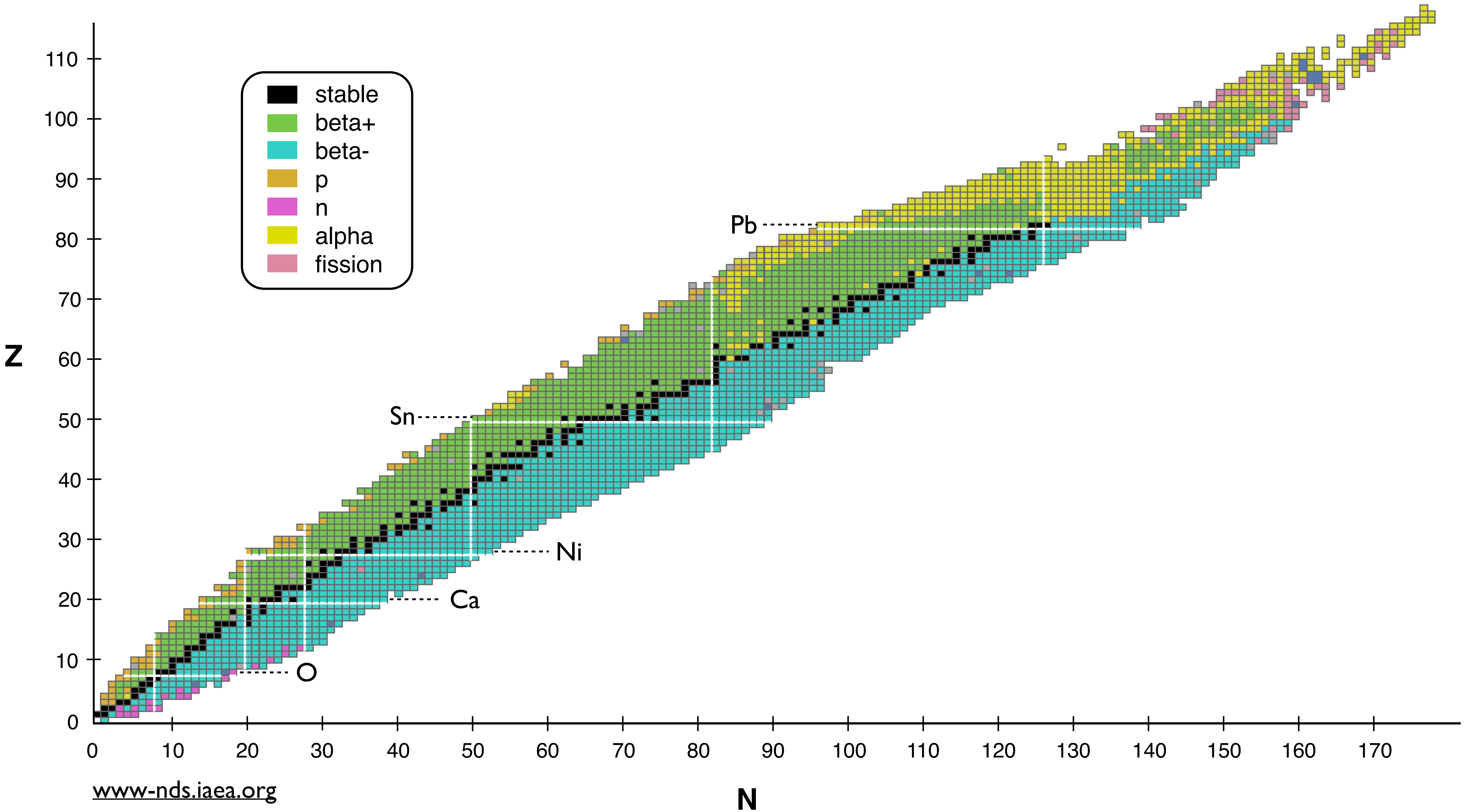
Perspectives on SFB EOS theory

3rd workshop of the SFB 1245

Kai Hebeler

Mainz, July 5, 2018

The nuclear landscape and nuclear matter



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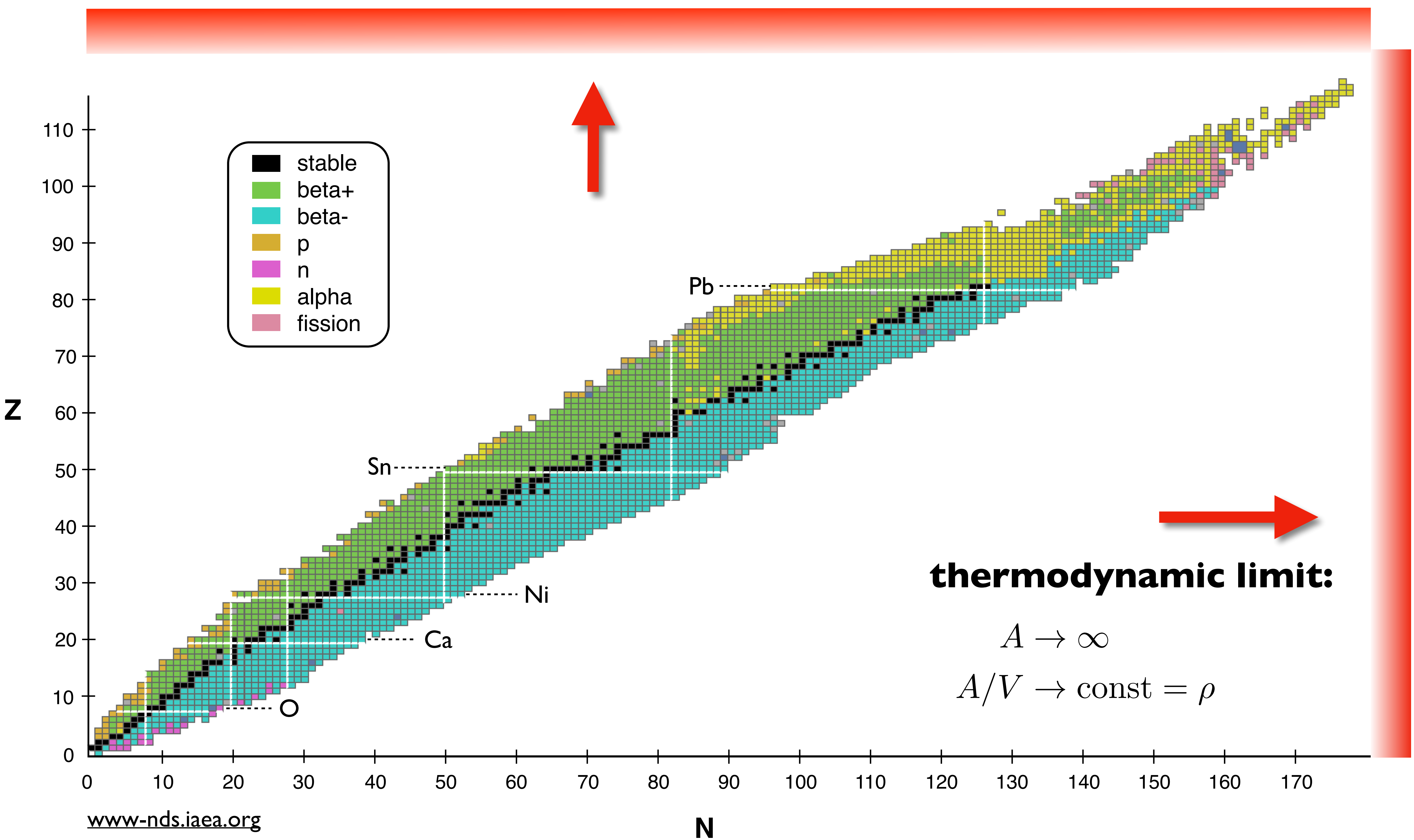
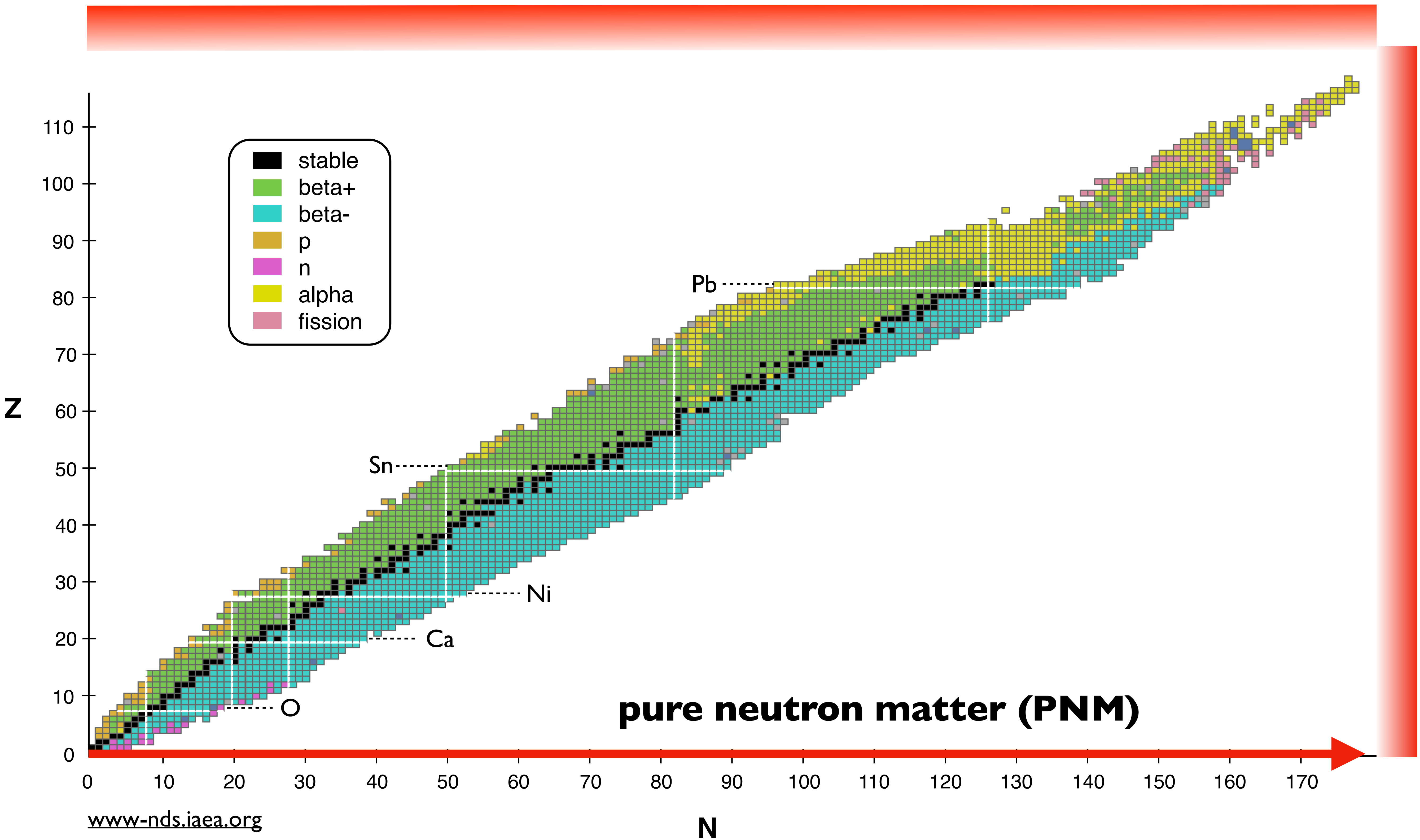


thermodynamic limit:

$$A \to \infty$$

$$A/V \to \text{const} = \rho$$

The nuclear landscape and nuclear matter



www-nds.iaea.org

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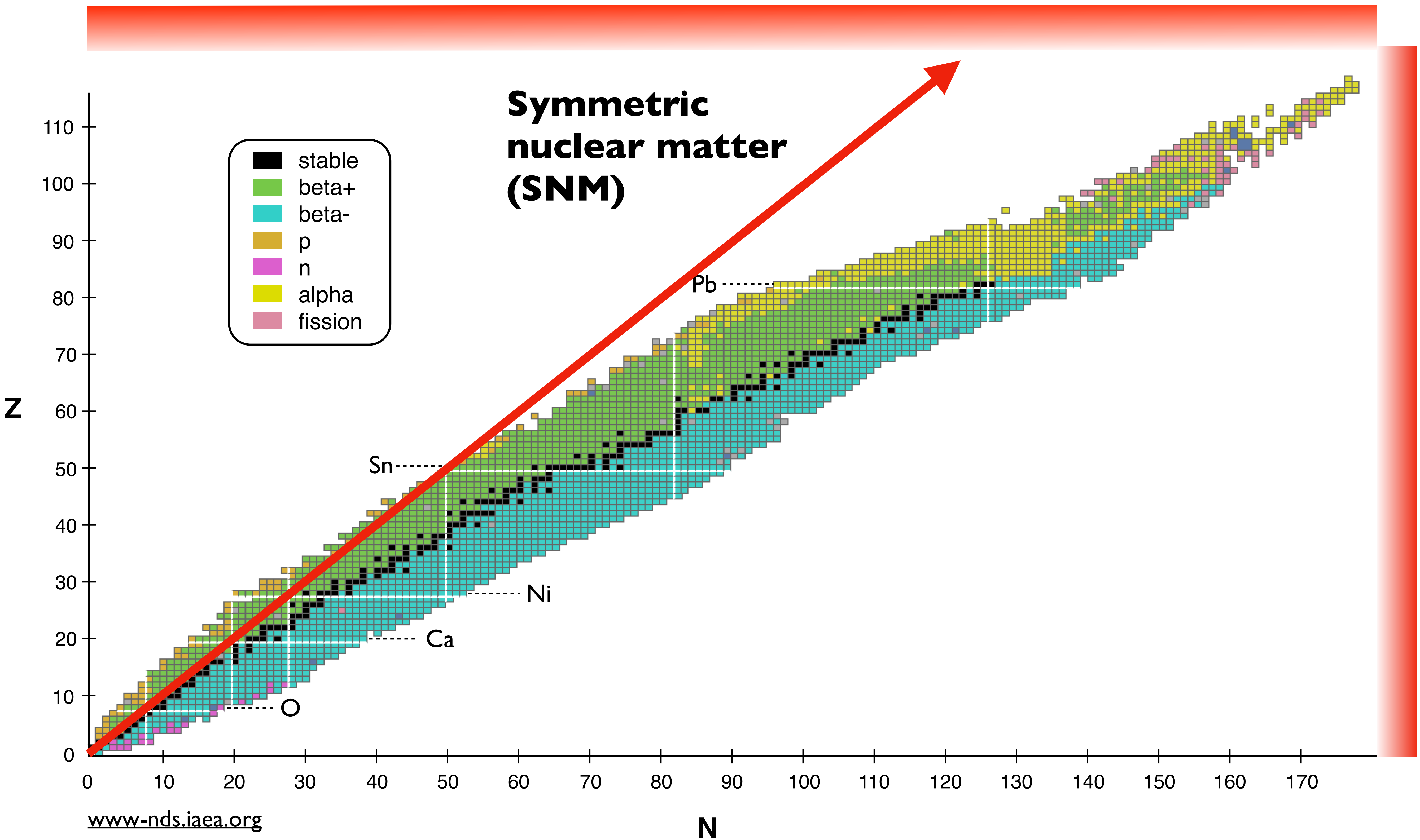

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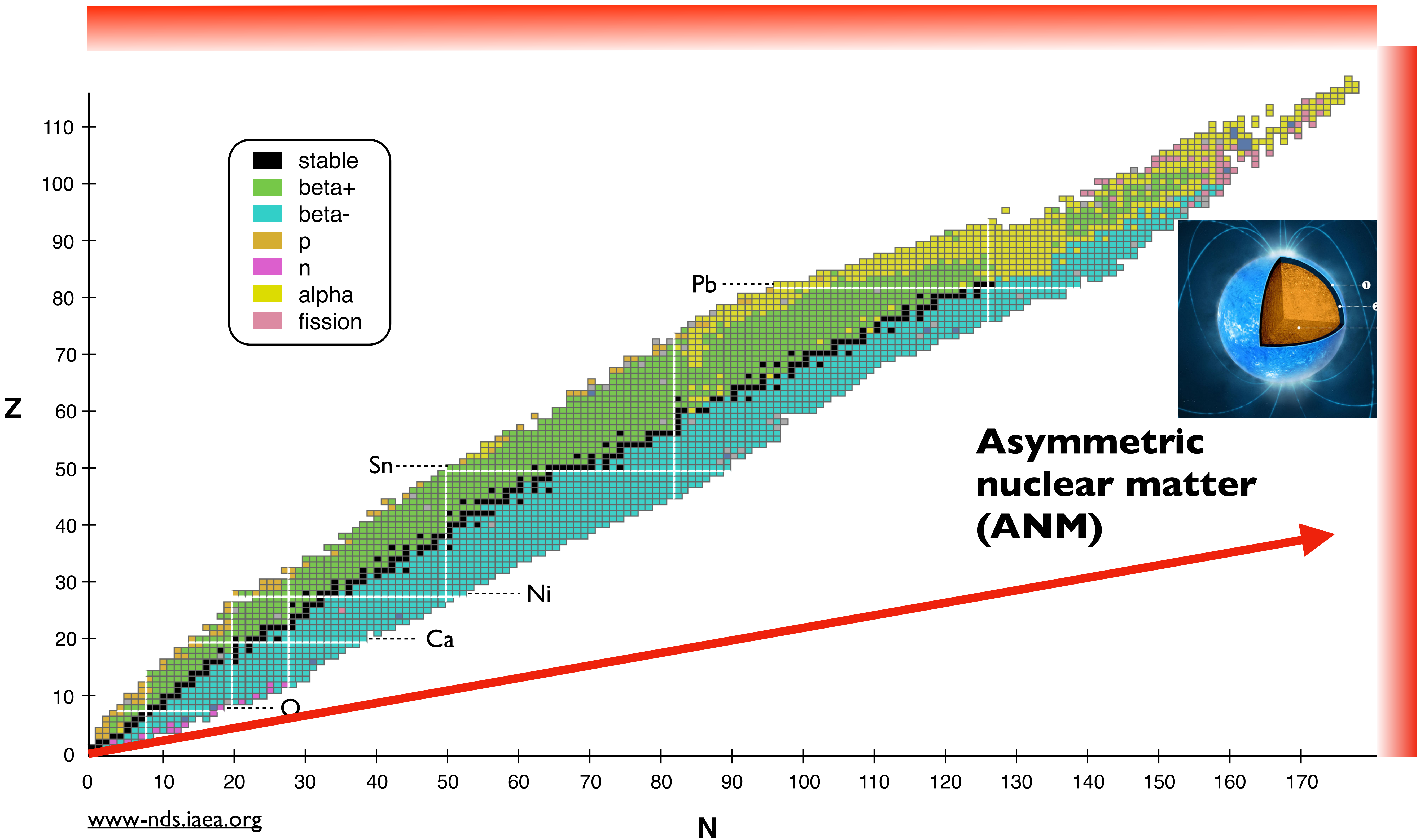
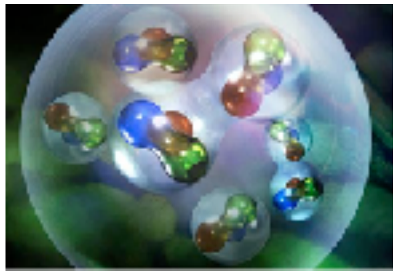
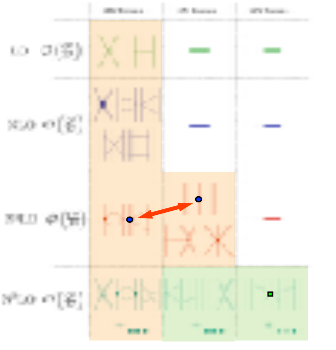


From the strong interaction to nuclear matter

nuclear interactions





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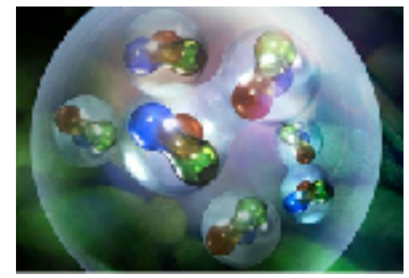

many-body frameworks

From the strong interaction to nuclear matter

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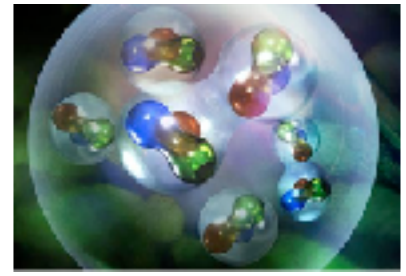
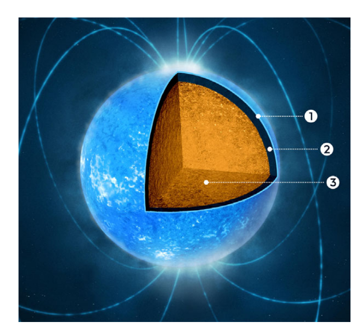
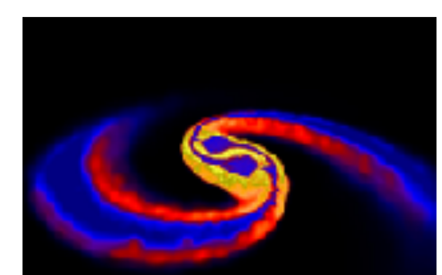


many-body frameworks

equation of state

$$E = E(\rho, T, Y_e, \ldots)$$
$$P = P(\rho, T, Y_e, \ldots)$$





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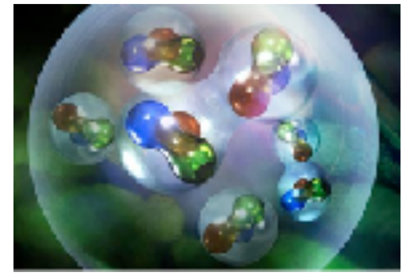

many-body frameworks

probing next-generation nuclear forces

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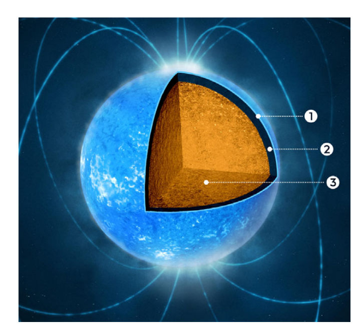
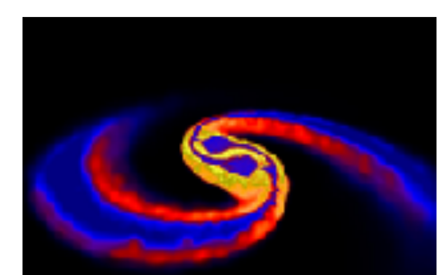
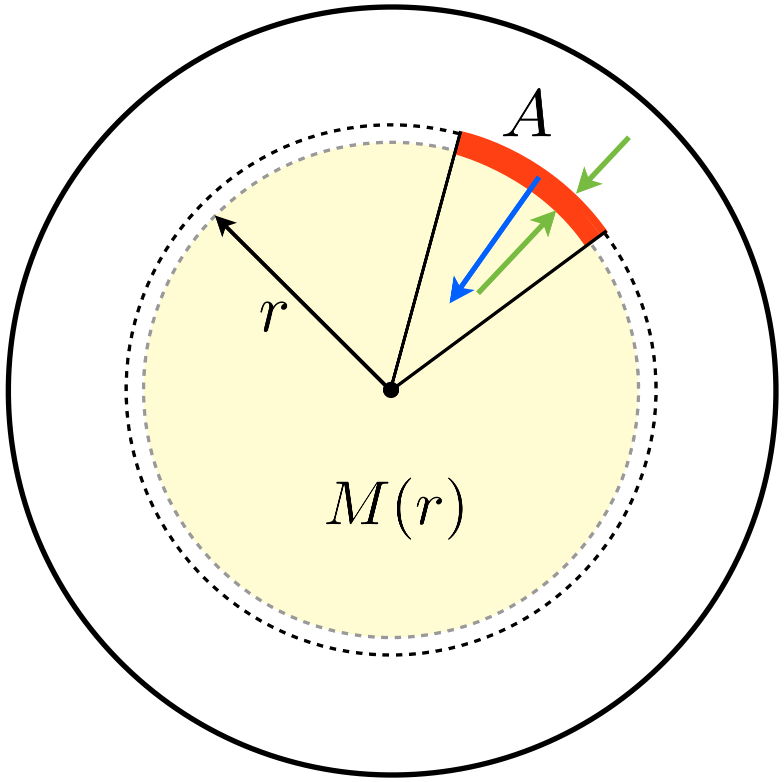




The EOS of high-density matter: neutron stars



- consider forces on a mass element:

gravity: $$F_g = -\frac{GM(r)\rho(r)A\,dr}{r^2}$$

pressure difference: $$F_p = A\left(p_{\rm out} - p_{\rm in}\right) = A\,dp$$

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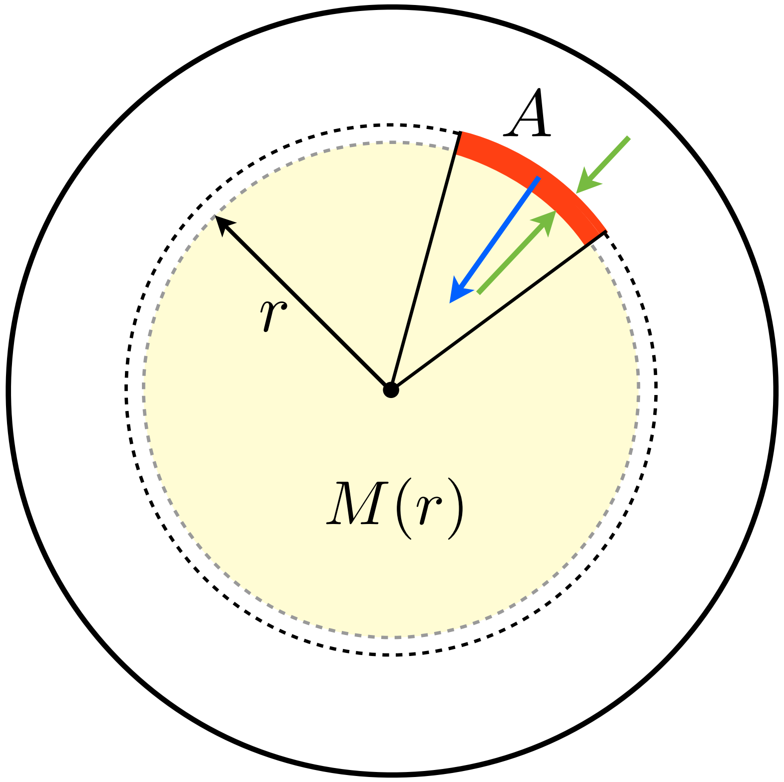

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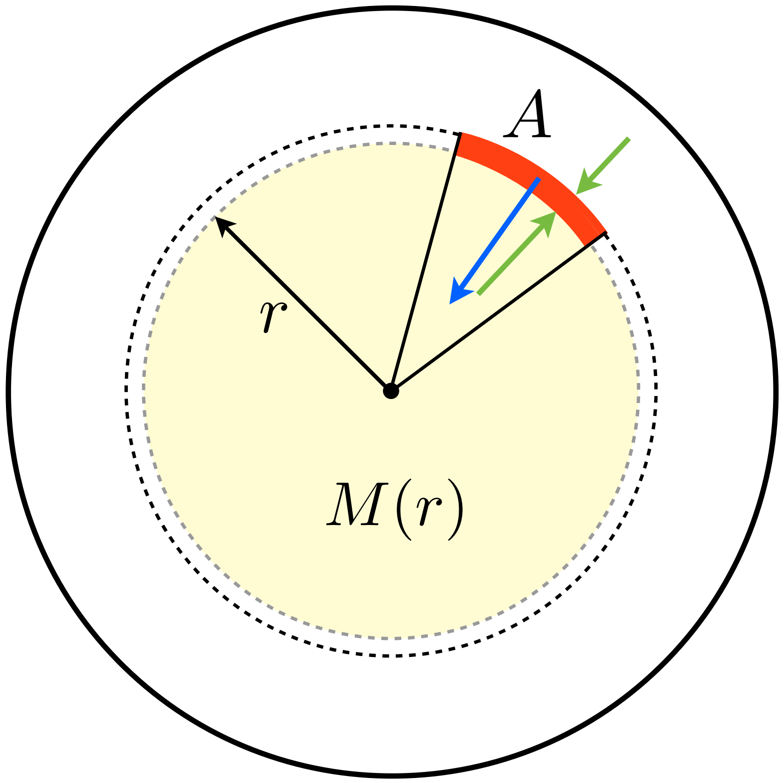

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- include general-relativistic corrections:

$$\frac{dp}{dr} = -\frac{GM(r)\varepsilon(r)}{r^2}\left[1+\frac{p(r)}{\varepsilon(r)c^2}\right]\left[1+\frac{4\pi r^3 p(r)}{M(r)c^2}\right]\left[1-\frac{2GM(r)}{c^2 r}\right]$$

'Tolman-Oppenheimer-Volkov' equation

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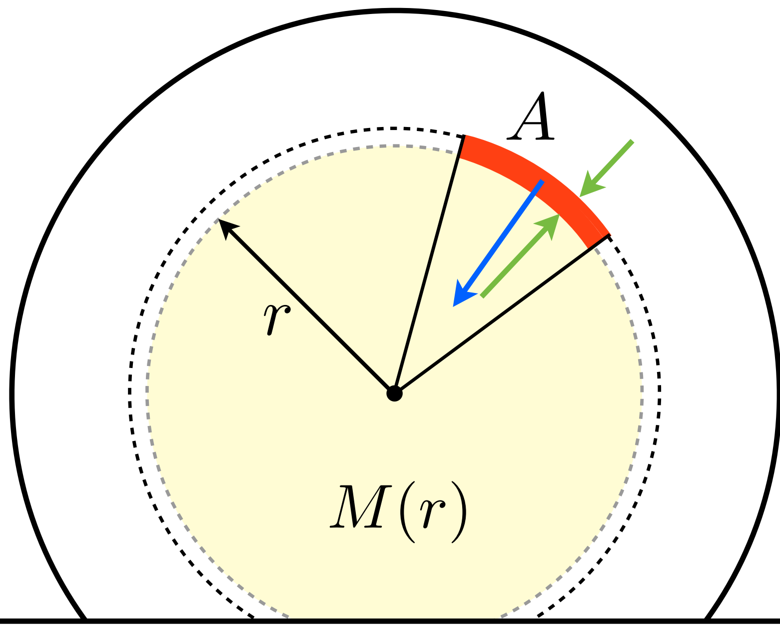

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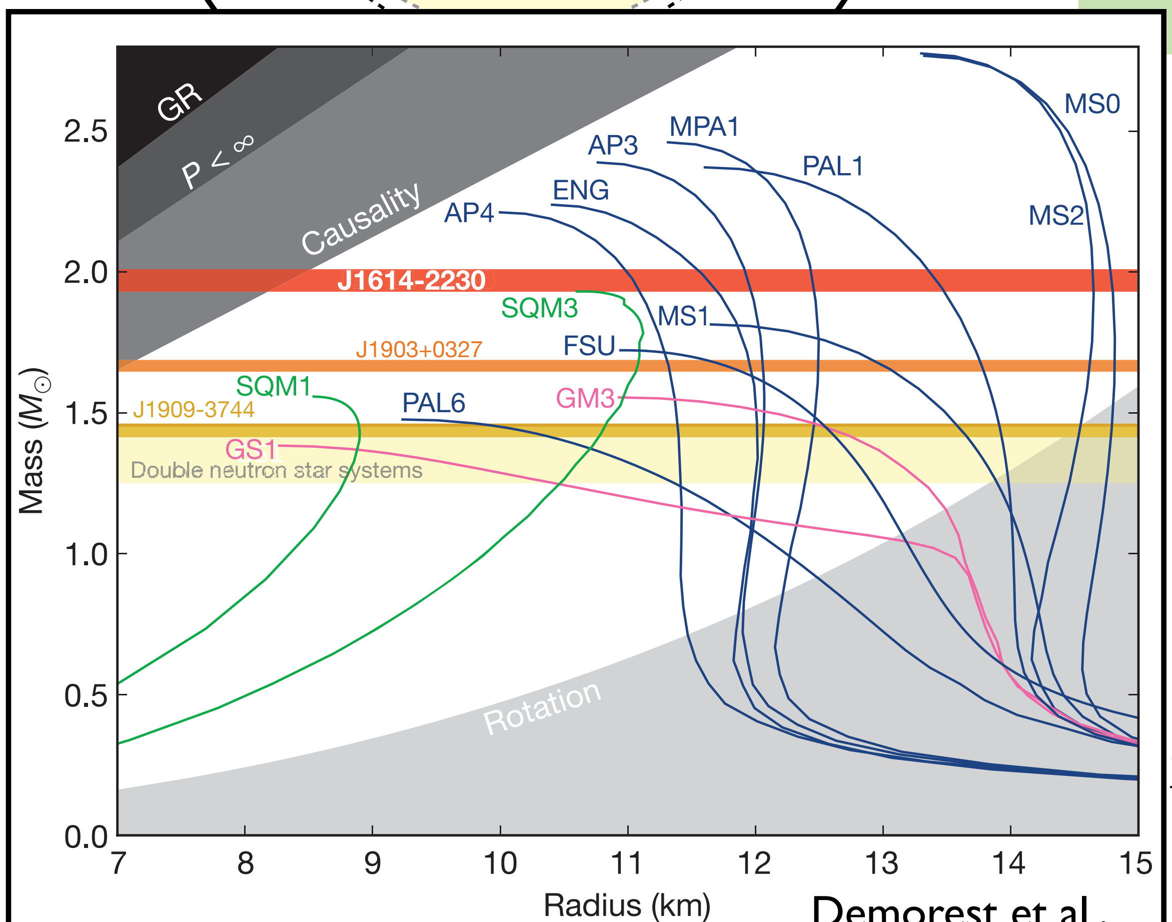
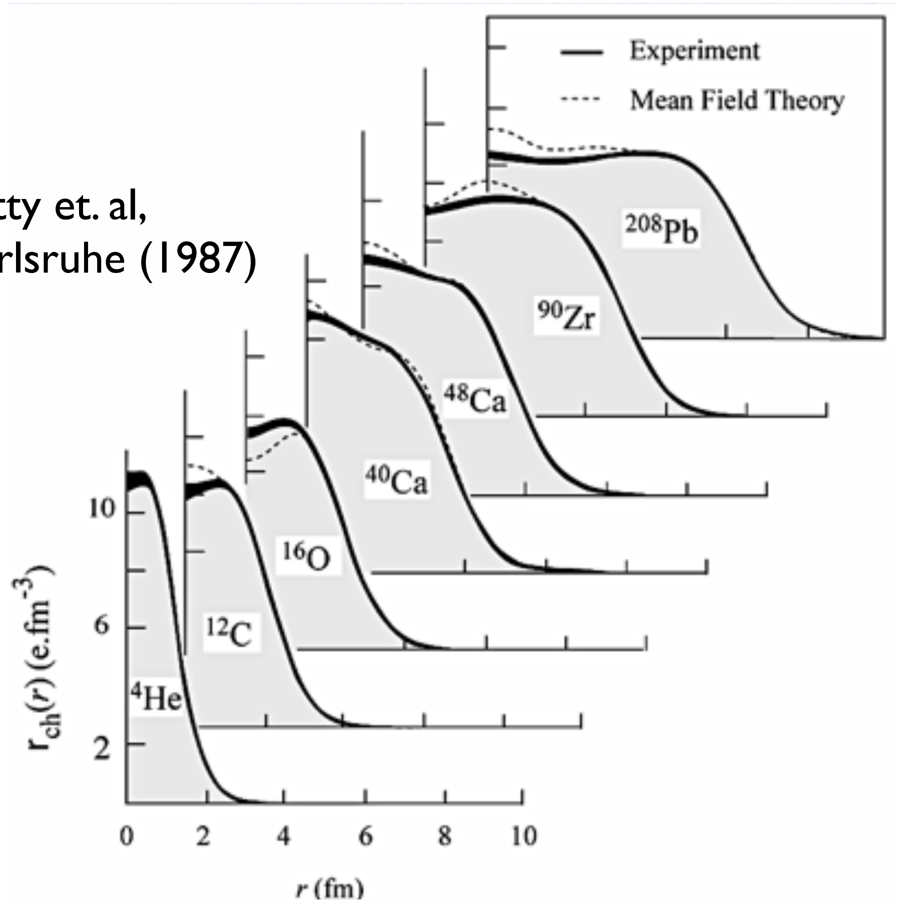


Demorest et al., Nature 467, 1081 (2010)

'Tolman-Oppenheimer-Volkov' equation

Nuclear saturation

Batty et. al,
Karlsruhe (1987)



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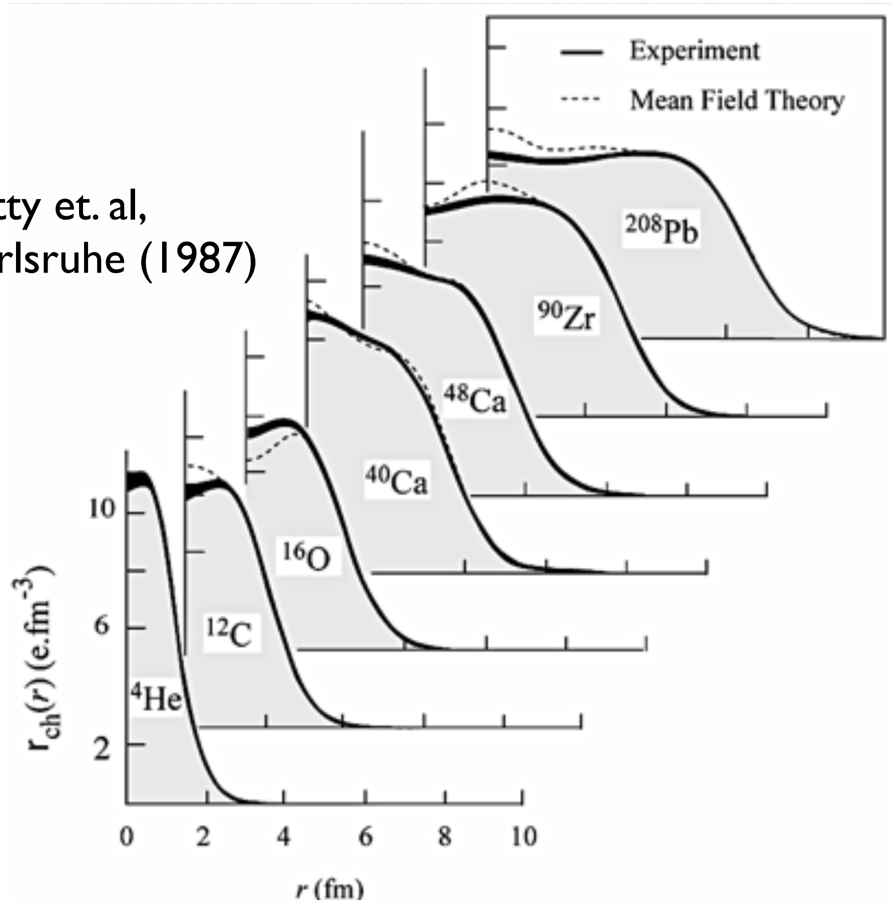

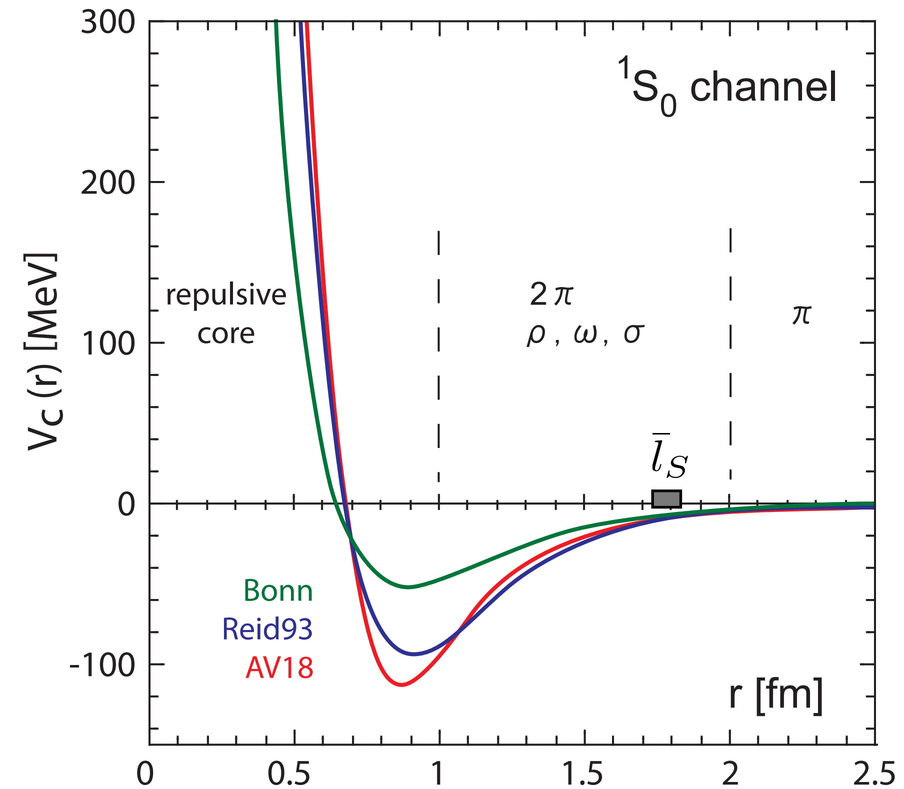

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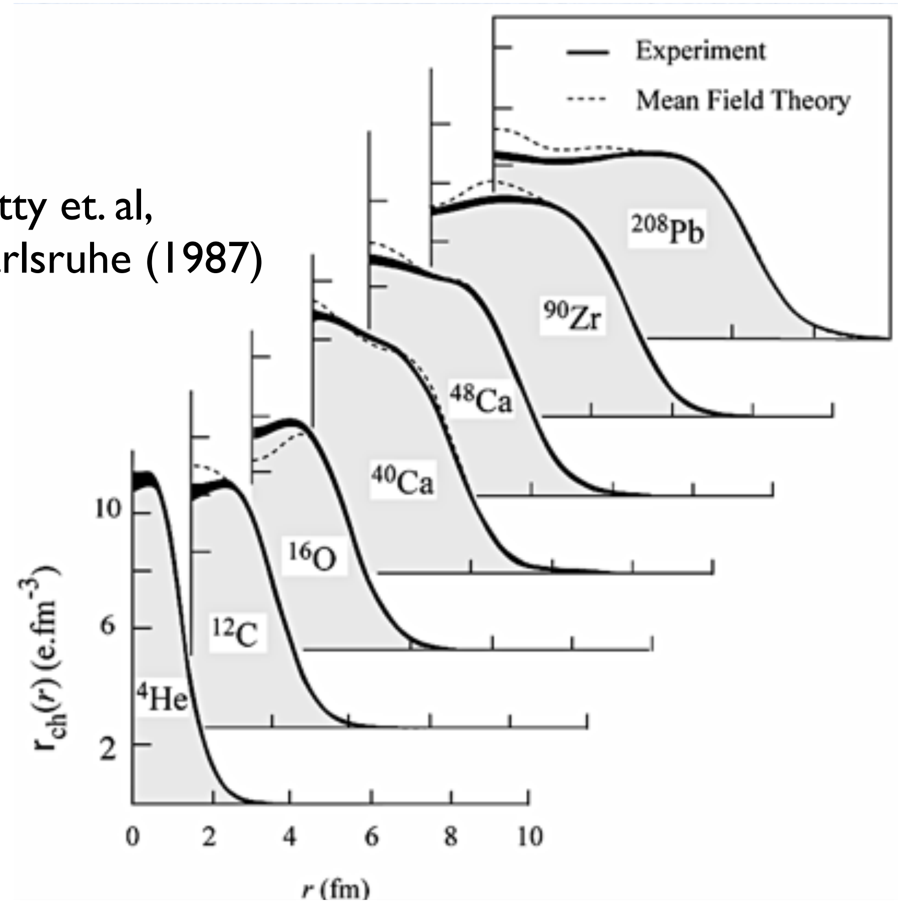
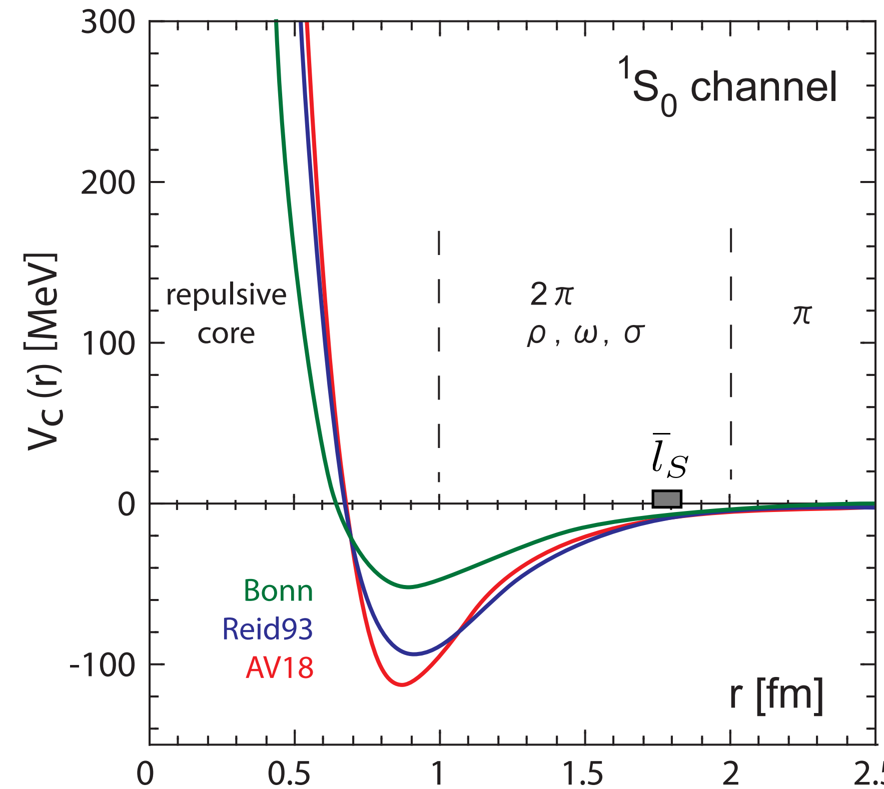
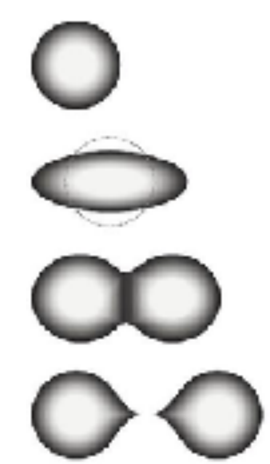




Semi-empirical mass formula:

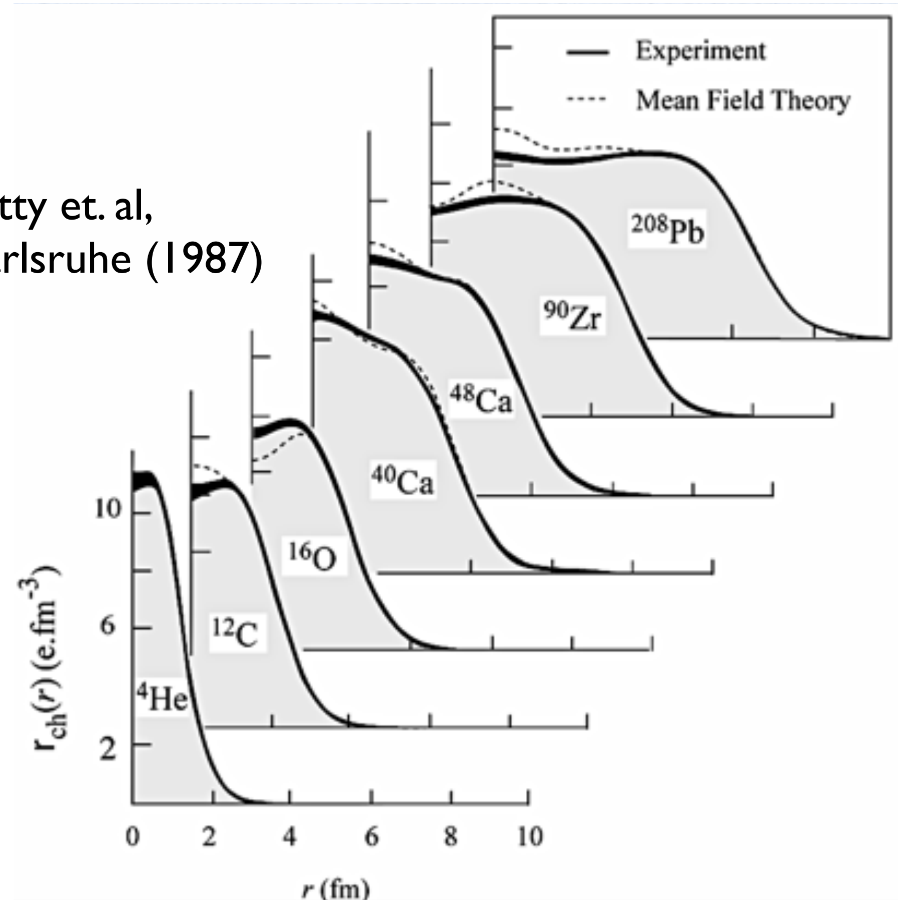
$$E_B = a_V A - a_S A^{2/3} - a_A \frac{(A-2Z)^2}{A^{1/3}} - a_C \frac{Z(Z-1)}{A^{1/3}} \pm \frac{a_p}{A^{1/2}}$$

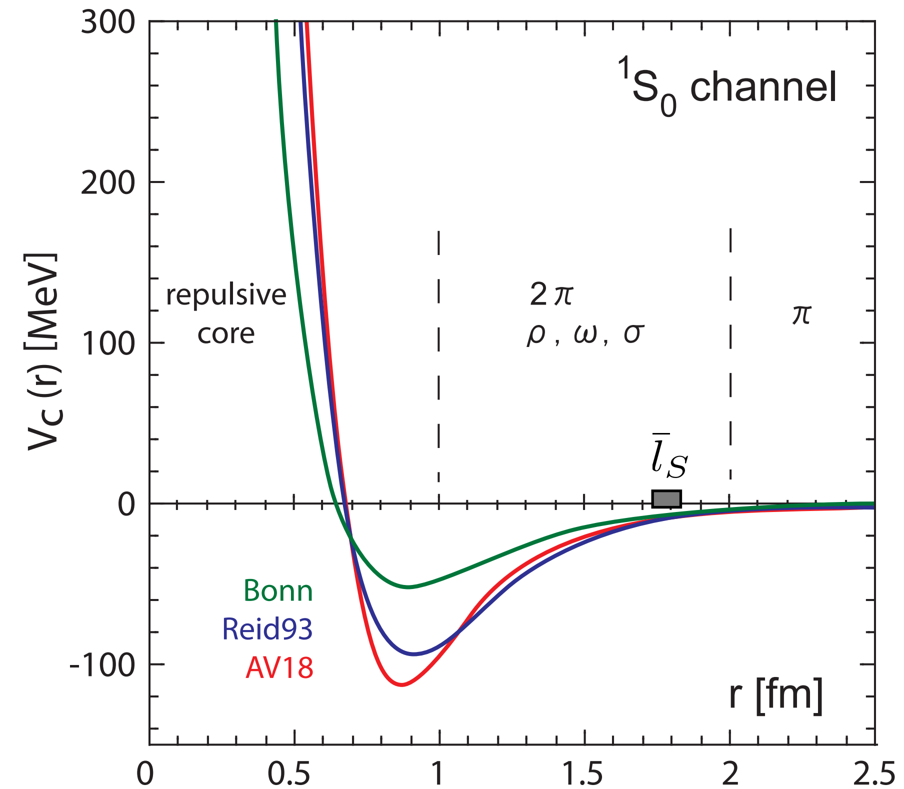
$$a_V \sim 16 \text{ MeV}, a_S \sim 18 \text{ MeV}, a_A \sim 23 \text{ MeV}, a_C \sim 0.7 \text{ MeV}, a_p \sim 12 \text{ MeV}$$



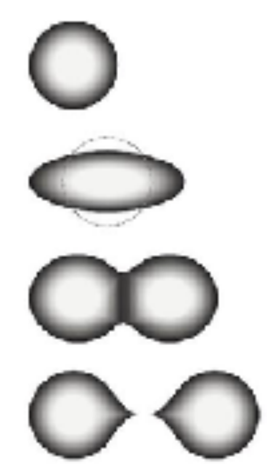
Nuclear saturation and the liquid drop model

Batty et. al, Karlsruhe (1987)





Semi-empirical mass formula: **thermodynamic limit** $A \to \infty$



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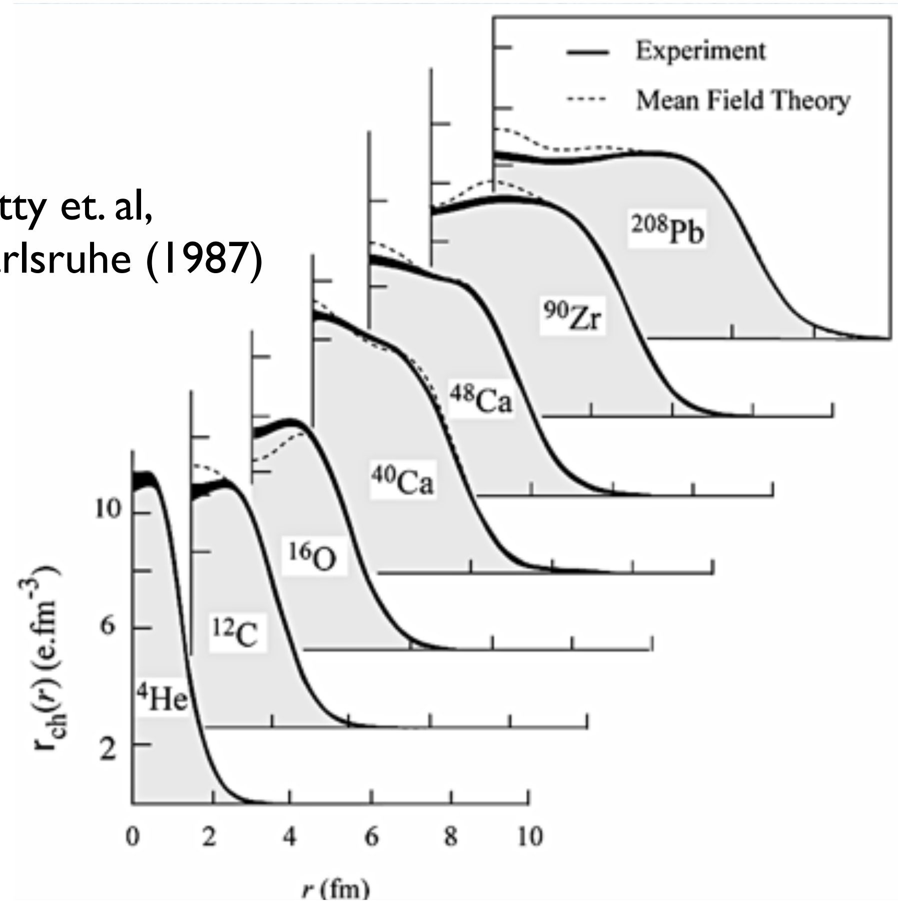

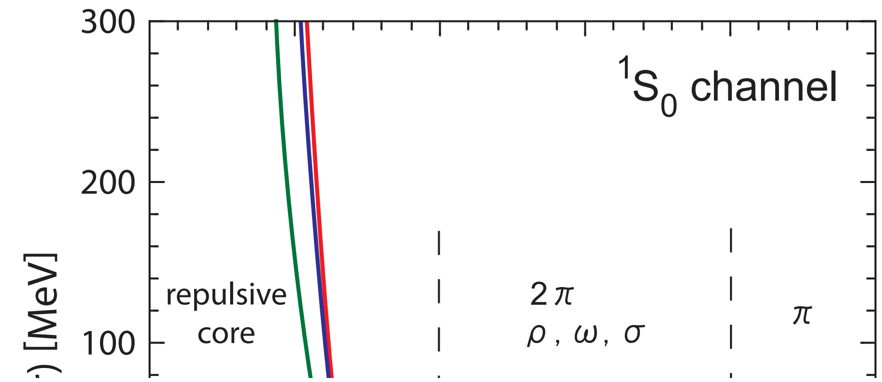

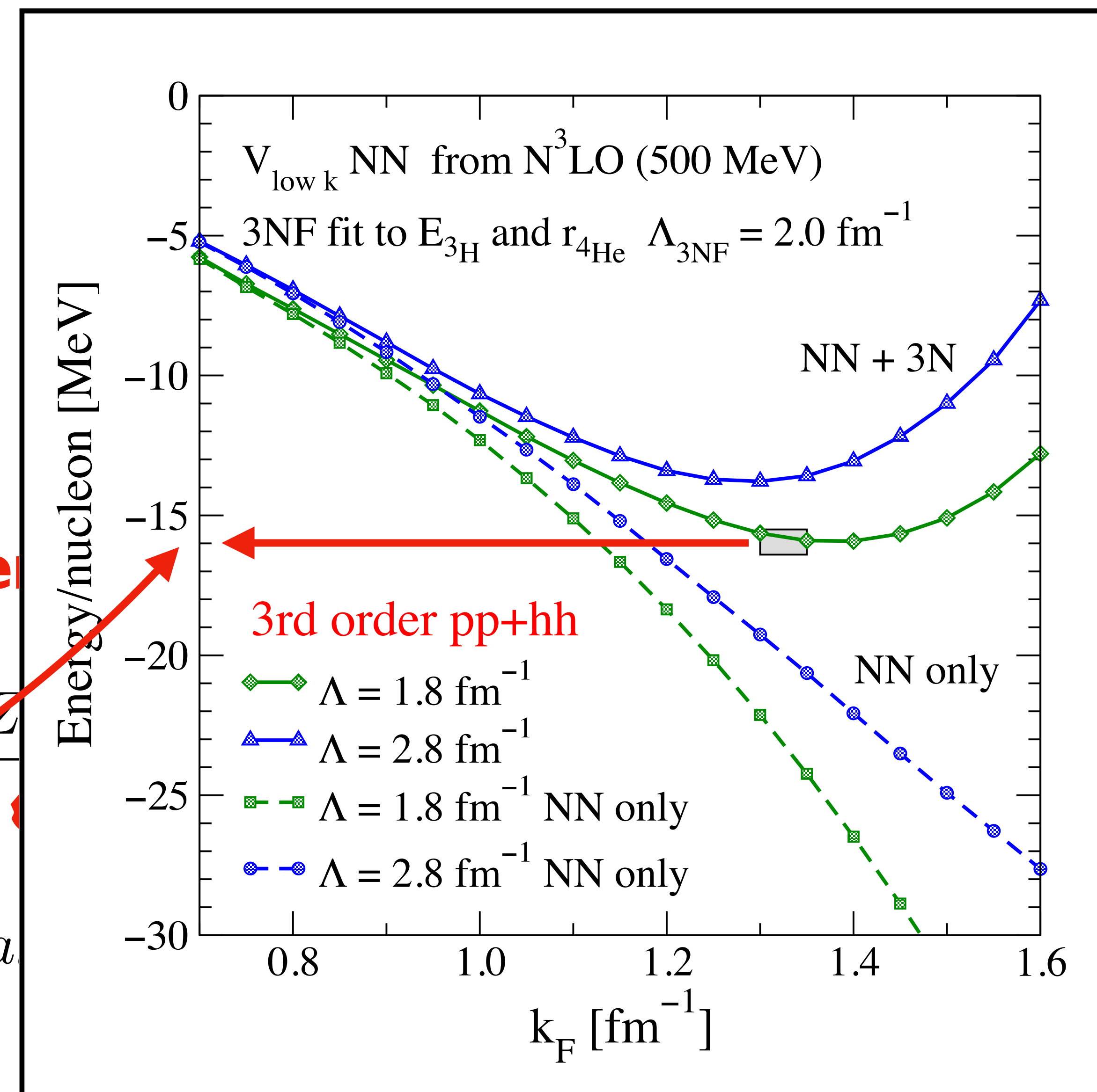

Semi-empirical mass formula: **the**

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Practical calculation of the EOS

Statistical mechanics reminder:

$$Z = \mathrm{Tr}\, e^{-\beta H},$$

$$F = -k_B T \log Z = E - TS,$$

$$P = k_B T \frac{\partial \log Z}{\partial V}$$

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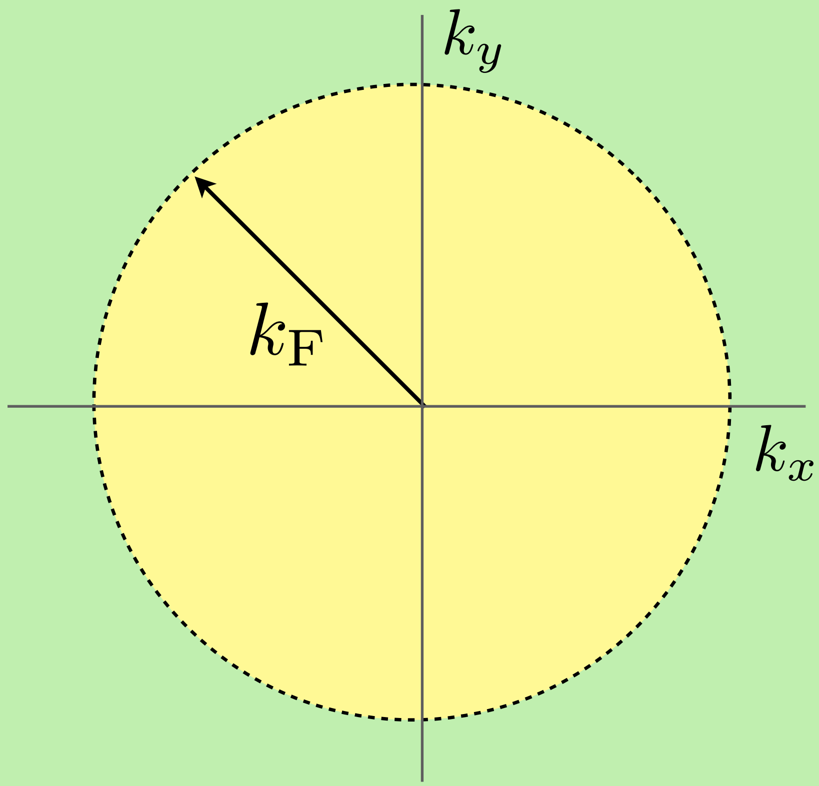
Hamiltonian:

$$H = T + V_{NN} + V_{3N}$$
$$= H_0 + \underbrace{(-H_0 + V_{NN} + V_{3N})}_{H_1}$$

H_0 defines reference state:
(e.g. free state or HF state)



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Evaluation of exact partition function in general highly nontrivial:

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One way to approximately evaluate Z:perturbation theory

$$\left\langle e^{-\beta H_1} \right\rangle_{H_0} = 1 - \beta \left\langle H_1 \right\rangle_{H_0} + \frac{\beta^2}{2!} \left\langle H_1^2 \right\rangle_{H_0} - \frac{\beta^3}{3!} \left\langle H_1^3 \right\rangle_{H_0} + \ldots$$

Many-body perturbation theory: Diagrammatic representation