

## 2<sup>nd</sup> Workshop of the SFB 1245

October 4 - 6, 2017

Schloss Waldthausen, Budenheim

**Marcel Schmidt**

Institute of Nuclear Physics

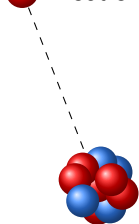
Technische Universität Darmstadt

Project A05

October 5, 2017



Neutron

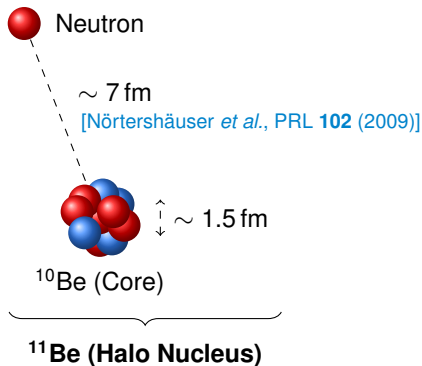


$^{10}\text{Be}$  (Core)

$^{11}\text{Be}$  (Halo Nucleus)

- ▶ Deeply-bound core vs. Weak valence binding

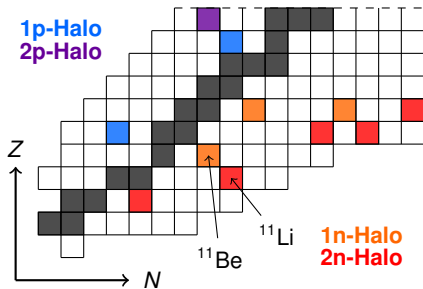
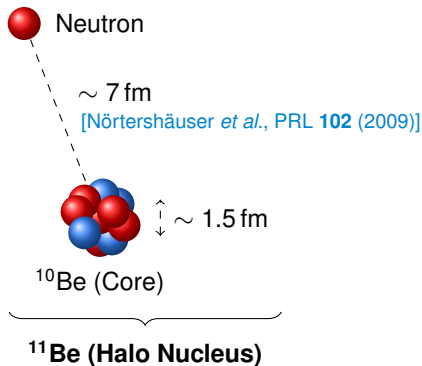




- ▶ Deeply-bound core vs. Weak valence binding  $\rightarrow$  Scale Separation!



# Motivation: *Halo Nuclei*



- ▶ Deeply-bound core vs. Weak valence binding  $\rightarrow$  Scale Separation!
- ▶ Many more near dripline

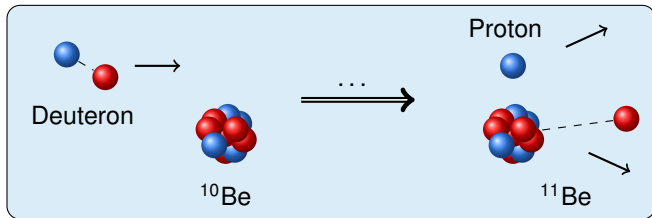
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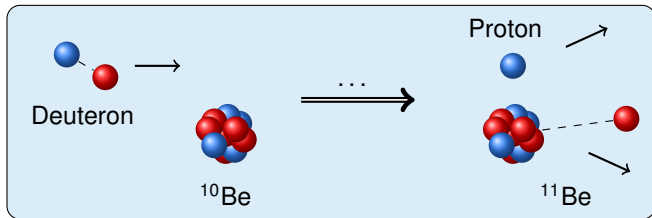
- ▶ Case study:



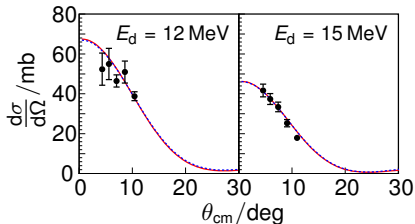
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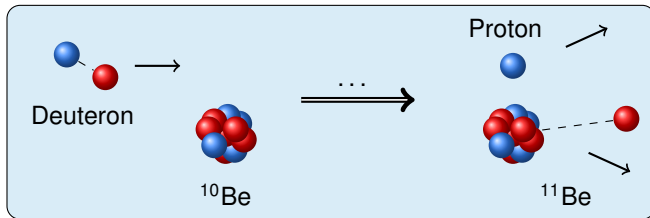


- ▶ Cross section data (ORNL) [Schmitt *et al.*, PRL **108** (2012)]

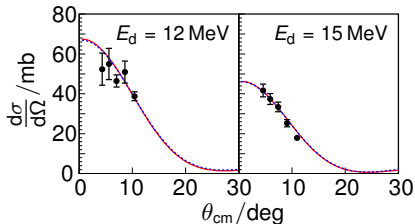


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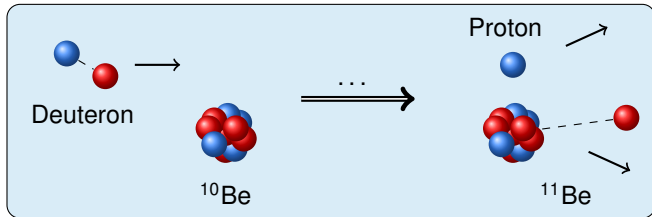
Traditional Reaction Models  
→ No uncertainty quantification



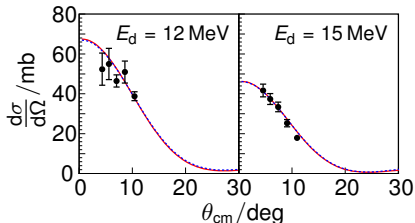


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**Halo EFT** [Bertulani *et al.*, NPA **712** (2002)]

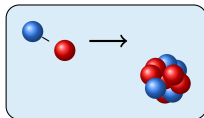
→ Power counting

→ Unify structure & reaction



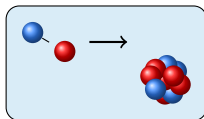
## I. Develop **EFT for halo reactions**. Focus on...

- ▶ ... 1n-halos  $\rightarrow$  Beryllium-11
- ▶ ... strong interaction



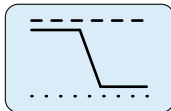
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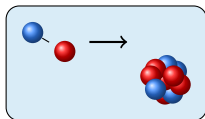
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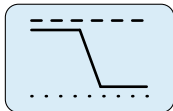
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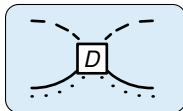
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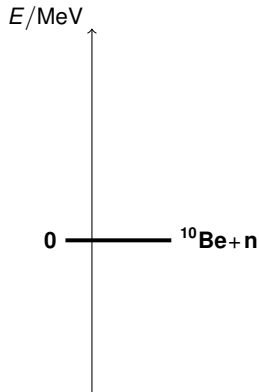


## III. Introduce **3-body forces** for...

- ▶ ... renormalization
- ▶ ... loss effects



# I. EFT for $^{10}\text{Be}(d, p)^{11}\text{Be}$ : *Phenomenology*

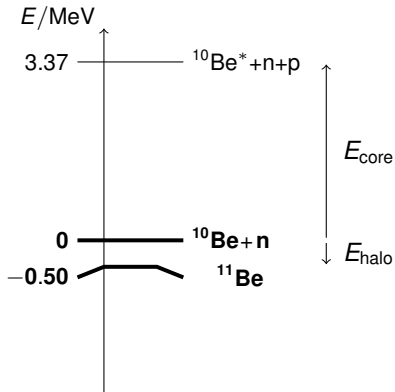


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▶  $^{11}\text{Be} \hat{=} (^{10}\text{Be}+n) (\frac{1}{2}^+)$  weakly bound

→ **Halo EFT**:  $R_{\text{core}}/R_{\text{halo}} \sim 0.4$

[Hammer & Phillips, NPA 865 (2011)]



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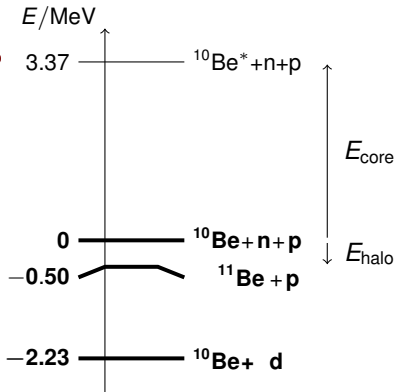
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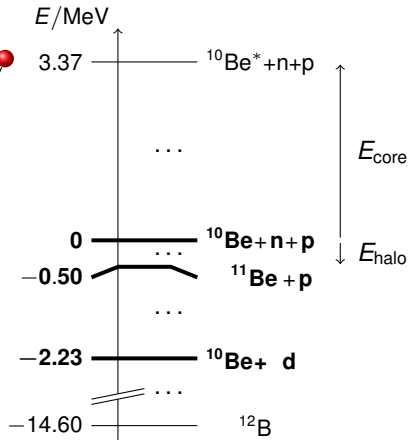
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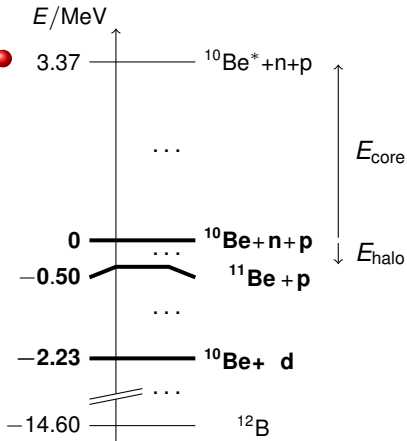
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Long-range physics  
**explicit**

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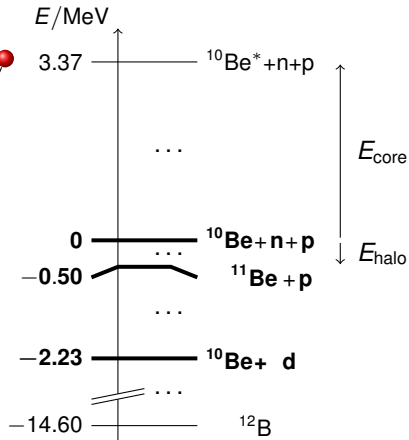
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Long-range physics  
**explicit**

vs.

Short-range physics  
**implicit**



# I. EFT for $^{10}\text{Be}(d, p)^{11}\text{Be}$ : *Lagrangian (LO)*

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▶ Full propagators:

$$\text{.....}^{(\text{full})} = \text{.....} + \text{---} \text{---}^{(\text{full})}$$

→ effective range expansion



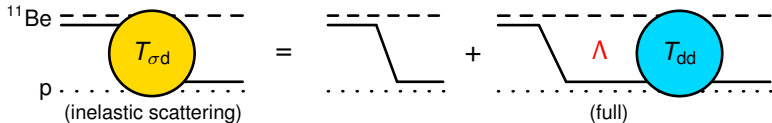
## II. Scattering Amplitude: *Construction*

- ▶ Iterate **neutron exchanges**:



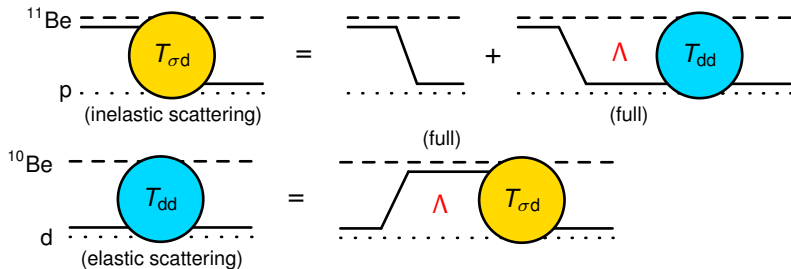
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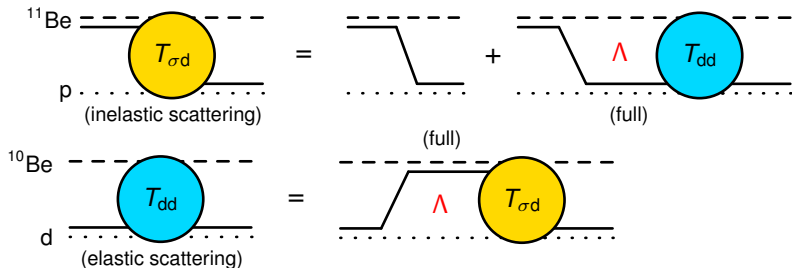
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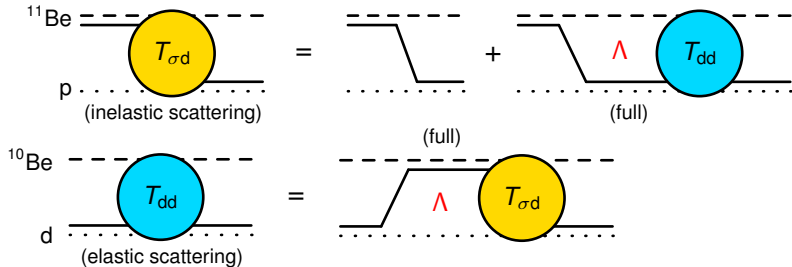
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$$\underbrace{\vec{T}_{J^\pi}(p, p')}_{\text{amplitude vector}} = - \underbrace{\vec{V}_{J^\pi}(p, p')}_{\text{potential vector}} + \int_0^\Lambda \frac{dq q^2}{(2\pi)^3} \underbrace{V_{J^\pi}(p, q)}_{\text{potential matrix}} \underbrace{\underline{G}(q)}_{\text{Propagator matrix}} \vec{T}_{J^\pi}(q, p')$$

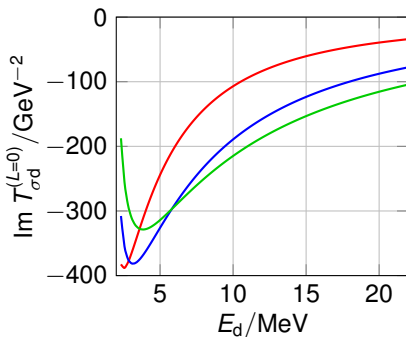
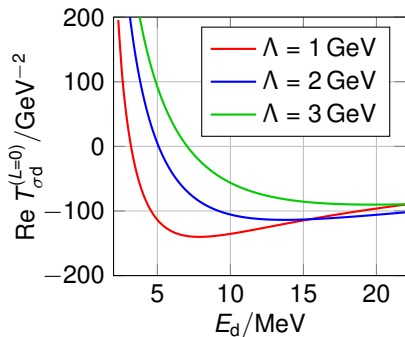


## II. Scattering Amplitude: *Cutoff-Dependence*

- ▶  $L > 0$  integrals converge as  $\Lambda \rightarrow \infty$ ,  $L = 0$  integrals don't

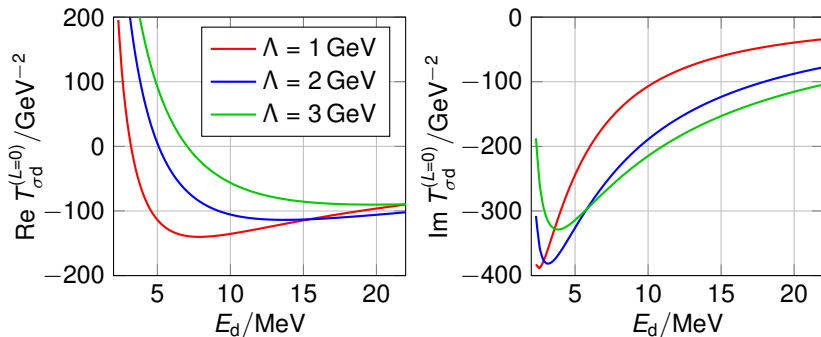
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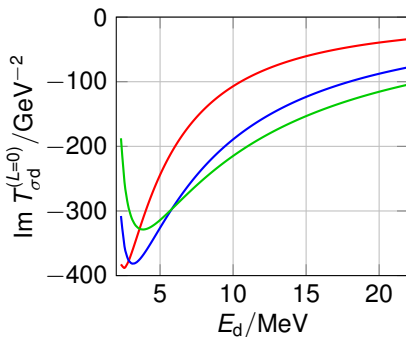
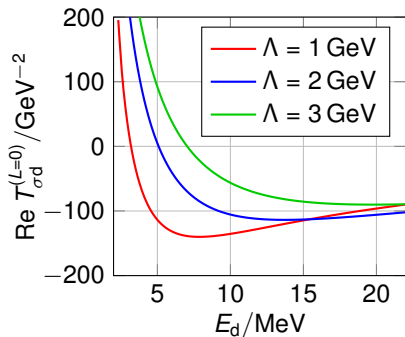
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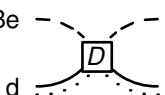
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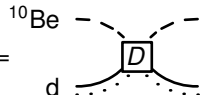
- ▶ Even more: Unphysical Efimov states
- ▶ Renormalize w/ **3-body force!**



### III. 3-Body Force: *Reproducing Data*


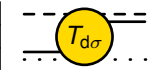
▶ Adjust 3-body force  $D(\Lambda) =$    $(L = 0)$

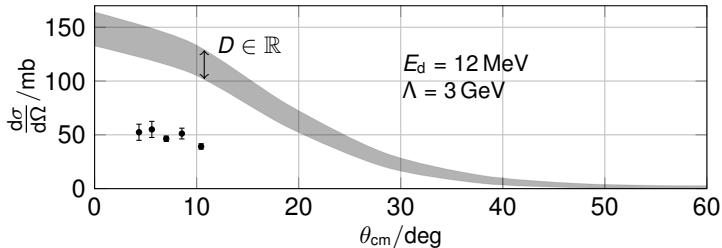
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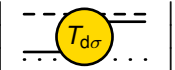
▶ **cross section**  $\frac{d\sigma}{d\Omega} \propto \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right|^2$   reproduces data!

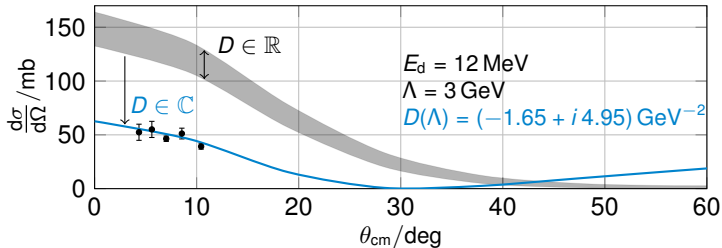
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- ▶ **Complex  $D$  needed!** Cross section  $\Lambda$ -independent!

### III. 3-Body Force: *Loss Channels*

- ▶  $\text{Im}(D)$  introduces **loss channels** outside the EFT's scope  
e.g.  $^{10}\text{Be}+d \rightarrow ^{12}\text{B}(1^+)$ :  $B_3 = 12.37 \text{ MeV}$  (deeply inelastic)



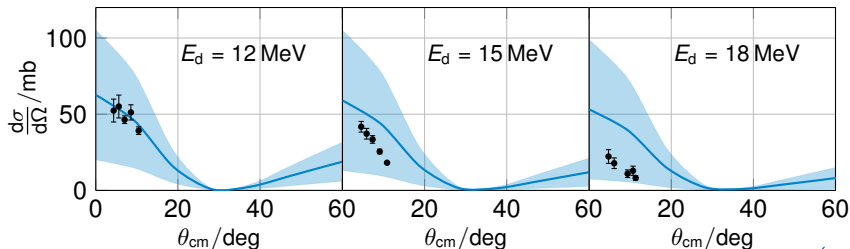
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- ▶ **Predictions at LO:**



**Agreement within NLO bands!**



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- ▶ LO: Auxiliary fields for  $^{11}\text{Be}$  and deuteron ✓





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- ▶ **Amplitude & cross section**

- ▶ Constructed amplitude in  $J^\pi$  channels ✓



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- ▶ Renormalization in  $L = 0$  channel ✓
- ▶ Consideration of loss effects in  $L = 0$  ✓



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- ▶ NLO: Intermediate states  $^{11}\text{Be}^*$ ,  $np(^1S_0)$

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### ▶ Amplitude & cross section

- ▶ Constructed amplitude in  $J^\pi$  channels ✓
- ▶ Include Coulomb (& NLO corrections) perturbatively

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- ▶ Losses in other  $L$  channels?



# Appendix

# I. EFT for $^{10}\text{Be}(d, p)^{11}\text{Be}$ : *Halo EFT Part*

[Hammer, Phillips, NPA 865 (2011)]

$$\mathcal{L}_{^{11}\text{Be}} = n_{\alpha}^{\dagger} \left( i\partial_0 + \frac{\nabla^2}{2m_{\text{N}}} \right) n_{\alpha} + c^{\dagger} \left( i\partial_0 + \frac{\nabla^2}{2m_{\text{C}}} \right) c$$

Non-relativistic fields





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Non-relativistic fields



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Non-relativistic fields



$$\left. \begin{array}{l} \text{Auxiliary} \\ \text{fields} \end{array} \right\} + \sigma_\alpha^\dagger \left[ - \left( i\partial_0 + \frac{\nabla^2}{2M_{Nc}} \right) + \Delta_\sigma + \dots \right] \sigma_\alpha - g_\sigma \left[ (n_\alpha c)^\dagger \sigma_\alpha + \text{h.c.} \right]$$

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Non-relativistic fields

$$\left. \begin{array}{l} \text{Auxiliary} \\ \text{fields} \end{array} \right\} \left\{ \overbrace{\sigma_\alpha^\dagger \left[ - \left( i\partial_0 + \frac{\nabla^2}{2M_{Nc}} \right) + \Delta_\sigma + \dots \right] \sigma_\alpha} - \overbrace{g_\sigma \left[ (n_\alpha c)^\dagger \sigma_\alpha + \text{h.c.} \right]} \right.$$

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Non-relativistic fields

Auxiliary fields

$$\left\{ \begin{array}{l} \overbrace{+ \sigma_\alpha^\dagger \left[ - \left( i\partial_0 + \frac{\nabla^2}{2M_{\text{Nc}}} \right) + \Delta_\sigma + \dots \right] \sigma_\alpha} - \overbrace{g_\sigma \left[ (n_\alpha c)^\dagger \sigma_\alpha + \text{h.c.} \right]} \\ + \pi_\alpha^\dagger \left[ \left( i\partial_0 + \frac{\nabla^2}{2M_{\text{Nc}}} \right) + \Delta_\pi + \dots \right] \pi_\alpha - g_\pi c^{\frac{1}{2}\alpha'}_{\frac{1}{2}\alpha, 1i} \left[ \left( n_{\alpha'} \overleftrightarrow{\nabla}_{i'} c \right)^\dagger \pi_\alpha + \text{h.c.} \right] \end{array} \right.$$

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Non-relativistic fields

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- Parameters:  $g_\sigma$ ,  $\Delta_\pi$ ,  $g_\pi$  (LO) and  $\Delta_\sigma$  (NLO)

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- ▶ Parameters:  $g_\sigma$ ,  $\Delta_\pi$ ,  $g_\pi$  (LO) and  $\Delta_\sigma$  (NLO)
- ▶ Fix with  $\gamma_\sigma$ ,  $\gamma_\pi$  and effective ranges  $r_\pi$ ,  $r_\sigma$  from experiment

# I. EFT for $^{10}\text{Be}(d, p)^{11}\text{Be}$ : *Full Two-Body Propagator*



- ▶ Full propagator: Resum two-body loops!





- ▶ Full propagator: Resum two-body loops!

$$\rightarrow {}^{11}\text{Be} (s\text{-wave}): \quad \overset{\text{(full)}}{\text{---}} = \text{---} + \text{---} \circlearrowleft \overset{\text{(full)}}{\text{---}}$$

The diagram shows the full propagator for  $^{11}\text{Be}$  in the s-wave. It is represented as a double line with a dashed center line. This is equal to the sum of two terms: a double line with a dashed center line, and a double line with a dashed center line connected to a loop. The loop consists of a solid line on top and a dashed line on the bottom, with the symbol  $\Sigma_\sigma$  inside. The loop is enclosed in a dashed circle. The right side of the loop is connected to a double line with a dashed center line, labeled "(full)".

- ▶ Full propagator: Resum two-body loops!

$$\rightarrow {}^{11}\text{Be} \text{ (s-wave): } \begin{array}{c} \text{(full)} \\ \text{---} \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{---} \text{---} \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{---} \begin{array}{c} \text{(full)} \\ \text{---} \\ \text{---} \end{array} \Sigma_{\sigma}$$

$$\rightarrow {}^{11}\text{Be}^* \text{ (p-wave): } \begin{array}{c} \text{(full)} \\ \text{---} \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{---} \text{---} \begin{array}{c} \text{---} \\ \text{---} \end{array} \text{---} \begin{array}{c} \text{(full)} \\ \text{---} \\ \text{---} \end{array} \Sigma_{\pi}$$

# I. EFT for $^{10}\text{Be}(d, p)^{11}\text{Be}$ : Full Two-Body Propagator

- ▶ Full propagator: Resum two-body loops!

→  $^{11}\text{Be}$  (*s*-wave):  $\text{---}^{\text{(full)}}\text{---} = \text{---}^{\text{(full)}}\text{---} + \text{---}^{\text{(full)}}\text{---} \circlearrowleft_{\Sigma_{\sigma}} \text{---}^{\text{(full)}}\text{---}$

→  $^{11}\text{Be}^*$  (*p*-wave):  $\text{---}^{\text{(full)}}\text{---} = \text{---}^{\text{(full)}}\text{---} + \text{---}^{\text{(full)}}\text{---} \circlearrowleft_{\Sigma_{\pi}} \text{---}^{\text{(full)}}\text{---}$

- ▶ Amplitudes:

$$T_{l=0} = \text{---}^{\text{(full)}}\text{---} \text{---}^{\text{(full)}}\text{---}$$

$$T_{l=1} = \text{---}^{\text{(full)}}\text{---} \text{---}^{\text{(full)}}\text{---}$$

# I. EFT for $^{10}\text{Be}(d, p)^{11}\text{Be}$ : *Full Two-Body Propagator*

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$$\rightarrow {}^{11}\text{Be}^* \text{ (p-wave): } \begin{array}{c} \text{(full)} \\ \text{---} \\ \text{---} \end{array} = \begin{array}{c} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{(full)} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{(full)} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \Sigma_{\pi}$$

- ▶ Amplitudes:

$$T_{l=0} = \begin{array}{c} \text{(full)} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \\ \sim \left( -a_{\sigma}^{-1} + r_{\sigma} \frac{k^2}{2} + \dots - ik \right)^{-1}$$

$$T_{l=1} = \begin{array}{c} \text{(full)} \\ \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \text{---} \\ \text{---} \end{array} \\ \sim k^2 \left( -a_{\pi}^{-1} + r_{\pi} \frac{k^2}{2} + \dots - ik^3 \right)^{-1}$$

## II. Scattering Amplitude: *Equations*

- ▶ Partial wave projected amplitudes for  $J \in \mathbb{N}_0$ ,  $L \in \{J - 1, J, J + 1\}$ :

$$\begin{pmatrix} T_{\sigma d}^{3L_J} \\ T_{dd}^{3L_J} \end{pmatrix}(\rho, \rho') = \frac{2\sqrt{\gamma_d \gamma_\sigma}}{\mu_{Nc}} \begin{pmatrix} I_L(\rho, \rho'; E) \\ 0 \end{pmatrix} + \int \frac{dq q^2}{(2\pi)^3} \begin{pmatrix} (1+y) \sqrt{\frac{\gamma_\sigma}{\gamma_d}} \frac{I_L(q, p; E)}{-\gamma_d + \sqrt{\frac{1+2y}{4} q^2 - m_N(E+i\epsilon)}} T_{dd}^{3L_J}(q, \rho') \\ 2\sqrt{\frac{\gamma_d}{\gamma_\sigma}} \frac{I_L(p, q; E)}{-\gamma_\sigma + \sqrt{\frac{1+2y}{(1+y)^2} q^2 - 2\mu_{Nc}(E+i\epsilon)}} T_{\sigma d}^{3L_J}(q, \rho') \end{pmatrix}$$

with  $\gamma_d = 46 \text{ MeV}$ ,  $\gamma_\sigma = 29 \text{ MeV}$ ,  $y \equiv \frac{m_N}{m_c} = 0.1$ ,  $\mu_{Nc} \equiv m_N m_c / (m_N + m_c)$  and

$$I_L(\rho, q; E) \equiv \frac{-2}{\rho q} Q_L \left( \frac{m_N E}{\rho q} - \frac{\rho}{q} - \sqrt{\frac{1+y}{2}} \frac{q}{\rho} + i\epsilon \right),$$

where  $Q_L$  is Legendre function of 2<sup>nd</sup> kind.

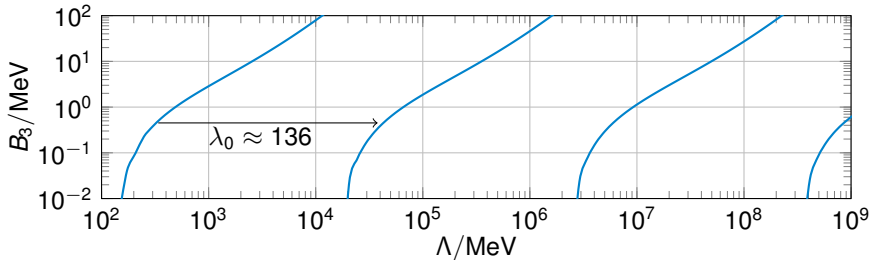


## II. Scattering Amplitude: *Efimov Effect*

- ▶  $L > 0$  integrals converge as  $\Lambda \rightarrow \infty$ ,  $L = 0$  integrals don't

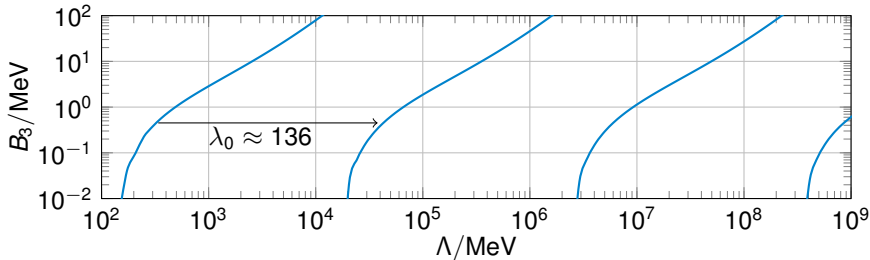
## II. Scattering Amplitude: *Efimov Effect*

- ▶  $L > 0$  integrals converge as  $\Lambda \rightarrow \infty$ ,  $L = 0$  integrals don't
- ▶ 3-body bound states (below d-<sup>10</sup>Be):



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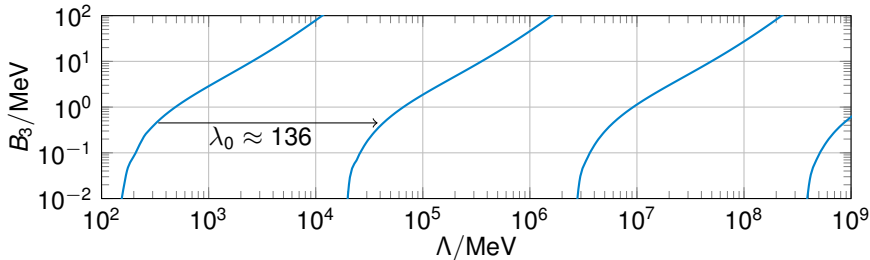
- ▶ (Unphysical) **Efimov states**: Discrete scale invariance  $\Lambda \rightarrow \lambda_0 \Lambda$   
[Efimov, PLB 33 (1970)]





## II. Scattering Amplitude: *Efimov Effect*

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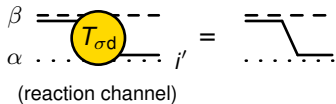


- ▶ (Unphysical) **Efimov states**: Discrete scale invariance  $\Lambda \rightarrow \lambda_0 \Lambda$   
[Efimov, PLB 33 (1970)]
- ▶ Renormalize w/ **3-body force**!



## II. Scattering Amplitude: *Intermediate States*

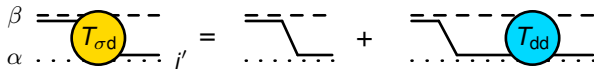
- Insert  $|\pi\rangle \equiv |^{11}\text{Be}^* + p\rangle$  and  $|\nu\rangle \equiv |^{10}\text{Be} + np(^1S_0)\rangle$  at NLO:



with  $\alpha, \beta \in \{-1/2, +1/2\}$  and  $i, i' \in \{-1, 0, +1\}$

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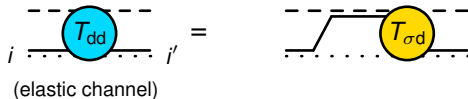
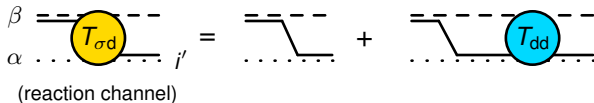


(reaction channel)

with  $\alpha, \beta \in \{-1/2, +1/2\}$  and  $i, i' \in \{-1, 0, +1\}$

## II. Scattering Amplitude: *Intermediate States*

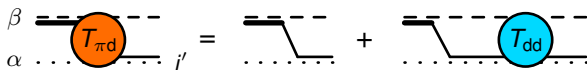
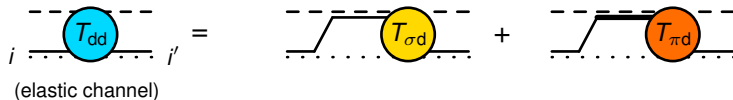
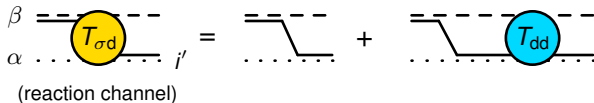
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






with  $\alpha, \beta \in \{-1/2, +1/2\}$  and  $i, i' \in \{-1, 0, +1\}$









## II. Scattering Amplitude: *Intermediate States*

- ▶ Insert  $|\pi\rangle \equiv |^{11}\text{Be}^* + p\rangle$  and  $|\nu\rangle \equiv |^{10}\text{Be} + np(^1S_0)\rangle$  at NLO:

$\beta$    $i'$  =  + 
  
 (reaction channel)

$i$    $i'$  =  + 
  
 (elastic channel)

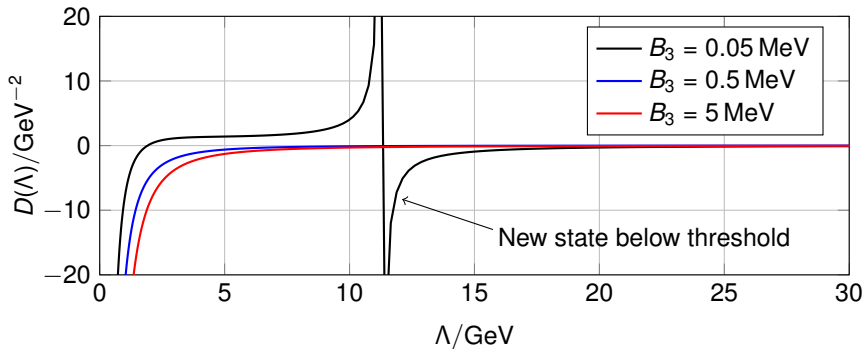
$\beta$    $i'$  =  + 

$i$    $i'$  =  + 

with  $\alpha, \beta \in \{-1/2, +1/2\}$  and  $i, i' \in \{-1, 0, +1\}$

### III. 3-Body Force: *Renormalization w/o Losses*

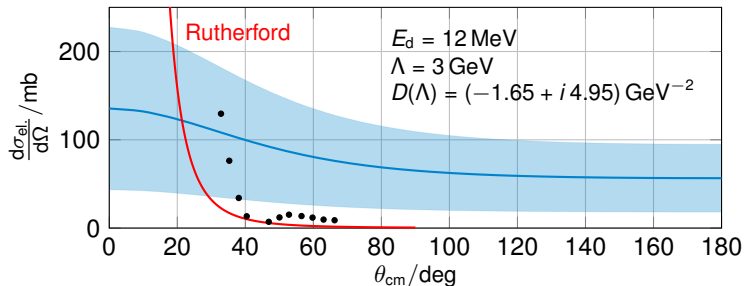
- ▶ Demand pole in  $T_{dd}$  below  $d^{-10}\text{Be}$  threshold at binding energy  $B_3$



### III. 3-Body Force: *Elastic Channel*

- ▶ Elastic scattering  $^{10}\text{Be}(d, d)^{10}\text{Be}$

- ▶ **cross section**  $\frac{d\sigma_{\text{el.}}}{d\Omega} \propto \left| \frac{\text{---} \circlearrowleft T_{dd} \text{---}}{\text{---} \text{---}} \right|^2$



- ▶ **Need Coulomb force!**

