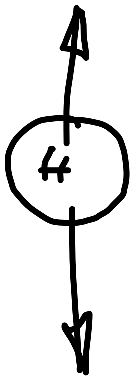


$$W = 6\pi \eta a u$$

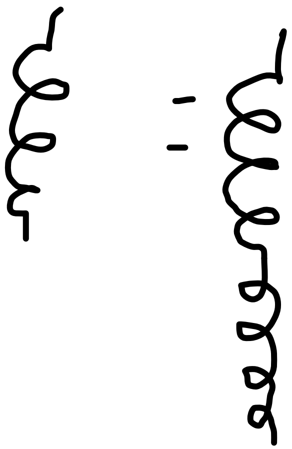
$$G = g \frac{4}{3}\pi a^3 (\rho_s - \rho)$$

$$\eta = 82 \text{ mPa sec.}$$

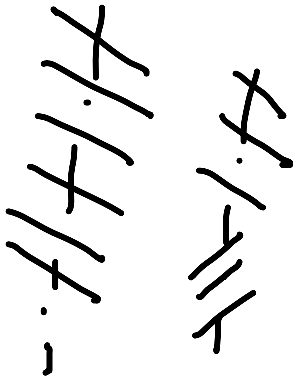
∇ Test $Re \ll 1$

$\Rightarrow W = G$

$$\eta_T = 3.2\,\%$$



$$P = \begin{pmatrix} 2A & 0 \\ 0 & 2D \end{pmatrix}$$



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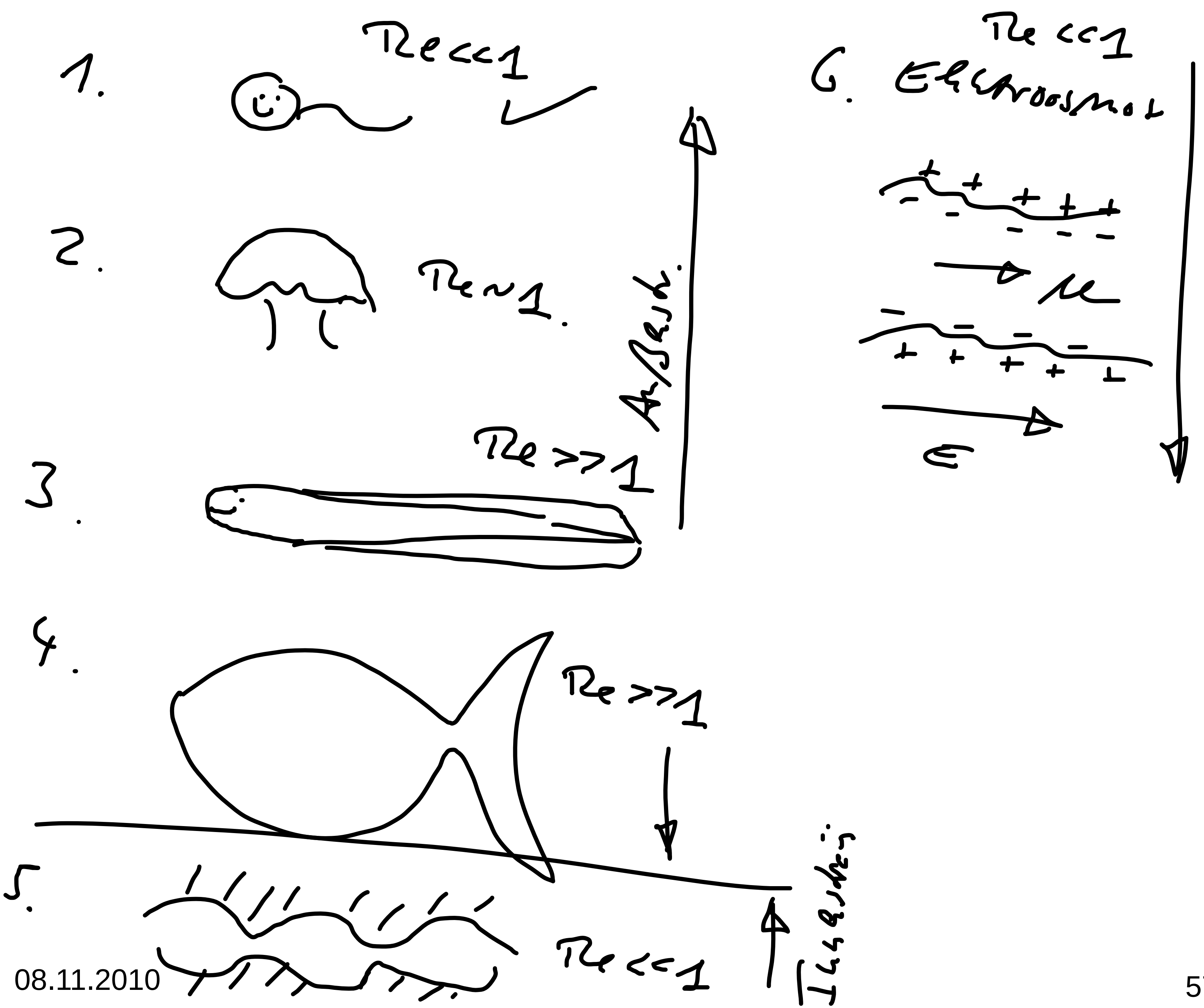

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Zu 3. Schwimmbewegung d. Aale.

Sir James Lighthill

Note on the swimming of slender-fish.

Journal of Fluid Mechanics 1960

JFM 1960.

siehe Bibliothek SLA.

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Applied Mathematics ≙ Mechanik.

James Lighthill Ernst Becker

„slender" ≙ • Theory schlanker Körper.

• klassische Aerodynamik.

$$\left(\frac{\text{Länge}}{\text{Dicke}}\right)^{-1} = \varepsilon \text{ ist klein.}$$

„perturbation Method":

• Störungsrechnung

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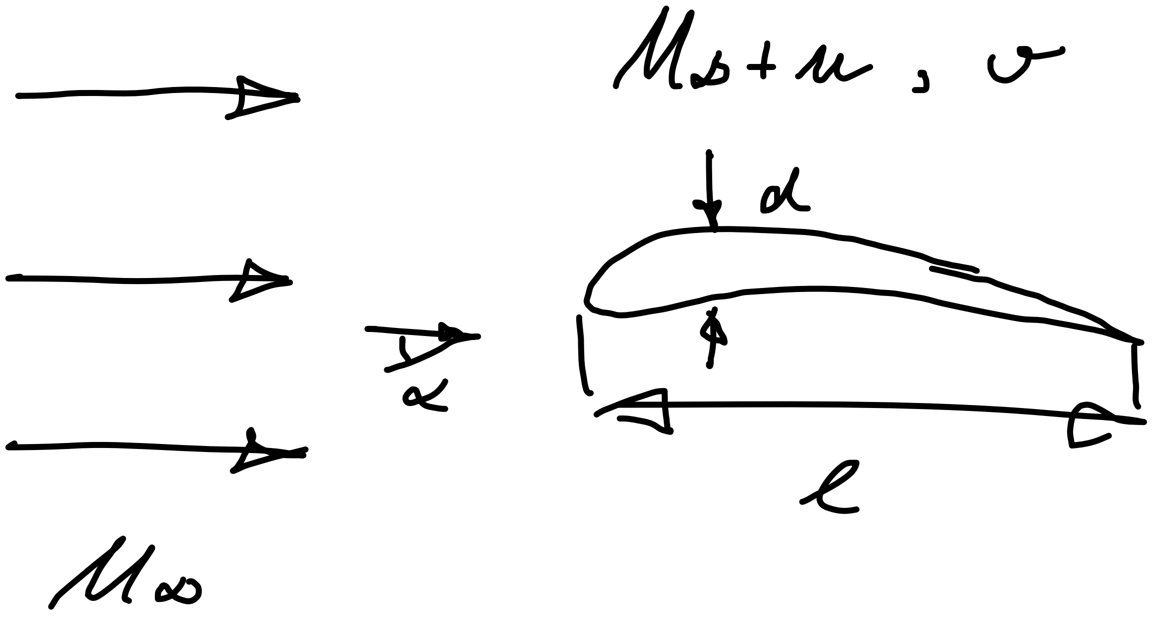

$$\frac{d}{l} = \varepsilon \ll 1$$

$$\alpha \ll 1.$$

u, v Störungsgeschwindigkeit

$u, v \ll U_\infty$.

⇝ Linearisierung von Bewegungsgleichung.
Linearisierung der Randbedingung.

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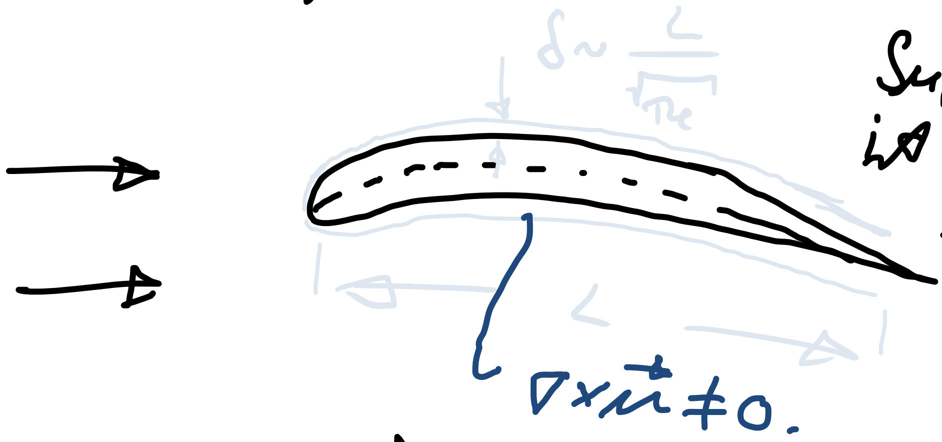
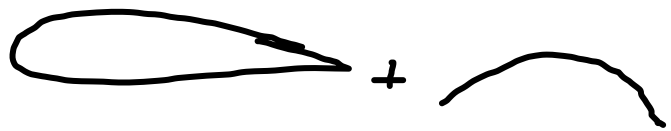
$\nabla \times \vec{u} = 0$



Superposition.
ist möglich, da $\Delta \Phi = 0$

$=$ 

$\nabla \cdot \vec{u} = 0.$

$\nabla \times \vec{u} = 0 \rightsquigarrow \vec{u} = \nabla \Phi$

$\nabla \cdot \nabla \Phi = 0$

$\Delta \Phi = 0$

$Re \gg 1: \delta \ll L$

Grenzschichtdicke ist Vernachlässigbar

Φ Geschwindigkeitspotential.

Randbedingung: kinematische Randbedingung. $\vec{u} \cdot \vec{n} = \vec{u}_w \cdot \vec{n}$ a.d.W.

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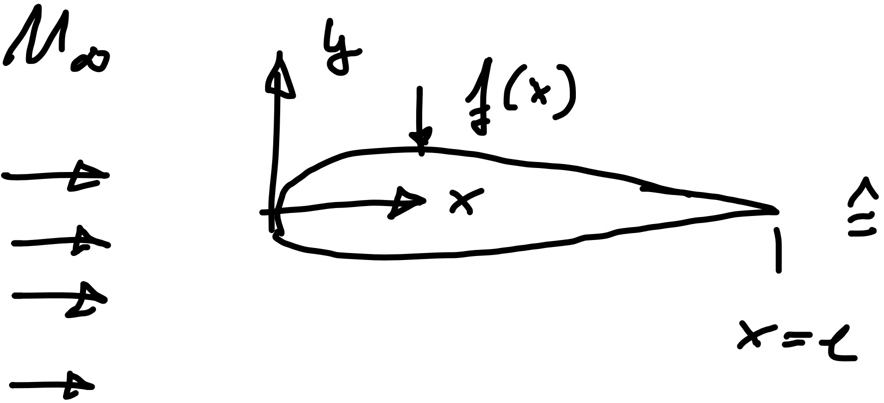
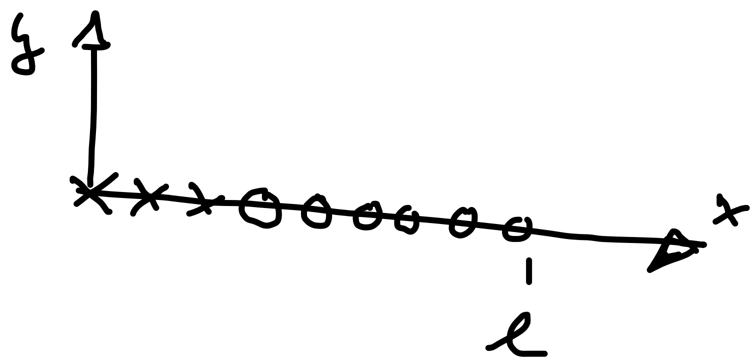
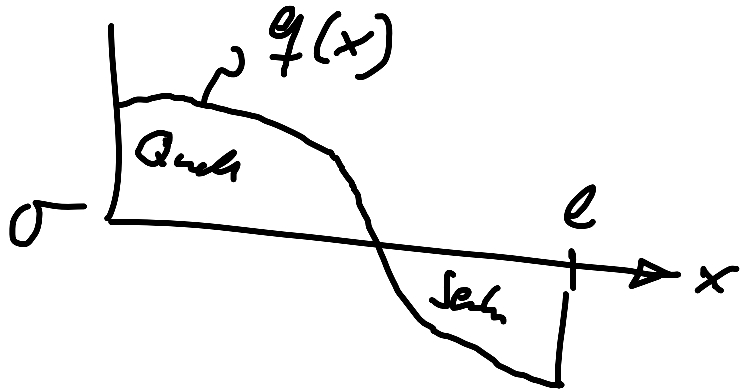
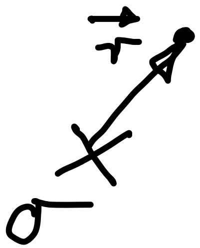






q(x) Quellstärke an der Koordinate x

Eine Quelle im unbegrenzten 2D-Raum.



$$\vec u = \frac{E}{2\pi r}\vec e_r \Rightarrow \Phi_E = \frac{E}{2\pi}\ln r$$

$$r = |\vec r|.$$

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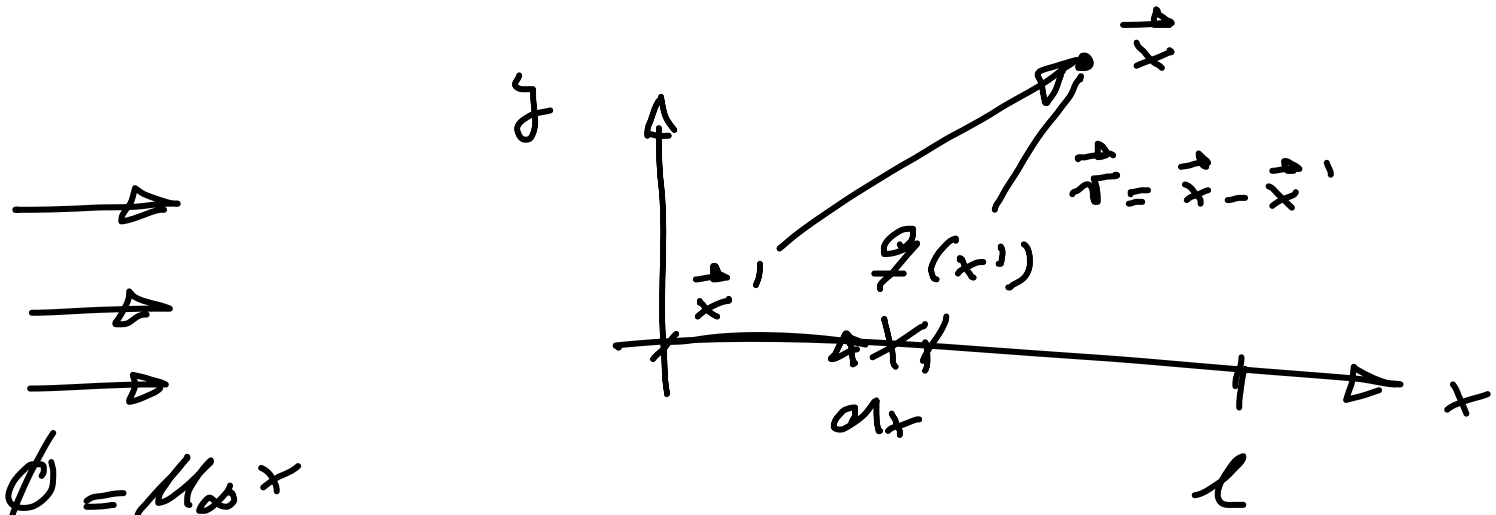

$$\phi_{u_\infty} = u_\infty x$$

$$d\phi_q = \frac{q(x')\,dx'}{2\pi} \ln|\vec{x} - \vec{x}'|$$

?

$$\phi = u_\infty x + \int_0^l \frac{q(x')\,dx'}{2\pi} \ln|\vec{x} - \vec{x}'|$$

$$\vec{x} - \vec{x}' = x\vec{e}_x + y\vec{e}_y - x'\vec{e}_x$$
$$= (x - x')\vec{e}_x + y\vec{e}_y$$

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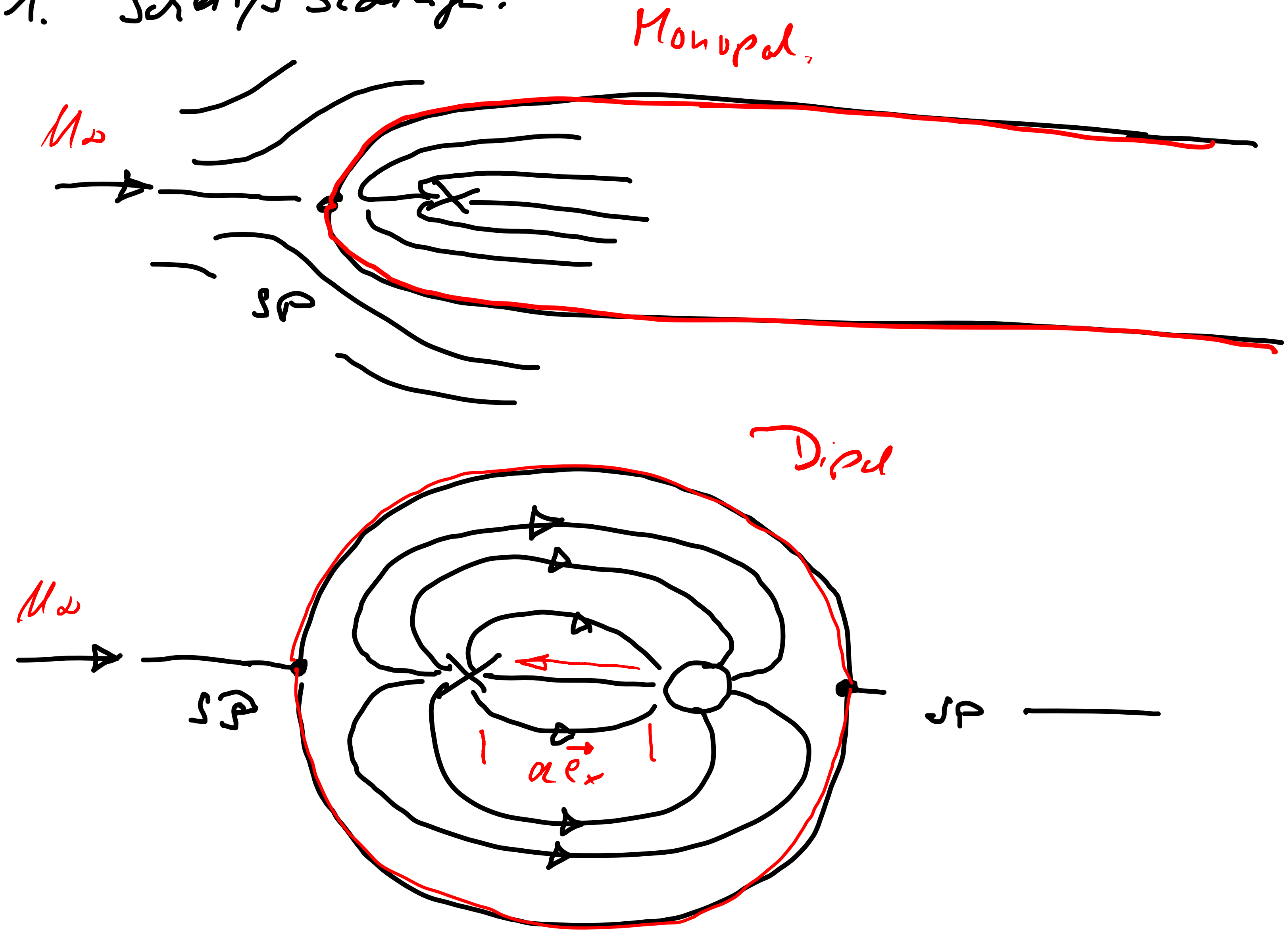
1. Schließbedingung.



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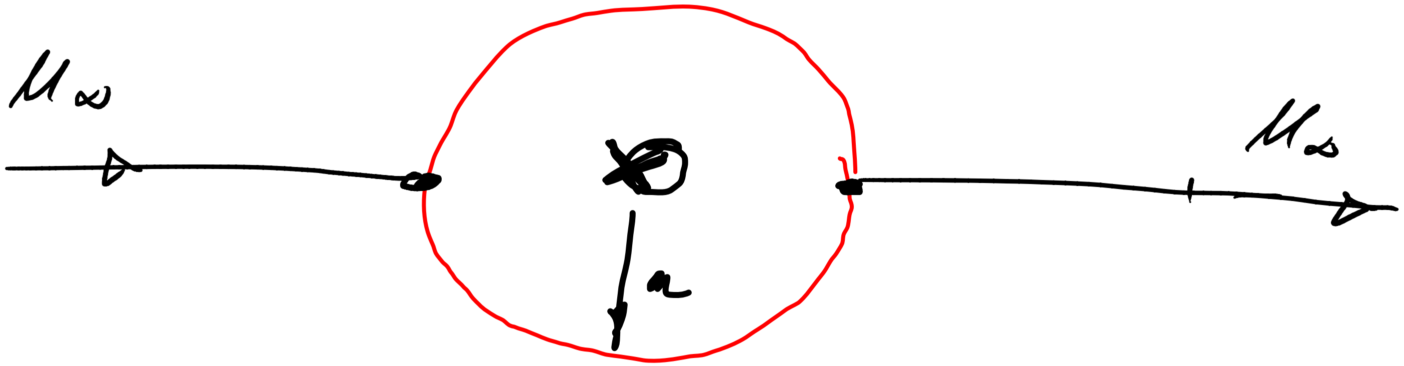
Grenzübergang

$$a\vec{e}_x \to 0$$

$$E \to \infty.$$



$v, u \sim \frac{1}{r^2}$ beim Dipol.

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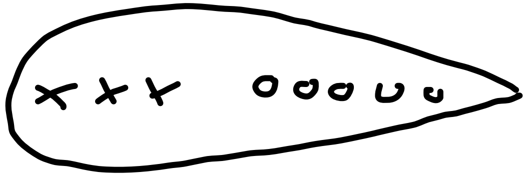
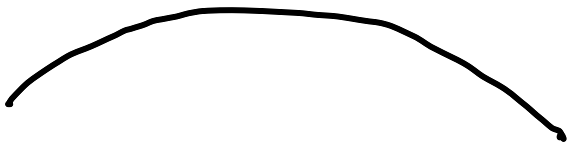
Schließbeding. $\int_0^l q(x)\,dx \stackrel{!}{=} 0.$



≙ Dipol. $\sim \frac{1}{r^2}$



≙ Wirbel $\sim \frac{1}{r}$

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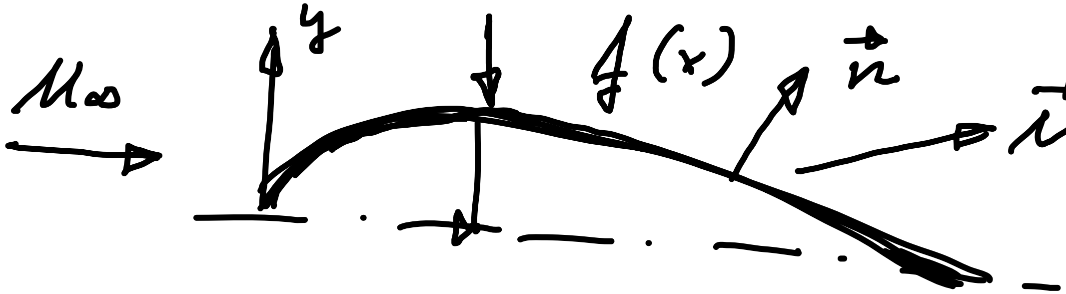
Bedingung für $g(x)$: $\vec{x}_W = x\,\vec{e}_x + f(x)\,\vec{e}_y$.

Kinematische Randbedingung

$$\vec{u}\cdot\vec{n} = \vec{u}_W\cdot\vec{n} \quad \text{a.d.W}$$



$$\vec{u} = (U_\infty + u)\,\vec{e}_x + v\,\vec{e}_y.$$

implizite Darstellung der Wand

$$F(x,y,\cancel{t}) := f(x,\cancel{t}) - y = 0$$

Zeit t spielt in der Aerodynamik keine Rolle.

Normalenvektor $\vec{n} \sim \nabla F = \frac{\partial f}{\partial x}\vec{e}_x - \vec{e}_y$ an der Wand.

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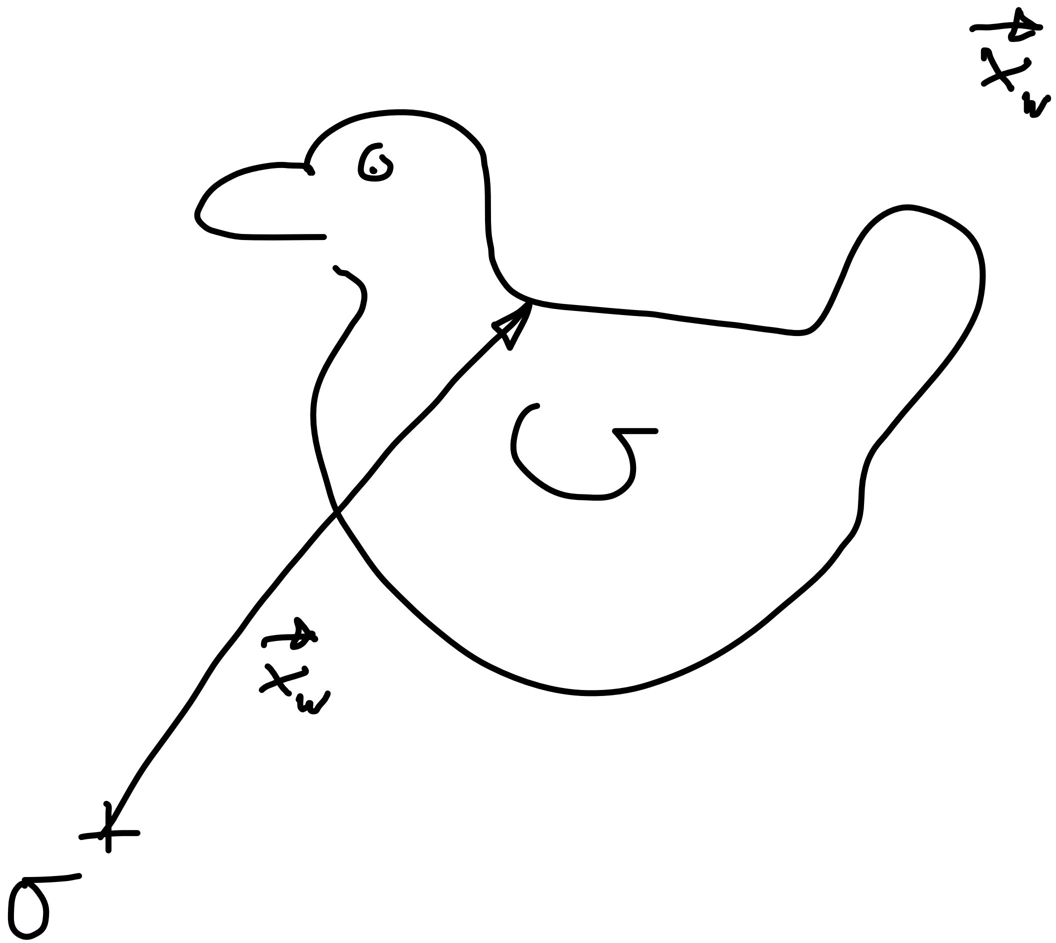

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$$\vec{u}\cdot\vec{n} = \vec{u}_w\cdot\vec{n} \quad \text{a.d.W.}$$

$$\left[(U_\infty+u)\,\vec{e}_x + v\,\vec{e}_y\right]\cdot\left[\frac{\partial f}{\partial x}\vec{e}_x - \vec{e}_y\right] = \begin{cases} 0 & \text{für das ruhende Profil} \\ \dfrac{\partial f}{\partial t} & \text{für den bewegten Körper.} \end{cases}$$

$$(U_\infty+u)\frac{\partial f}{\partial x} - v = \begin{cases} 0 \\ \dfrac{\partial f}{\partial t} \end{cases} \quad \text{a.d.W.}$$

Theorie schlanker Körper.

$$u, v \ll U_\infty \qquad \frac{u, v}{U_\infty} \sim \varepsilon \qquad \frac{d}{l} \sim \varepsilon$$

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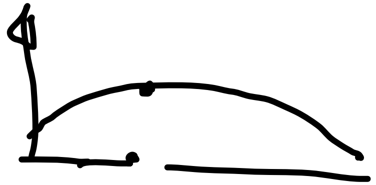
Linearisierung der Randbedingung

$$U_\infty \frac{\partial f}{\partial x} + u \frac{\partial f}{\partial x} - v = \begin{cases} 0 \\ \frac{\partial f}{\partial t} \end{cases}$$

$\sim \varepsilon \qquad \sim \varepsilon^2 \;(\approx 0.) \qquad \sim \varepsilon$



$$U_\infty \frac{\partial f}{\partial x} - v = \frac{\partial f}{\partial t}$$ Randbedingung a.d. W

$y = f(x).$

Linearisieren am Ort $y = 0.$

⇝ Taylorentwicklung.

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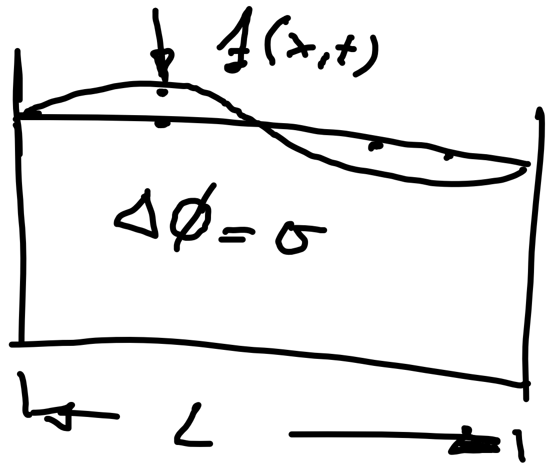
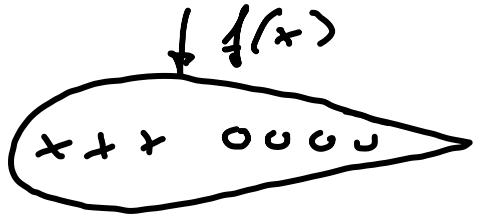
1. Schwallwelle



$f \ll L$

2.

$$\frac{v}{u_\infty} = \frac{df}{dx} \quad \text{an } y = 0$$ für die schlanke Körper.



$$\Phi = u_\infty x + \int_0^l \frac{q(x')}{2\pi} \ln|\vec{x} - \vec{x}'|\, dx'$$

$$\frac{df}{dx} = \frac{\partial}{\partial y} \int_0^l \frac{q(x')}{2\pi} \ln|\vec{x} - \vec{x}'|\, dx'$$

Integralgleichung.

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⇝ Im Prinzip: Boundary Element Method

BEM.

Randelement methode.

⊕ Geringe Diskretisierungsaufwand.

⊕ Sehr gut geeignet für Außenströmung.
(Aerodynamik und Akustik).

⊖ Nichtlineares System.

⊖ Innenströmung.

2D

Monopol $\sim \frac{1}{r}$

Dipol $\sim \frac{1}{r^2}$

Wirbel $\sim \frac{1}{r}$

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