Description of ^{31}Ne in Halo EFT

TECHNISCHE UNIVERSITÄT DARMSTADT

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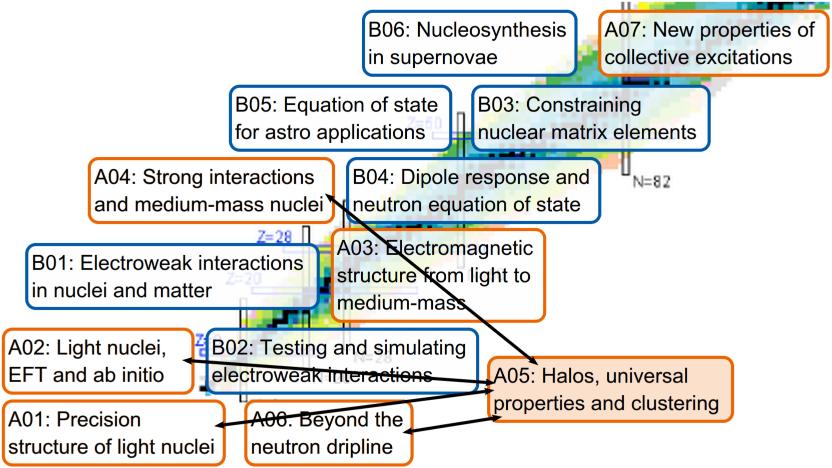
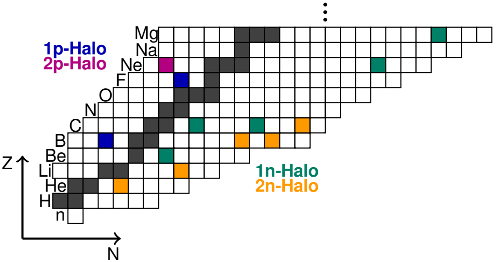
Program Areas



Introduction and Motivation

- Only small number of neutron halo nuclei have been identified
  - Most of them are neutron-rich light isotopes of He through C
- Until now, the heaviest 1*n*-halo nuclei are ^{37}Mg and ^{31}Ne

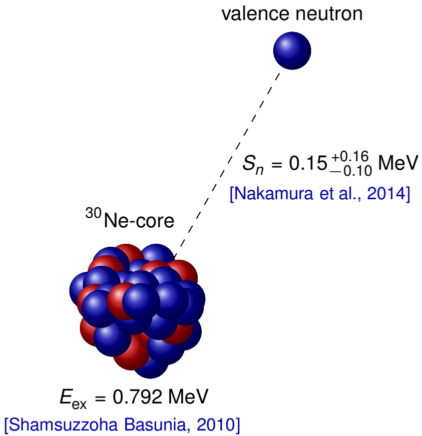


- 1*n*-removal reactions on C and Pb targets revealed: [Nakamura et al., 2014] ^{31}Ne deformed nucleus with a significant *P*-wave halo component

⇒ ^{31}Ne offers a prototype to study deformation-driven halos and understand emergent properties in heavier-near-drip-line nuclei

Introduction and Motivation

- One-neutron halo nuclei are exotic nuclear states
- Degrees of freedom: tightly bound core and a loosely bound valence neutron
- In general: valence neutron bound in a low-ℓ wave
- Quantum numbers determined: $J^{\mathrm{P}} = \frac{3}{2}^{-}$ [Nakamura et al., 2014]



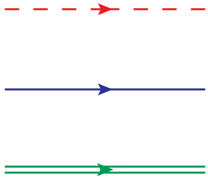
Effective Lagrangian

$$\mathcal{L} = c^\dagger \left[i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_c} \right] c + n_\alpha^\dagger \left[i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_n} \right] n_\alpha + \pi_\beta^\dagger \left[\eta \left(i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2M_{nc}} \right) + \Delta \right] \pi_\beta$$

$$- g \left[\left(c \overleftrightarrow{\nabla}_i \, n_\alpha \right) \pi_\beta^\dagger \, C^{\frac{3}{2}\beta}_{(1i)(\frac{1}{2}\alpha)} + \text{H.c.} \right],$$

where $M_{nc} = m_n + m_c$ (total mass of neutron and core)

Feynman rules:



EM Interaction – Minimal Substitution

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Photons are included via minimal substitution:

$$\partial_\mu \to D_\mu = \partial_\mu + ie\hat{q}A_\mu \quad (e > 0)$$

- $\hat{q}$ is the charge operator acting on a c or n field
  - $\hat{q}n = 0$
  - $\hat{q}c = q_c c$ with $q_c = 10$ for Ne
- Gauge-invariant operators involving $\vec{B}$ and $\vec{E}$ might contribute to electromagnetic observables within our power counting scheme

Scalar Component of EM Current

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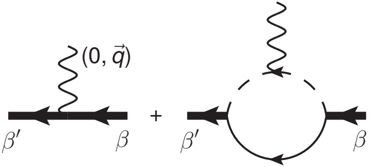
$$i\mathcal{A} = \langle \pi_{\beta'}(\vec{p}') | \, J_0 \, | \pi_{\beta}(\vec{p}) \rangle =$$



$$= -i q_{\mathrm{tot}} G_{\mathrm{E0}}(q) \sqrt{4\pi} q^0 Y_{00}(\vec{e}_{\vec{q}}) \tilde{T}^{00}_{\beta'\beta} - i Q G_{\mathrm{E2}}(q) 2 \sqrt{\frac{4\pi}{5}} q^2 \sum_M Y_{2M}(\vec{e}_{\vec{q}}) \tilde{T}^{2M}_{\beta'\beta}$$

$\lim_{q \to 0} G_{\mathrm{E0}}(q) \equiv 1$ (Charge conservation given by gauge-invariance)

$\lim_{q \to 0} G_{\mathrm{E2}}(q) \equiv 1 \Rightarrow Q$ (this limit defines Q)

Electric Form Factors

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Monopole and Quadrupole Form Factors

$$G_{E0}(q) = \left[1 + \frac{\gamma}{|r_1|} - \frac{y^2q^2 + 2\gamma^2}{yq|r_1|} \arctan\left(\frac{yq}{2\gamma}\right)\right]$$

$$G_{E2}(q) = \frac{1}{Q}\frac{q_{\text{tot}}}{8|r_1|yq^3}\left[2\gamma yq + \left(y^2q^2 - 4\gamma^2\right)\arctan\left(\frac{yq}{2\gamma}\right)\right]$$

with $y = \frac{m_n}{M_{nc}} = \frac{1}{31}$, $\gamma \hat{=}$ binding momentum

and estimate P-wave effective momentum: $r_1 \sim M_{\text{hi}}$

- Consistency check: $\lim_{q\to 0} G_{E0}(q) \equiv 1$ ✓
- $\lim_{q\to 0} G_{E2}(q) \equiv 1 \Rightarrow \boxed{Q = \frac{y^2 q_{\text{tot}}}{3\gamma|r_1|}} \quad \Rightarrow Q \in [0.17, 0.28]\, e\text{fm}^2$

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Electric Form Factors

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Charge and quadrupole radii are defined by expanding the form factors in q^2:

$$G_{E0}(q) \approx 1 - \frac{1}{6} < r_{E0}^2 > q^2 + \ldots$$

$$G_{E2}(q) \approx 1 - \frac{1}{6} < r_{E2}^2 > q^2 + \ldots$$

Now compare to the expansion of the calculated form factors:

Charge & Quadrupole Radii

$$\Rightarrow < r_{E0}^2 > = \frac{5y^2}{2\gamma|r_1|} \qquad \Rightarrow \sqrt{< r_{E0}^2 >} \in [0.35, 0.46]\,\text{fm}$$

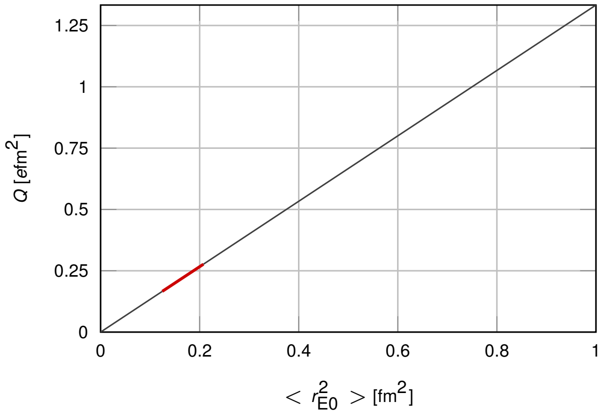
$$\Rightarrow < r_{E2}^2 > = \frac{3y^2}{5\gamma^2} \qquad \Rightarrow \sqrt{< r_{E2}^2 >} = 0.30\,\text{fm}$$

Correlations

$Q \longleftrightarrow < r_{E0}^2 >$:

- $< r_{E0}^2 >= \frac{5y^2}{2\gamma|r_1|}$
- $Q = \frac{y^2 q_{tot}}{3\gamma|r_1|}$

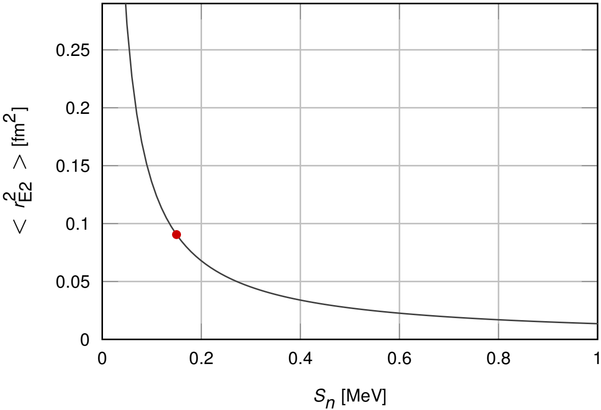
$$\Rightarrow \boxed{Q = \frac{2}{15} q_c e < r_{E0}^2 >}$$



Correlations

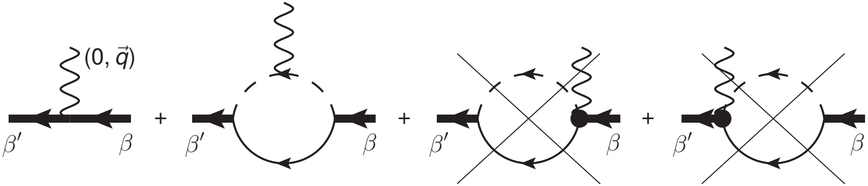
$< r^2_{\mathrm{E2}} > \longleftrightarrow S_n$:

$$< r^2_{\mathrm{E2}} > = \frac{3y^2}{10\mu}\frac{1}{S_n}$$

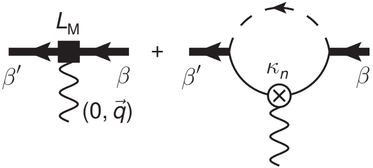


Vector Component of EM Current

Contributions to $\langle \pi_{\beta'}(\vec{p}')| J_k |\pi_\beta(\vec{p})\rangle$ due to minimal substitution:



Contributions to $\langle \pi_{\beta'}(\vec{p}')| J_k |\pi_\beta(\vec{p})\rangle$ due to magnetic moment coupling:



Magnetic Form Factors

Octupole Form Factor

$$G_{M3}(q) = \frac{3\gamma}{2(1-y)^3 q^3}\left[2\gamma(1-y)q + \left((1-y)^2 q^2 - 4\gamma^2\right)\arctan\left(\frac{(1-y)q}{2\gamma}\right)\right]$$
$$= G_{E2}[y \rightarrow (1-y)]$$

Octupole Moment and Radius

$$o_M = \frac{(1-y)^2 \kappa_n \mu_N}{10\sqrt{6}\gamma|r_1|} \qquad \Rightarrow o_M \in [-5.75, -3.45]\,\mu_N \mathrm{fm}^2$$

$$< r_{M3}^2 > = \frac{3(1-y)^2}{10\mu}\frac{1}{S_n} \qquad \Rightarrow \sqrt{< r_{M3}^2 >} = 8.95\ \mathrm{fm}$$

Correlations

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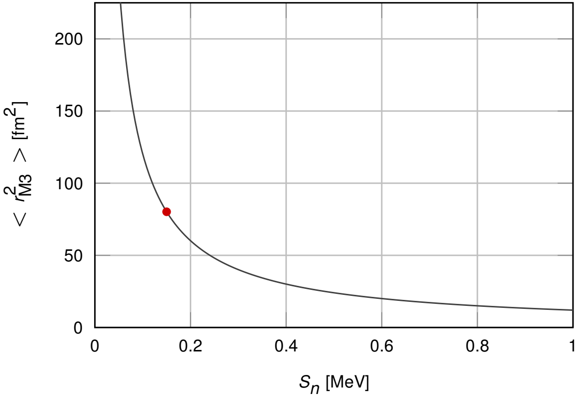
$< r^2_{\mathrm{M3}} > \longleftrightarrow S_n$:

$$< r^2_{\mathrm{M3}} > = \frac{3(1-y)^2}{10\mu}\frac{1}{S_n}$$



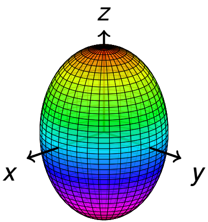
Nuclear Deformation

Consider a quadrupolar deformed nucleus with sharp edge at radius:

$$R_{\text{def}} = R_0 \left[1 + \beta_2 Y_{20}\right] / N$$

where $R_0 \hat{=}$ equilibrium radius,

$N \hat{=}$ volume normalization constant



$\Rightarrow$ Quadrupole moment:

$$Q(3/2) = \frac{1}{5} Q_0 = \frac{1}{5}\sqrt{\frac{16\pi}{5}} \frac{3}{4\pi} Ze\, R_0^2\, \beta_2$$

$$= \sqrt{\frac{1}{5\pi}} Ze\, \beta_2 < r_{E0}^2 >$$

Compare to EFT result $\longrightarrow$ $\beta_2 = 0.53$

Outlook

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- Matter Radii
- Coulomb Breakup
- Improvement of Power Counting

  $\Rightarrow$ Incorporation of Additional Field(s)

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Thank you for your attention!

References

Hammer, H. W. and Phillips, D. R. (2011).
Electric properties of the Beryllium-11 system in Halo EFT.
Nucl. Phys., A865:17–42.

Nakamura, T. et al. (2014).
Deformation-Driven *P*-Wave Halos at the Drip Line: Ne^{31}.
Phys. Rev. Lett., 112(14):142501.

Shamsuzzoha Basunia, M. (2010).
Nuclear Data Sheets for A = 30.
Nucl. Data Sheets, 111:2331–2424.

Backup

Introduction & Motivation

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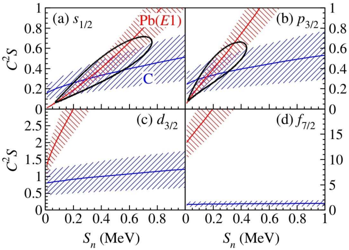
- Quantum numbers determined: $J^P = \frac{3}{2}^-$ [Nakamura et al., 2014]

  $\Rightarrow$ *P*-wave one-neutron halo with a spinless ^{30}Ne-core ($J = 0$)

  $\Rightarrow$ Coupling of $\ell = 1$ and neutron spin to $J = \frac{3}{2}$



- Low neutron separation energy $S_n = 0.15^{+0.16}_{-0.10}$ MeV [Nakamura et al., 2014]
  First excited state of the ^{30}Ne-core at $E = 792$ keV [Shamsuzzoha Basunia, 2010]

  $\Rightarrow$ Separation of scales

  $\Rightarrow$ Use Halo EFT in order to describe ^{31}Ne

Full Dimeron Propagator

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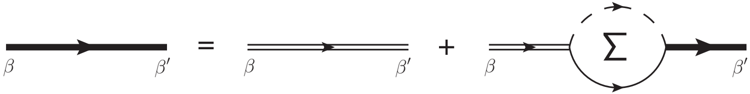
Full dimeron propagator:

Dress bare propagator with a geometric series of dimeron self-energies

$\Rightarrow$ Dyson equation:

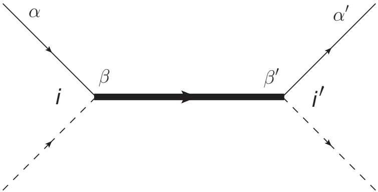


$$D_{\beta\beta'}(p_0, p) = \frac{\delta_{\beta\beta'}}{\eta\left(p_0 - p^2/(2M_{nc})\right) + \Delta - \Sigma + i\epsilon}$$

$$\Sigma = \frac{-\mu g^2 2\mu\left(p_0 - p^2/(2M_{nc})\right)}{6\pi}\left(\frac{3}{2}\Lambda^{\text{PDS}} - \sqrt{-2\mu\left(p_0 - p^2/(2M_{nc})\right) - i\epsilon}\right),$$

where $\mu \hat{=}$ reduced mass

Scattering Amplitude: Renormalization



$$T_{\alpha'\alpha}(\vec{p}',\vec{p}) = \frac{6\pi}{\mu}\frac{\frac{2}{3}\vec{p}'\vec{p}\delta_{\alpha'\alpha} + \frac{i}{3}\left(\sum_l \sigma_l\left(\vec{p}'\times\vec{p}\right)_l\right)_{\alpha'\alpha}}{\left(\frac{-6\pi\Delta}{\mu g^2} - \frac{3\pi\eta}{\mu^2 g^2}p^2 - \Lambda p^2 - ip^3\right)}$$

$$\equiv \frac{6\pi}{\mu}\frac{\frac{2}{3}\vec{p}'\vec{p}\delta_{\alpha'\alpha} + \frac{i}{3}\left(\sum_l \sigma_l\left(\vec{p}'\times\vec{p}\right)_l\right)_{\alpha'\alpha}}{\left(\frac{-6\pi\Delta^R}{\mu\left(g^R\right)^2} - \frac{3\pi\eta}{\mu^2\left(g^R\right)^2}p^2 - ip^3\right)}$$

$$\Rightarrow \quad \frac{-3\pi\eta}{\mu^2 g^2} - \Lambda = \frac{-3\pi\eta}{\mu^2\left(g^R\right)^2}, \qquad \frac{\Delta}{g^2} = \frac{\Delta^R}{\left(g^R\right)^2}$$

Matching

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Now compare renormalized amplitude to effective range expansion of amplitude:

$$
\left.\begin{aligned}
T_{\alpha'\alpha}(\vec{p}', \vec{p}) &= \frac{6\pi}{\mu} \frac{\frac{2}{3}\vec{p}'\vec{p}\delta_{\alpha'\alpha} + \frac{i}{3}\left(\sum_l \sigma_l \left(\vec{p}' \times \vec{p}\right)_l\right)_{\alpha'\alpha}}{\left(\frac{-6\pi\Delta^{\mathrm{R}}}{\mu\left(g^{\mathrm{R}}\right)^2} - \frac{3\pi\eta}{\mu^2\left(g^{\mathrm{R}}\right)^2}p^2 - ip^3\right)} \\
&= \frac{6\pi}{\mu} \frac{\frac{2}{3}\vec{p}'\vec{p}\delta_{\alpha'\alpha} + \frac{i}{3}\left(\sum_l \sigma_l \left(\vec{p}' \times \vec{p}\right)_l\right)_{\alpha'\alpha}}{\left(-\frac{1}{a_1} + \frac{r_1}{2}p^2 - ip^3\right)}
\end{aligned}\right\} \Rightarrow
\boxed{\begin{aligned}
a_1 &= \frac{\mu\left(g^{\mathrm{R}}\right)^2}{6\pi\Delta^{\mathrm{R}}} \\
r_1 &= -\frac{6\pi\eta}{\mu^2\left(g^{\mathrm{R}}\right)^2}
\end{aligned}}
$$

Note:

$a_1 \hat{=}$ scattering volume

$r_1 \hat{=}$ P-wave effective momentum

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Effective Range Parameters

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Determine effective range parameters by using measured observables:

- Demand a pole in the amplitude at $E = -S_n = -\gamma^2/(2\mu)$ ($\gamma > 0 \hat{=}$ binding momentum)

$$\Rightarrow \left(-\frac{1}{a_1} + \frac{r_1}{2}p^2 - ip^3\right)\Bigg|_{p=i\gamma} = 0$$

$$\Rightarrow a_1 = -\frac{2}{\gamma^2\,(r_1 + 2\gamma)}$$

- Assuming only Δ/g^2 to be fine-tuned (shallow P-wave state) [Hammer and Phillips, 2011]

$\Rightarrow$ scattering volume a_1 is enhanced by $1/(M_{\text{lo}}^2 M_{\text{hi}})$

$\Rightarrow$ P-wave effective momentum $r_1 \sim M_{\text{hi}}$ (breakdown scale of the theory)

Vector Component of EM Current

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$$\langle \pi_{\beta'}(\vec{p}')| \, J_k \, |\pi_\beta(\vec{p})\rangle$$

$$= \left(i q_{\text{tot}} G_{\text{E0}}(q) \sqrt{\frac{4\pi}{1}} q^0 \, Y_{00}(\vec{e}_{\vec{q}}) \, \tilde{T}^{00}_{\beta'\beta} + i Q G_{\text{E2}}(q) 2 \sqrt{\frac{4\pi}{5}} q^2 \sum_M Y_{2M}(\vec{e}_{\vec{q}}) \, \tilde{T}^{2M}_{\beta'\beta} \right) \frac{(\vec{p}' + \vec{p})^*_k}{2M_{nc}}$$

$$+ i \mu_{\text{M}} G_{\text{M1}}(q) \sqrt{\frac{4\pi}{3}} q^1 \sum_M \sqrt{2} \, C^{1M+k}_{(1k)(1M)} Y^*_{1M+k}(\vec{e}_{\vec{q}}) \left[\tilde{T}^{1M}_{\beta'\beta} \right]^\dagger$$

$$+ i o_{\text{M}} G_{\text{M3}}(q) 2 \sqrt{\frac{4\pi}{7}} q^3 \sum_M \sqrt{2} \, C^{3M+k}_{(1k)(3M)} Y^*_{3M+k}(\vec{e}_{\vec{q}}) \left[\tilde{T}^{3M}_{\beta'\beta} \right]^\dagger$$

Note:

For $J = 1/2$ the electric quadrupole and the magnetic octupole moment is not observable!

Magnetic Form Factors

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Dipole Form Factor

$$G_{\mathrm{M1}}(q) = \frac{1}{r_1}\left[\frac{aL_{\mathrm{M}} + b\kappa_n + c\frac{q_c}{A_c}}{\mu_{\mathrm{M}}}\right]\mu_N \text{ , with}$$

$$a = a(r_1, \Lambda)$$
$$b = b(q, \gamma, (1-y), \Lambda)$$
$$c = c(q, \gamma, y, \Lambda)$$

Dipole Moment and Radius

$$\mu_{\mathrm{M}} = \left[L_{\mathrm{M}} + \frac{3(\Lambda-\gamma)}{r_1}\left(L_{\mathrm{M}} - \kappa_n + y\frac{q_c}{A_c}\right)\right]\mu_N$$

$$< r^2_{\mathrm{M1}} > = \left[\frac{3}{2}y^3\frac{q_c}{A_c} - \frac{27}{10}(1-y)^2\kappa_n\right]\frac{\mu_N}{\mu_{\mathrm{M}}}\frac{1}{\gamma r_1}$$

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Nuclear Deformation

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Relation between spectroscopic and intrinsic quadrupole moment (bandhead):

$$Q(J) = \frac{J(2J-1)}{(J+1)(2J+3)} Q_0$$

$$\Rightarrow Q(3/2) = \frac{1}{5} Q_0$$

$$= \sqrt{\frac{1}{5\pi}} Ze\beta_2 < r_{E0}^2 >$$

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