{r1} \vec{e}_r}_{\vec{c}_1} \cdot \underbrace{\vec{e}_r}_{\vec{n}} \, dS = -\int_0^{2\pi} \int_0^{H} c_{r1} R_a \, dz \, d\varphi$$

$$\Rightarrow Q = -2\pi R_a H c_{r1}$$

$$\Rightarrow c_{r1} = -\frac{Q}{2\pi R_a H}$$

$$\vec{c}_1 = c_{r1} \vec{e}_r = -\frac{Q}{2\pi R_a H} \vec{e}_r$$

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b) $\vec{c}_2 = ?$ $\qquad \underbrace{c_{2u}\, e_\varphi \cdot \vec{n}}_{0}$

$$\underline{\vec{c}_2} = c_{2r}\,\vec{e}_r + \underline{c_{2u}}\,\vec{e}_u$$

$u \hat{=} \varphi$

$\vec{e}_\varphi \hat{=} \vec{e}_u$

c_{r2} : über Volumenstrom

$$Q = (+)\int \vec{c}\cdot\vec{n}\, dS \qquad \vec{n} = (-)\,\vec{e}_r$$

$$dS = R_i\, d\varphi\, dz$$

Einsetzen: $Q = \int_0^{2\pi}\int_0^{H} -c_{r2}\, R_i\, dz\, d\varphi$

$$\Rightarrow c_{r2} = -\frac{Q}{R_i\, 2\pi H}$$

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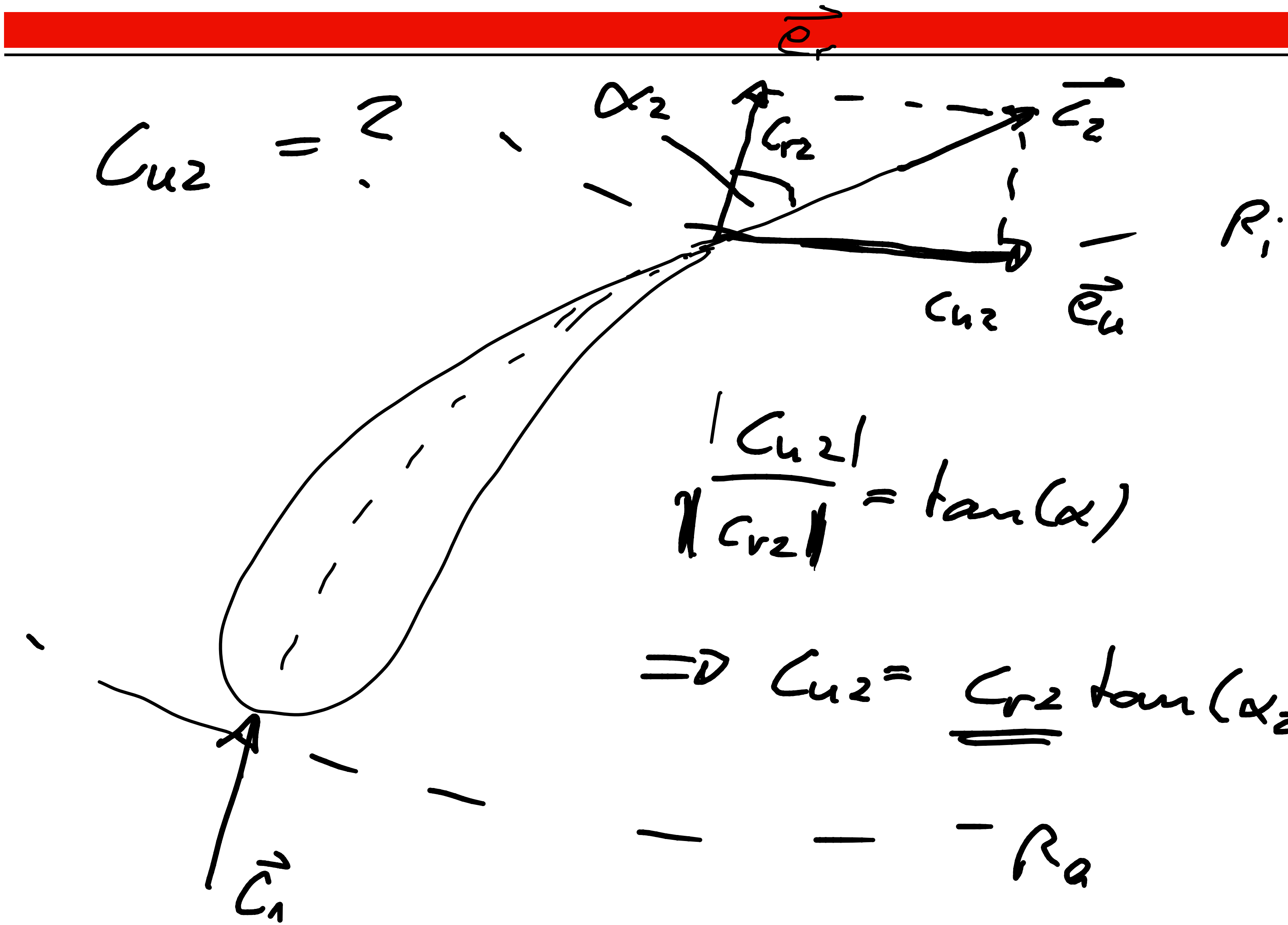
$$c_{u2} = ?$$



$$\frac{|c_{u2}|}{\|c_{r2}\|} = \tan(\alpha)$$

$$\Rightarrow c_{u2} = \underline{\underline{c_{r2}}} \tan(\alpha_2)$$

$$\Rightarrow c_{u2} = \frac{Q}{R_i \, 2\pi H} \tan(\alpha_2)$$

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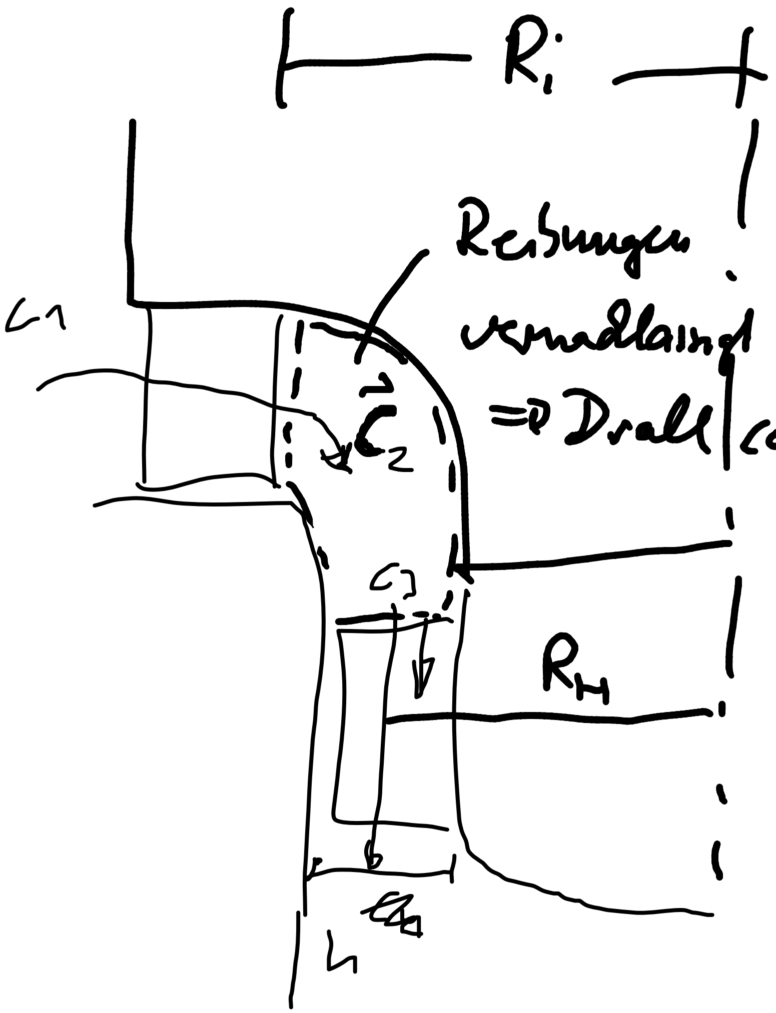

Reibungen vernachlässigt $\Rightarrow$ Drall constant

$$\underbrace{0 = \cancel{\dot m}\,(r_2 c_{u2} - r_1 c_{u1})}_{\text{Eulersche Turbinengl.}}$$

$$c_{u1} \hat{=} c_{2u}$$

$$c_{u2} \hat{=} c_{3u}$$

$$\Rightarrow R_2 c_{u3} - R_i c_{u2}) = 0$$

$$\Rightarrow c_{u3} = \frac{R_i}{R_2} c_{u2}$$

$$h \ll R_H$$

$$c_3 = c_{u3}\vec{e}_u + c_{r3}\vec{e}_r$$

$$\Rightarrow c_{u3} = \frac{Q}{2\pi R_i H} \tan(\alpha_2)$$

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c_{r3}:

$$Q = \int_{S_3} \vec{c} \cdot \vec{n} \, dS$$

$$\vec{n} = -\vec{e}_r \qquad dS = R_L \, dr \, d\varphi$$

$$\vec{c} = \vec{c}_3 = c_{u3} \vec{e}_\varphi + c_{r3} \vec{e}_r$$

$$\Rightarrow \quad Q = -2\pi R_L h \, c_{r3}$$

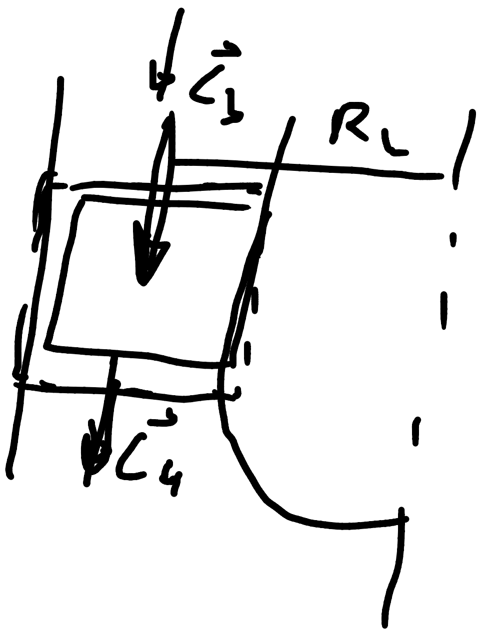
$$\Rightarrow \quad c_{r3} = -\frac{Q}{2\pi R_L h}$$

c_3 ✓

Abgenommene Turbinenleistung P_T

$$\vec{P} = \vec{\Omega} \cdot \vec{M} = -\vec{P}_T$$

$$\vec{\Omega} = \Omega \vec{e}_\Omega \qquad \vec{M} = \dot{m}\left(r_1 c_{u1} - r_2 c_{u2}\right)\vec{e}_\Omega$$



$$\left(R_L c_{u3} - R_L c_{u4}\right)$$

$c_{u4} = 0$, da drallfreie Abströmung

$$\Rightarrow \vec{P}_T = P_T \vec{e}_\Omega = \Omega \dot{m} R_L c_{u3}$$

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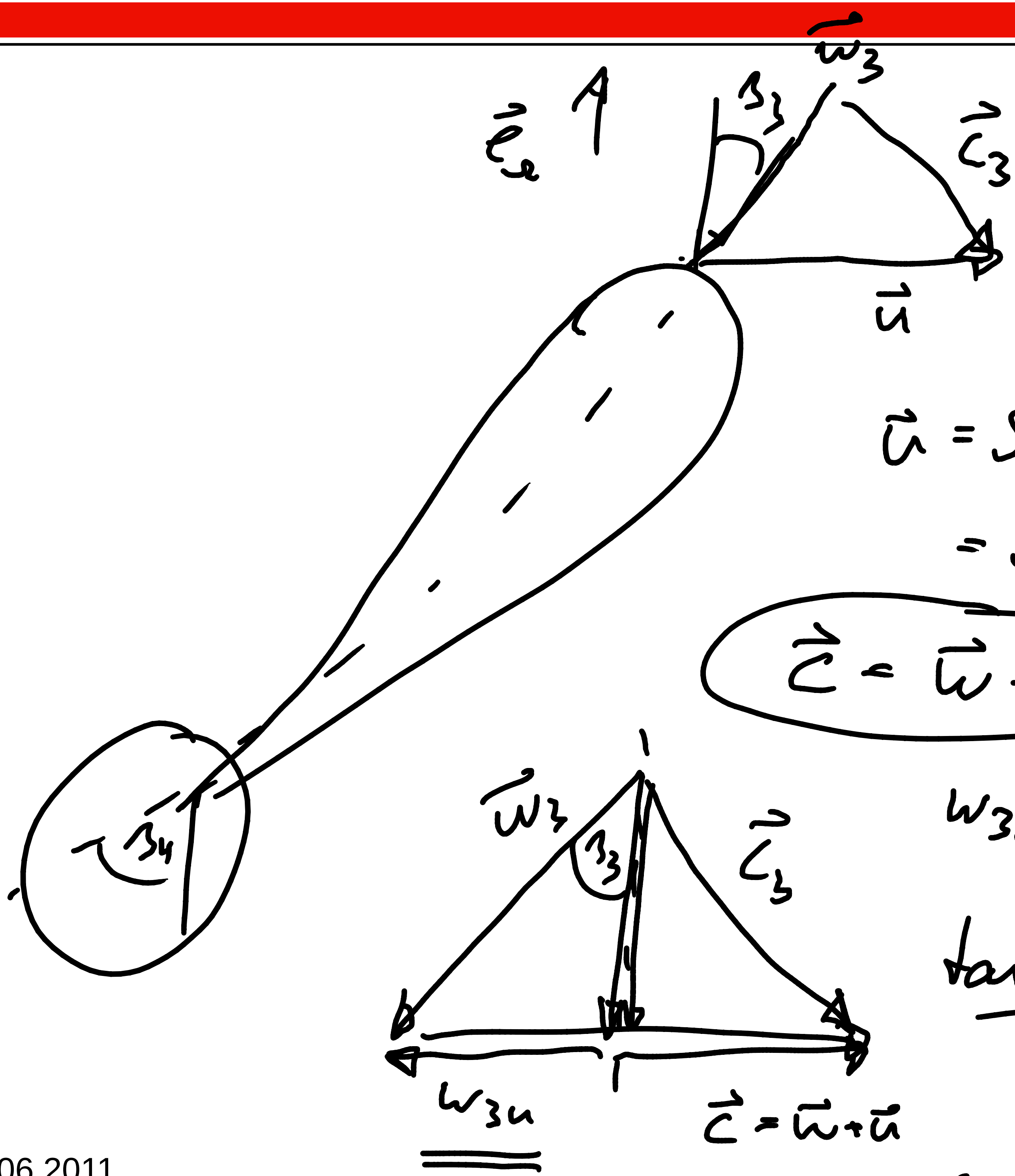

$$\vec u = \Omega \vec e_\Omega \times R_L \vec e_r$$
$$= \Omega R_L \vec e_u$$

$$\vec c = \vec w + u$$

$$w_{3\Omega} = c_{3\Omega}$$

$$\tan(\beta_3) = \frac{w_{3u}}{c_{3\Omega}}$$

$$\vec c = \vec w + \vec u$$

$$w_{u3} = c_{u3} - \Omega R_L$$

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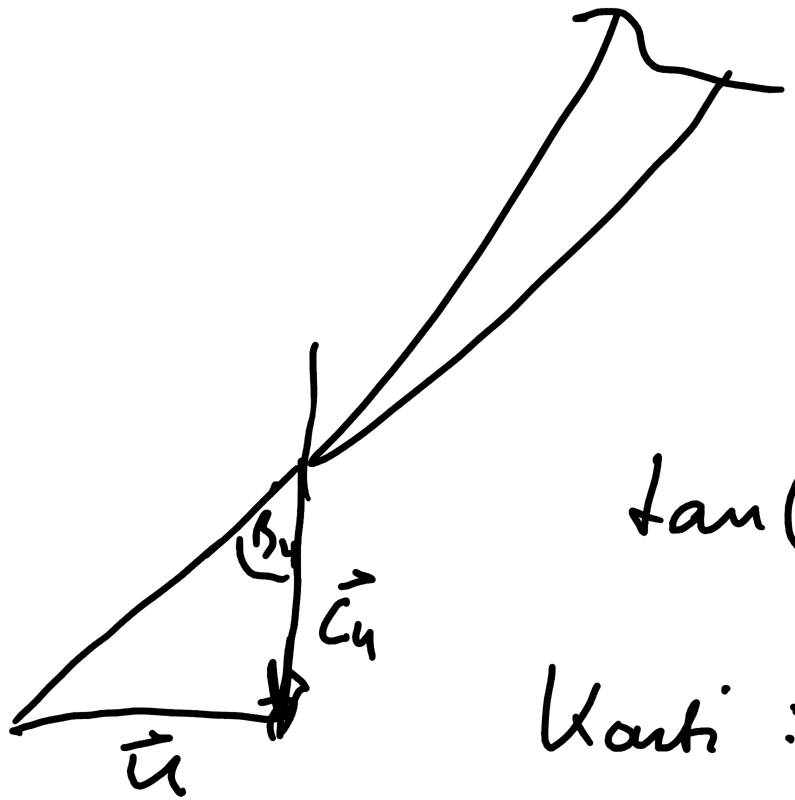
$$\tan(\beta_3) = \frac{|(c_{u3} - \Omega R_L)|}{|c_{\Omega 3}|} \quad \ldots$$



$$\tan(\beta_4) = \frac{|\vec{u}|}{|\vec{c}_4|}$$

$$\vec{u} = \Omega R_L \vec{e}_u$$

$$\vec{c}_4 = c_{\Omega 4} \vec{e}_\Omega$$

Konti: $c_{\Omega 4} = c_{\Omega 3}$

$$\Rightarrow \tan(\beta_4) = \frac{\Omega R}{c_{\Omega 3}} = \frac{2\pi R_1^2 \Omega 4}{Q}$$

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