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nn scattering length from the ^{6}He($p, p\alpha$)nn reaction

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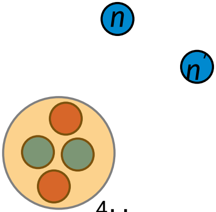
Theory for obtaining the

nn scattering length from the ^{6}He$(p, p\alpha)nn$ reaction



project A05 and $^{6}He(p, p\alpha)nn$

project A05: Halos and clustering in nuclei

- explore cluster degrees of freedom of exotic nuclei & few-neutron systems
- use reactions with radioactive beams and (halo) effective field theory

project A05 and ^{6}He($p, p\alpha$)nn

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nn scattering length from ^{6}He($p, p\alpha$)nn

- exploit the experimental and theoretical tools in this field to measure the *nn* scattering length
- experiment: reaction in inverse kinematics
  - will be conducted by Aumann & SAMURAI collaboration @ RIKEN RIBF
  - Marco Knösel works on detector design & data analysis
- theory: describe reaction in Halo EFT

Outline

Motivation & approach

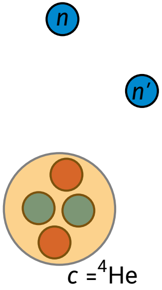
Halo EFT for ^{6}He

Benchmarking the ground-state spectrum

Reaction theory for $^6\text{He}(p, p\alpha)nn$

Results for the E_{nn} spectrum

Conclusion & Outlook



Motivation & approach

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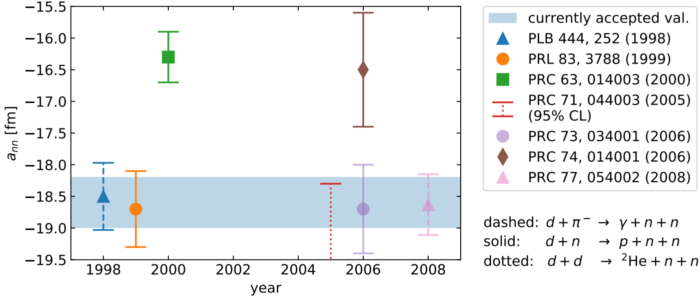
- **motivation:** no high-precision value for *nn* scattering length available



dashed: $d + \pi^- \rightarrow \gamma + n + n$
solid: $d + n \rightarrow p + n + n$
dotted: $d + d \rightarrow {}^2\mathrm{He} + n + n$

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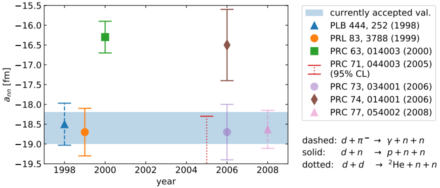

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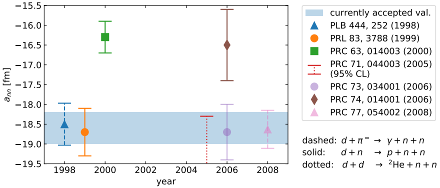

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  - different from the previous methods ➔ not the same difficulties
  - final *nn* pair has high center-of-mass velocity in the lab system ➔ avoids problems with detection efficiency

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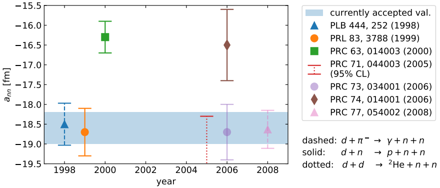

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- experiment proposal from Aumann & SAMURAI collaboration approved by RIKEN RIBF NP2012-SAMURAI55R1 (2020)

Obtaining the E_{nn} spectrum of ^{6}He

- **approach:** ^{6}He in Halo EFT
  1. calculate wave function $\Psi_c(p, q)$ (& do comparisons with model calc.)
  2. take final state interaction (FSI) into account
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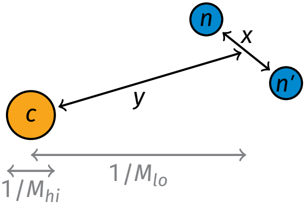
  - $\not\pi$EFT
  - core & valence nucleons as degrees of freedom
  - results are expanded in k/M_{hi}
    → systematic improvement possible



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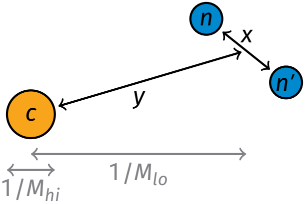

- **properties of ^{6}He**
  - Borromean $2n$ halo
  - separation of scales: $S_{2n} = 0.975\,\text{MeV} \;<\; E^*_\alpha \approx 20\,\text{MeV}$
  - quantum numbers: $J^\pi = 0^+$ (^{4}He: $J^\pi = 0^+$)
  - leading-order (LO) Halo EFT interaction channels:
    – nn: $^1S_0$
    – nc: $^2P_{3/2}$ (not at LO: $^2P_{1/2}$, $^2S_{1/2}$)

Halo EFT for ^{6}He formulated in Ji, Elster, Phillips, PRC 90 (2014)
review of Halo EFT in Hammer, Ji, Phillips, JPG 44 (2017)

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Faddeev equations

use EFT in dimer formalism

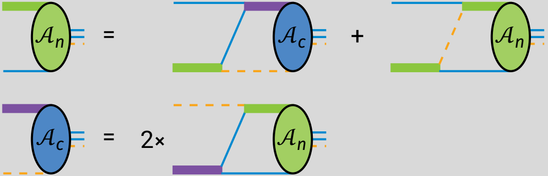
1. step: obtain dressed dimer propagators

& renormalize using **input values**

a_1, r_1

a_0

2. step: set up equations for Faddeev transition amplitudes



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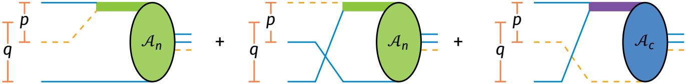
obtain wave functions by multiplying amplitudes with propagators

$\Psi_n(p, q) =$



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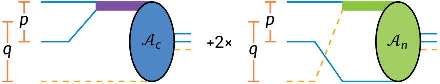
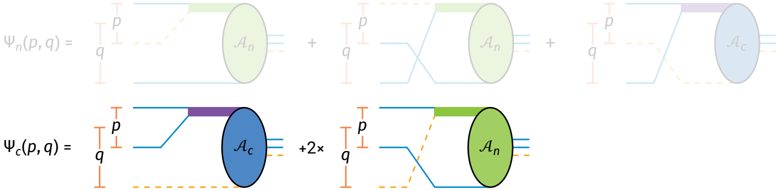


Calculation of wave functions & probability densities

obtain wave functions by multiplying amplitudes with propagators



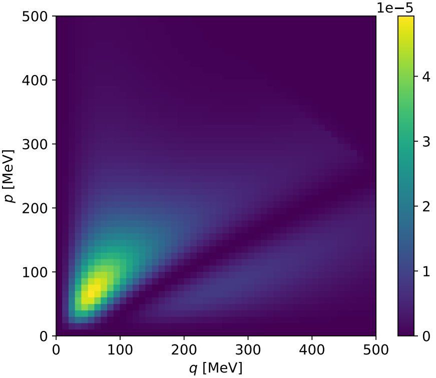
➔use wave function with core as spectator (Ψ_c)

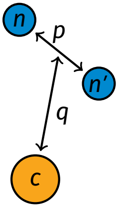
Results for the wave function

calculated ground-state wave functions and probability densities in Halo EFT

Göbel, Hammer, Ji, Phillips, FBS 60 (2019)

$\Psi_c^2(p,q)p^2q^2$





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- compare ground-state distributions $\rho(p_{nn}) \approx \int \mathrm{d}q\, q^2\, |\Psi_c(p_{nn}, q)|^2$
- $\nexists$ published results for $\rho(p_{nn})$

→ use FaCE Thompson, Nunes, Danilin, Comput.Phys.Commun. 161 (2004)

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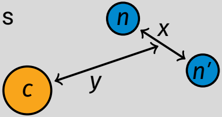
computer code: Faddeev with Core Excitations (FaCE)

- solves the Schrödinger equation of three-body cluster models
- *input:*
  - local l-dependent two-body potentials (central or spin-orbit)
  - phenomenological three-body force
- *output:* hyperspherical wave function components $\chi^S_{K,l}(\rho)$ with $\rho^2 = x^2 + y^2$



Compare ground-state distribution from EFT with model calculations: defining the model

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the two-body potentials in use

use local, l-dependent Gaussian potentials

- central pot.: $\langle r; l, s | V_c^{(\tilde{l})} | r'; l', s' \rangle \coloneqq \delta_{l,l'} \delta_{l,\tilde{l}} \frac{\delta(r'-r)}{r'^2} \bar{V}_c^{(l)} \exp\left(-r^2 / \left(a_{c;l}^2\right)\right)$
- spin-orbit pot.

"standard setting": local Gaussian model 1 (LGM1)

- nn interaction: $V_c^{(0)}$
- nc interaction: $V_c^{(0)}, V_c^{(1)}, V_{SO}^{(1)}, V_c^{(2)}, V_{SO}^{(2)}$
- phenomenological three-body force

Compare ground-state distribution from EFT with model calculations: Results I

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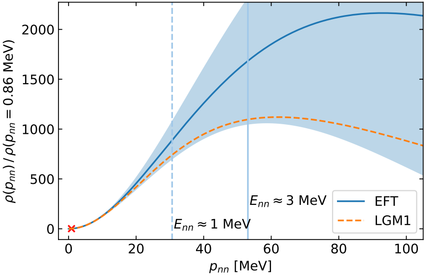

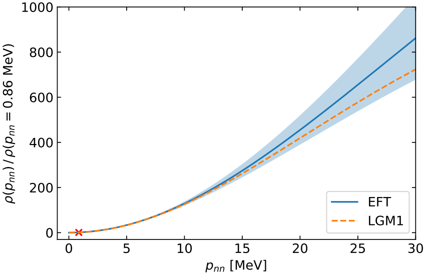

Göbel, Aumann, Bertulani, Frederico, Hammer, Phillips, arXiv:2103.03224 (2021)

- model calc. within uncertainty band of EFT ✔
- especially for small p_{nn} agreement is good

➔ investigate sources of discrepancies by doing additional model calculations

Compare ground-state distribution from EFT with model calculations: Results II

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additional model calculations

- LGM2: same as LGM1, but *nc* int. only in ${}^2P_{3/2}$
- LGM3: same as LGM2, but no three-body force

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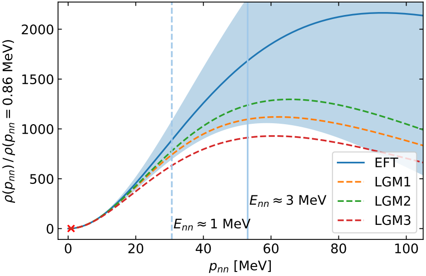

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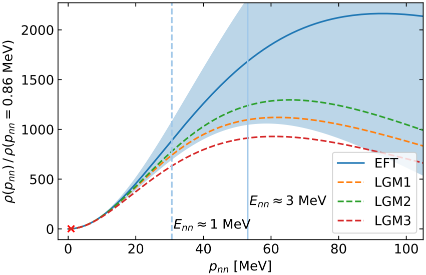

conclusions so far

- additional *nc* int. channels are important
- three-body force important for obtaining the correct S_{2n}
- range of three-body force with tuned strength not so important

Compare ground-state distribution from EFT with model calculations: Results III

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additional model calculations II

- turn three-body Halo EFT calc. into model calc. by using different t
- use t with Yamaguchi form factors → good phase shifts

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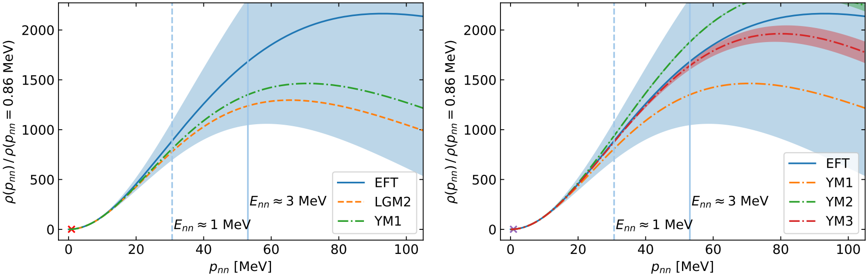

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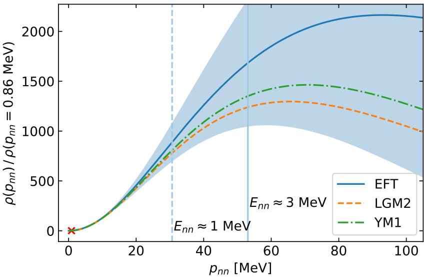

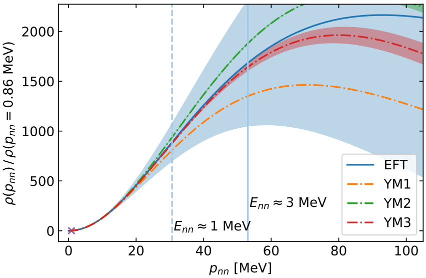

conclusions

- *nc* int. accounts for most of the discrepancies between LO EFT and model calc.
- especially the unitarity term is important

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General information on ^{6}He$(p, p\alpha)nn$

- ^{6}He$(p, p\alpha)nn$ is a knock-out reaction, in which the α core of ^{6}He is removed by a p
- **initial state:** ^{6}He bound state $|\Psi\rangle$

$$\left(K_{nn} + K_{c(nn)} + V_{nn} + V_{nc} + V_{3B}\right)|\Psi\rangle = -B_3\,|\Psi\rangle$$

- **final state:** $|p, q\rangle_c$ all particles are free (state of definite momentum!)

$$\left(K_{nn} + K_{c(nn)}\right)|p, q\rangle_c = (-B_3 + E_{KO})\,|p, q\rangle_c$$

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- ∃ multiple FSIs in ^{6}He$(p, p\alpha)nn$: V_{nn}, V_{nc}, V_{np}, V_{3B}, etc.

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Calculating the wave function after FSI

- treatments of FSI are based on two-potential scattering theory

  Goldberger, Watson, "Collision Theory" (1964)
- two approaches are available:
  - approximation by using FSI enhancement factors
  - exact calculation

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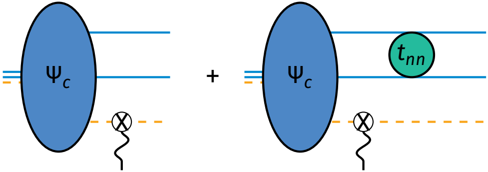
  - exact calculation
- exact calculation is based on t_{nn}



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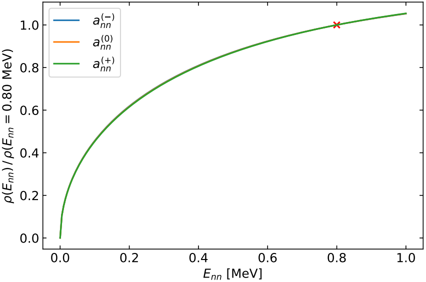
E_{nn} spectrum before and after FSI

obtain distribution by using $\rho^{(t)}(p) = \int \mathrm{d}q\, p^2 q^2 \left|\Psi_c^{(\mathrm{wFSI})}(p,q)\right|^2$

variation of a_{nn}: $a_{nn}^{(-)} = -20.7$ fm, $a_{nn}^{(0)} = -18.7$ fm, $a_{nn}^{(+)} = -16.7$ fm



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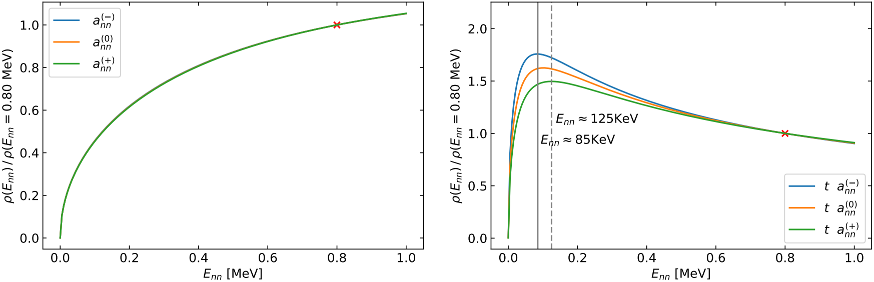

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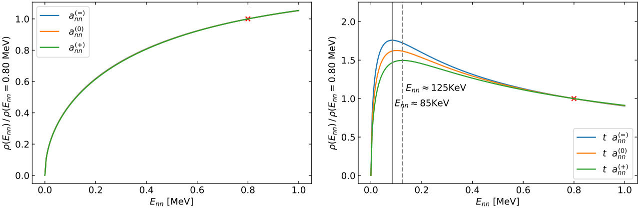

conclusions

- significant sensitivity on *nn* scattering length
- sensitivity almost entirely caused by FSI ➔ ^{6}He is simply a suitable neutron source (nevertheless, ^{6}He wave function is an important ingredient)

Sensitivity of the E_{nn} spectrum on effective range

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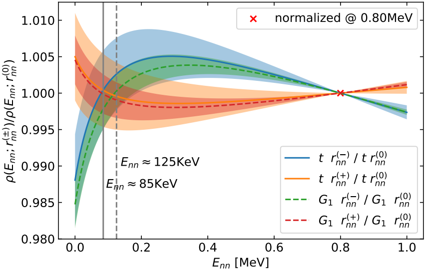
variation of r_{nn}: $r_{nn}^{(-)}$ = 2.0 fm, $r_{nn}^{(0)}$ = 2.73 fm, $r_{nn}^{(+)}$ = 3.0 fm



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outlook

- formulate an EFT for the reaction
- go to NLO

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  - used two-potential scattering theory for calculating FSI effects
- E_{nn} spectrum is sensitive to the nn scattering length
- E_{nn} spectrum is almost insensitive to the nn effective range

outlook

- formulate an EFT for the reaction
- go to NLO

Thank you for your attention!