Nuclear equation of state for arbitrary proton fraction and temperature

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with Kai Hebeler and Achim Schwenk

arXiv:2204.14016

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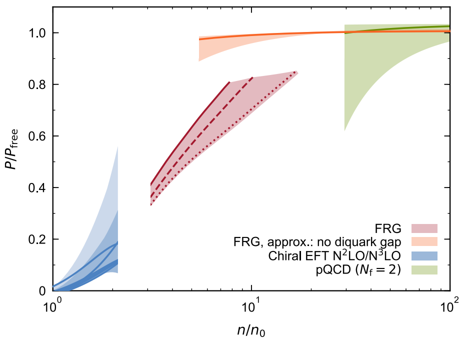
Introduction

- Nuclear matter: idealized system of neutrons and protons in thermodynamic limit (no surface effects, homogeneous, ...)
- Key input for astrophysics
- This talk: nuclear EOS from chiral EFT
- So far EOS studied often for $T = 0$ for PNM ($x = 0$) or SNM ($x = 0.5$)
- Thermal effects matter for astro applications
  e.g. Yasin et al., Phys. Rev. Lett. 124 (2020)
- Astrophysical systems are neutron rich or in β-equilibrium

Leonhardt et at.,
Phys. Rev. Lett. 125 (2020)



$\longrightarrow$ Chiral EFT EOS for finite temperature and asymmetric matter JK, Hebeler, Schwenk, arXiv:2204.14016

Method

Nuclear interaction

Chiral EFT: $H = H_0 + V_{NN} + V_{3N}$

Grand-canonical potential

$$\Omega(T, \mu_n, \mu_p) = -\frac{1}{\beta} \ln \mathrm{Tr} \left(e^{-\beta\left(H - \mu_n N_n - \mu_p N_p\right)} \right)$$

Approximation strategy

Many-body perturbation theory, MC integration

Gaussian process emulation

$$\{F(x_i, T_j, n_k) + \Delta_{ijk}\}_{ijk} \xrightarrow{\text{GP}} F(x, T, n)$$

Equation of state (EOS)

$F(x, T, n), P(x, T, n), \ldots$

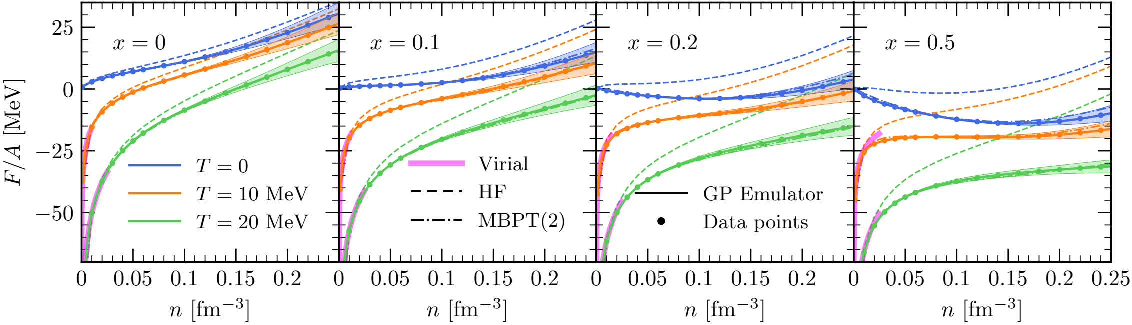
Free energy

- EMN interaction at N^{3}LO ($\Lambda = 450\,\mathrm{MeV}$) Entem, Machleidt, Nosyk, Phys. Rev. C 96 (2017)
- Bands are order-by-order EFT uncertainty estimates Epelbaum et al., Eur. Phys. J. A 51 (2015)

  $$\Delta X^{(j)} = Q \cdot \max\left(\left|X^{(j)} - X^{(j-1)}\right|, \Delta X^{(j-1)}\right)$$

- Excellent reproduction of data by GP, good MBPT convergence, no MC noise
- Virial EOS: model independent fugacity $z_t = e^{\mu_t/T}$ expansion Horowitz, Schwenk, Nucl. Phys. A 776 (2006)

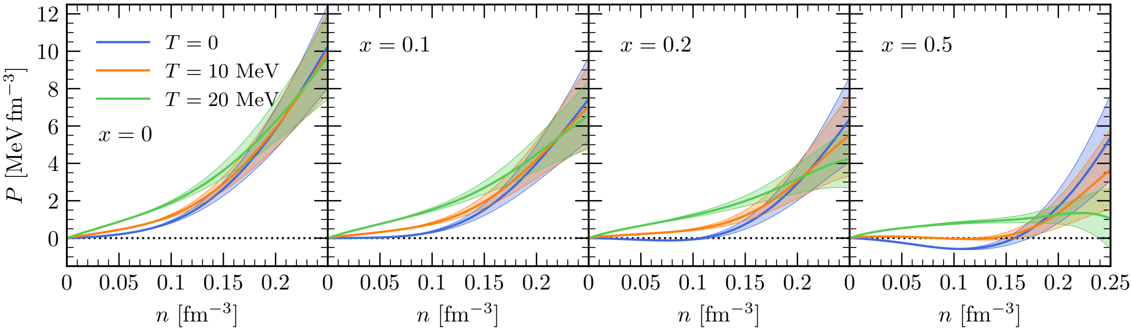
JK, Hebeler, Schwenk, arXiv:2204.14016



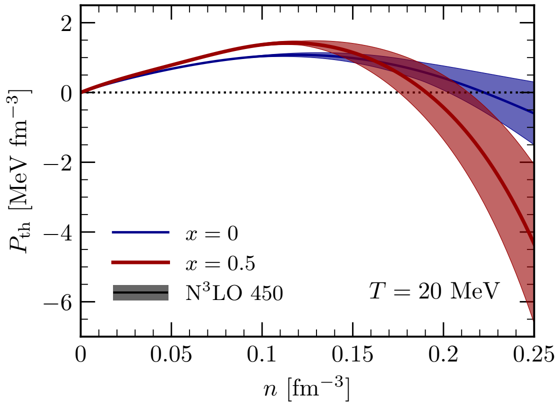
Pressure

- Calculate $P = n^2 \partial_n F/A$ using GP emulator
- Thermodynamically consistent
- For very neutron-rich conditions depends weakly on temperature for $n \gtrsim n_0$
- Pressure isothermals cross at higher density (negative thermal expansion)

JK, Hebeler, Schwenk, arXiv:2204.14016



Thermal pressure



- $P_{\rm th}(T) = P(T) - P(T=0)$
- Pressure isothermals cross if $P_{\rm th}(T) = 0$
- For NM associated with increasing effective neutron mass m_n^* (three-nucleon interactions)

Carbone, Schwenk, Phys. Rev. C 100 (2019)
JK, Wellenhofer, Hebeler, Schwenk, Phys. Rev. C 103 (2021)

Thermal pressure

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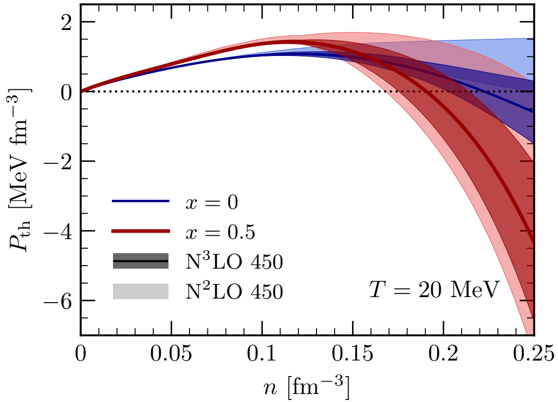

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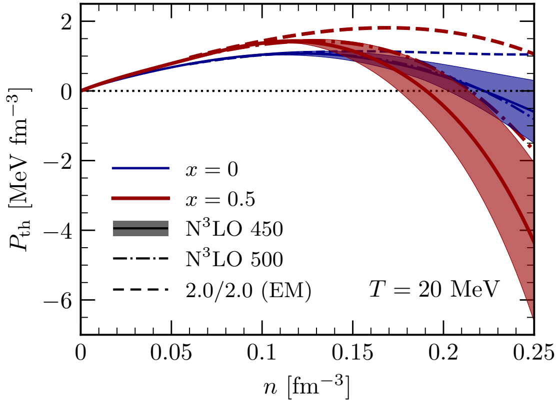

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- For 2.0/2.0 interactions consistent with non-perturbative SCGF calculations

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Thermal pressure

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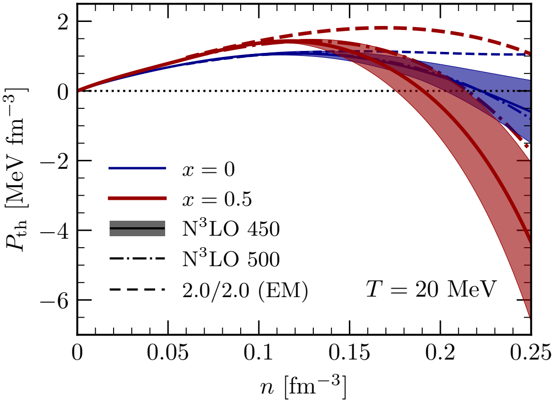

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- For 2.0/2.0 interactions consistent with non-perturbative SCGF calculations
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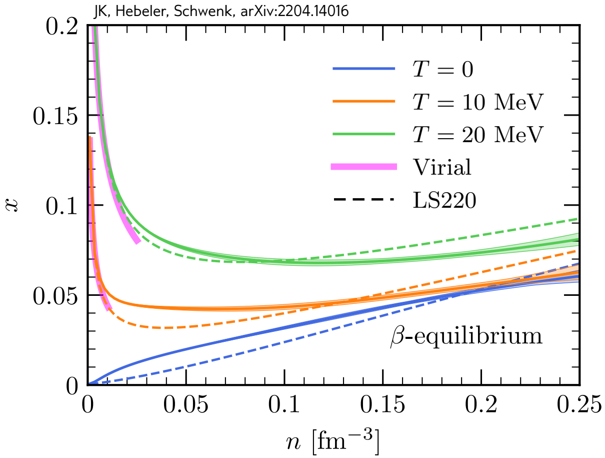
- Decreasing P_{th} for different chiral orders, cutoffs, and interactions

Neutron star matter



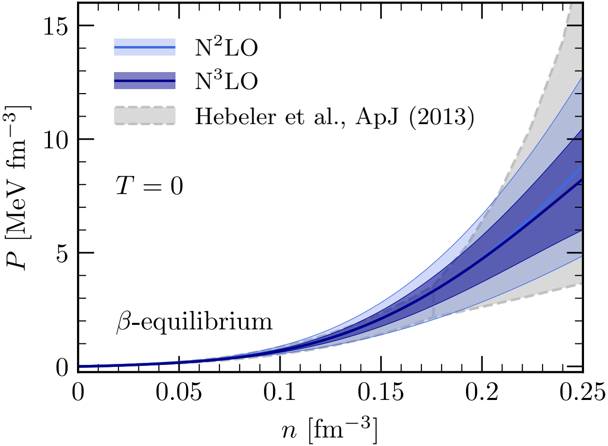
- Determine x by β-eq.

$$m_n + \mu_n = (m_p + \mu_p) + (m_e + \mu_e)$$

- Ultra-rel. e^- with $n_e = n_p$
- Key input $\hat{\mu} = \mu_n - \mu_p = -\frac{\partial}{\partial x}\frac{F}{A}$
- Use GP emulator for derivatives
- Reasonable agreement with LS EOS, our results exhibit weaker density dependence

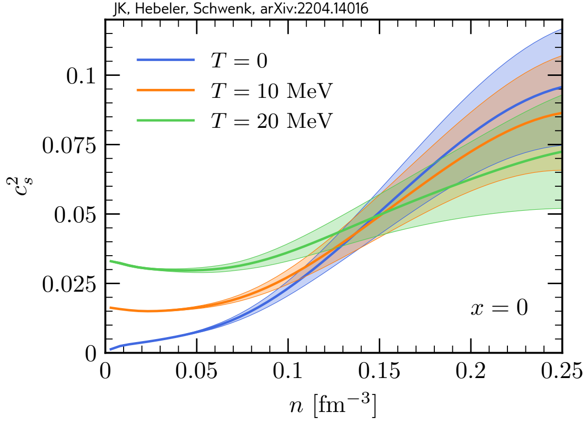
Neutron star pressure

JK, Hebeler, Schwenk, arXiv:2204.14016



- In beta-eq. $P(n, x_{\beta\text{-eq.}}(n, T), T = 0)$
- Improvement over older calculations that use parametrization for beta eq.
  Hebeler et al., Astrophys. J. 773 (2013)
- Higher pressure around saturation density
- Compatible, although older band has different meaning (not EFT uncertainty estimates)
- Natural behavior of EFT uncertainty bands

Speed of sound



- Pressure derivative at constant entropy

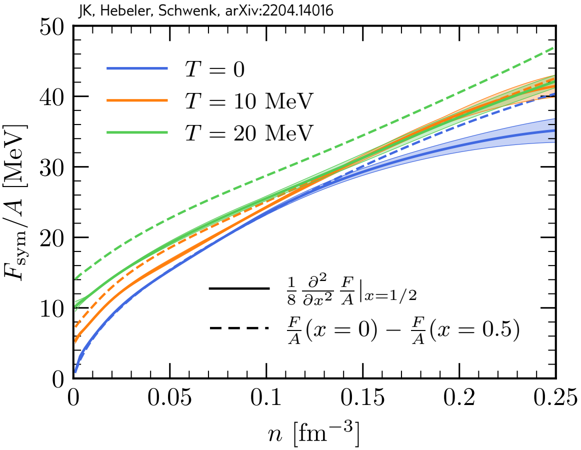
$$c_s^2 = \left.\frac{\partial P}{\partial \epsilon}\right|_{S,x}$$

- With internal energy density

$$\epsilon = n\left(\frac{E}{A} + m_n\right)$$

- At $T = 0$ monotonic increase, increase is weaker at finite T
- Decreases at higher densities with increasing T (like P)

Symmetry energy



- Compare two definitions of the symmetry energy ($\beta = (n_n - n_p)/n$)

$$\frac{F}{A}(n,\beta) \approx \frac{F}{A}(n,\beta=0) + \frac{F_{\rm sym}}{A}\beta^2$$

- Differences due to contributions beyond quadratic x dependence
- Kinetic part has non-quadratic contributions

Summary

- Calculations of EOS around saturation density using chiral EFT
- Developed calculations for $T > 0$ and arbitrary x
- Constructed emulator for free energy
- EFT dominates over MBPT uncertainties for neutron rich matter
- Pressure at higher densities increases with decreasing T (negative thermal expansion)
- EOS in beta equilibrium directly without parameterizations between PNM and SNM
- Application: speed of sound and symmetry energy