EFT developments for nuclear reactions

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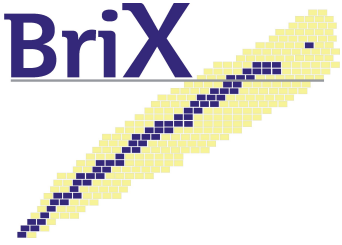


5 October 2017

Halo nuclei

Exotic nuclear structures are found far from stability
In particular halo nuclei with
peculiar quantal structure :

- Light, n-rich nuclei
- Low S_n or S_{2n}

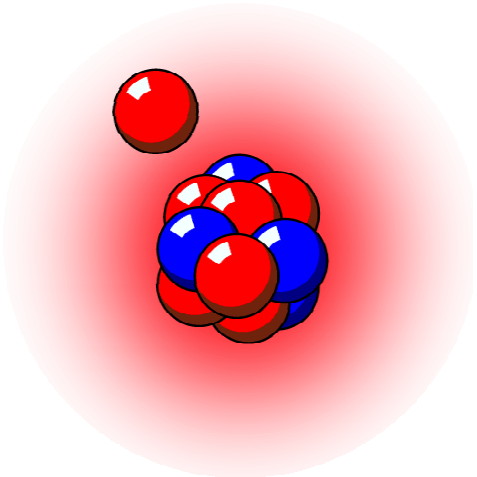


Exhibit large matter radius
due to strongly clusterised structure :
neutrons tunnel far from the core and form a halo

One-neutron halo

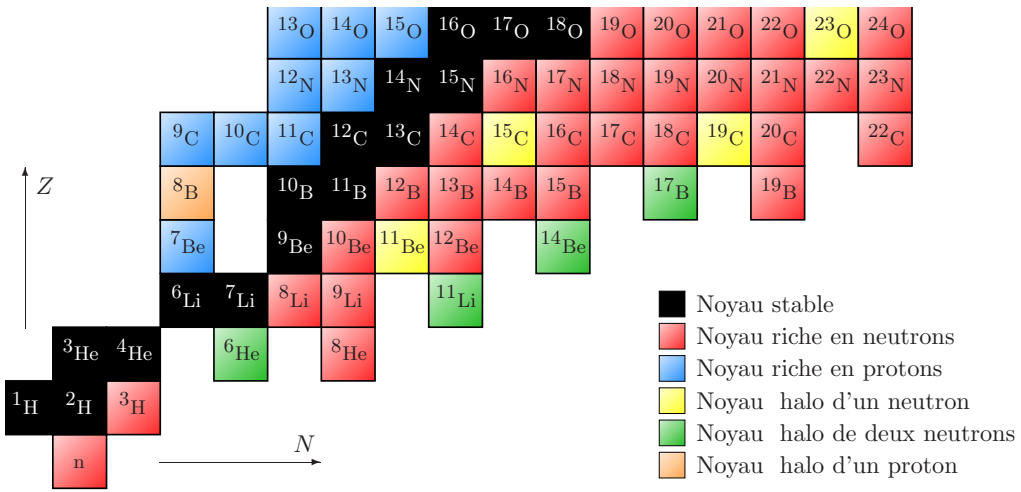
$^{11}Be \equiv {}^{10}Be + n$

$^{15}C \equiv {}^{14}C + n$

Two-neutron halo

$^{6}He \equiv {}^{4}He + n + n$

$^{11}Li \equiv {}^{9}Li + n + n$



Proton halos are possible but less probable : ^{8}B, ^{17}F

Reactions with halo nuclei

Halo nuclei are fascinating objects
but difficult to study [$\tau_{1/2}(^{11}\text{Be}) = 13$ s]

$\Rightarrow$ require indirect techniques, new probes, like reactions :

Elastic scattering
Breakup $\equiv$ dissociation of halo from core
by interaction with target

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$\Rightarrow$ require indirect techniques, new probes, like reactions :

Elastic scattering
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Need good understanding of the reaction mechanism
(i.e. a good reaction model)
to know to what the probe is sensitive
(i.e. what nuclear-structure information it provides)
have reliable inputs for the model
(i.e. optical potentials to describe the interactions with target)

We address these issues using EFT

1. Reaction model

2. Including halo-EFT within reaction models
   - EFT description of ^{11}Be @ NLO
   - Breakup calculations of ^{11}Be into ^{10}Be+n

3. Optical potentials
   - Double-folding potential from χEFT NN interactions
   - ^{16}O-^{16}O calculations

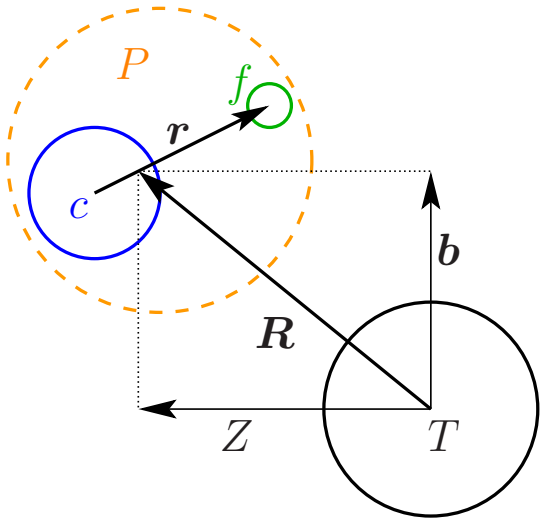
4. Summary

Framework

Projectile (P) modelled as a two-body quantum system :
core (c)+loosely bound nucleon (f) described by

$$H_0 = T_r + V_{cf}(\boldsymbol{r})$$

V_{cf} effective interaction
describes the quantum system
with ground state Φ_0



Target T assumed structureless

Interaction with target simulated by optical potentials
$\Rightarrow$ breakup reduces to three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with initial condition $\Psi(\boldsymbol{r},\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ}\Phi_0(\boldsymbol{r})$

We use the Dynamical Eikonal Approximation (DEA)

[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

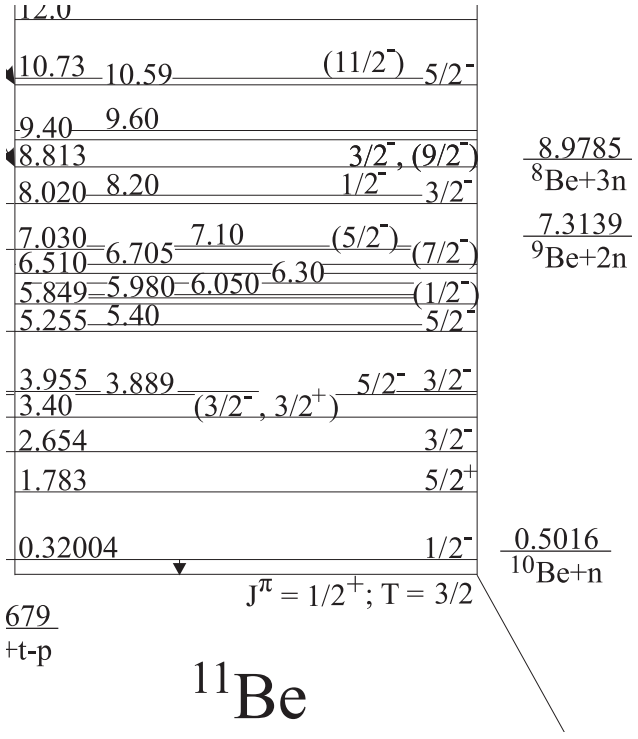
Usual phenomenological description

In reaction models, projectile ≡ two-body system :

$$H_0 = T_r + V_{cf}(\boldsymbol{r}),$$

where V_{cn} is a phenomenological Woods-Saxon that reproduces the basic nuclear properties of the projectile (binding energy, J^π,...)

$^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be} \otimes \mathrm{n}$



- $\frac{1}{2}^+$ ground state :
  $\epsilon_{\frac{1}{2}^+} = -0.503$ MeV
  In our model, seen as $1s_{\frac{1}{2}}$ neutron bound to $^{10}\mathrm{Be}(0^+)$
- $\frac{1}{2}^-$ bound excited state :
  $\epsilon_{\frac{1}{2}^-} = -0.184$ MeV
  In our model, seen as $0p_{\frac{1}{2}}$ neutron bound to $^{10}\mathrm{Be}(0^+)$

^{10}Be-n potential

Replace the ^{10}Be-n interaction by effective potentials in each partial wave

Use halo EFT : clear separation of scales (in energy or in distance)
$\Rightarrow$ provides an expansion parameter (small scale / large scale)
along which the low-energy behaviour is expanded

[H.-W. Hammer, C. Ji, D. R. Phillips JPG 44, 103002 (2017)]

Use narrow Gaussian potentials

$$V_{lj}(r) = V_0\, e^{-\frac{r^2}{2\sigma^2}} + V_2\, r^2 e^{-\frac{r^2}{2\sigma^2}}$$

Fit V_0 and V_2 to reproduce ϵ_{lj} and C_{lj} (@ NLO for bound states)

σ = 1.2, 1.5 or 2 fm is a parameter used to evaluate the sensitivity of the calculations to this effective model

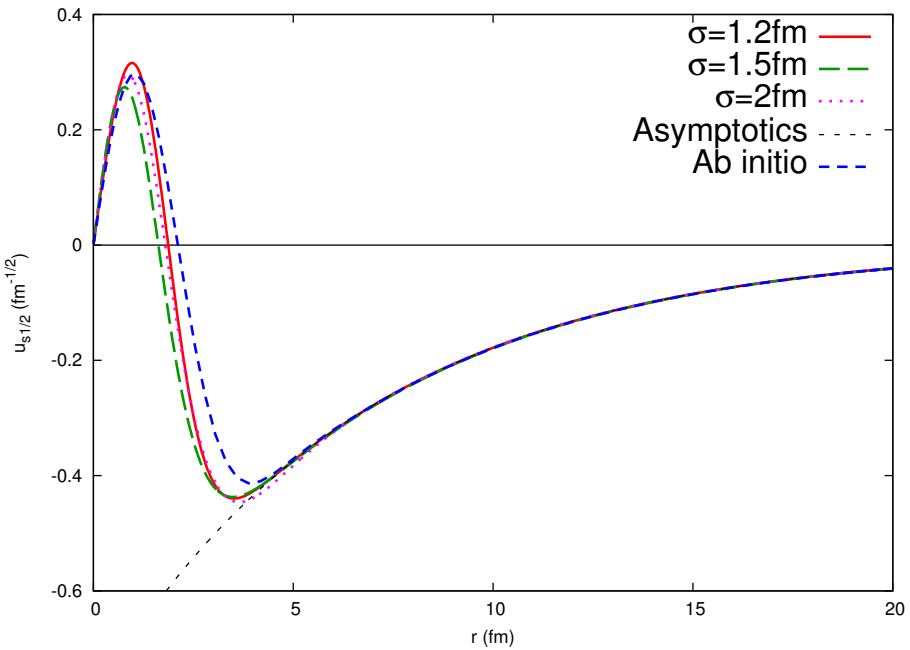
ϵ_{lj} is known experimentally, but what about C_{lj} ?
Fortunately, for ^{11}Be, we've got the ab initio calculation of Calci *et al.*

[A. Calci *et al.* PRL 117, 242501 (2016)]

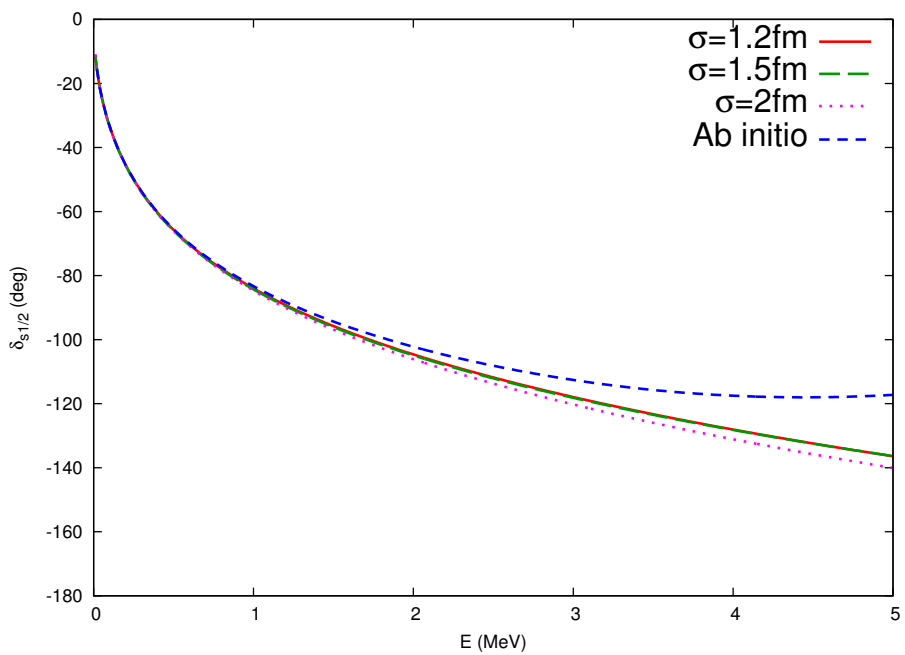
$s\frac{1}{2}$: @ NLO potentials fitted to $\epsilon_{\frac{1}{2}^+}$ and $C_{\frac{1}{2}^+}$

Potentials fitted to $\epsilon_{1s\frac{1}{2}} = -0.503$ MeV and $C_{1s\frac{1}{2}} = 0.786$ fm$^{-1/2}$

Ground-state wave function



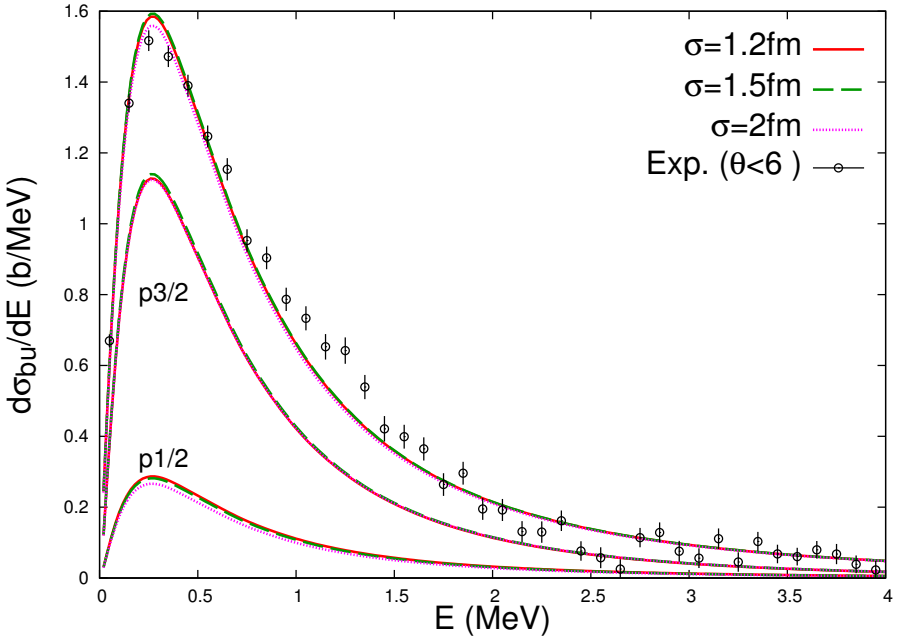
$s\frac{1}{2}$ phaseshifts



- Wave functions : same asymptotics but different interior
- $\delta_{s\frac{1}{2}}$: all effective potentials are in good agreement with ab initio up to 1.5 MeV (same effective-range expansion)
- Similar results obtained for $p\frac{1}{2}$ (excited bound state)
- In higher partial waves ($lj \geq p3/2$) $V_{lj} = 0$

NLO analysis of ^{11}Be+Pb$\rightarrow^{10}$Be+n+Pb @ $69A$MeV

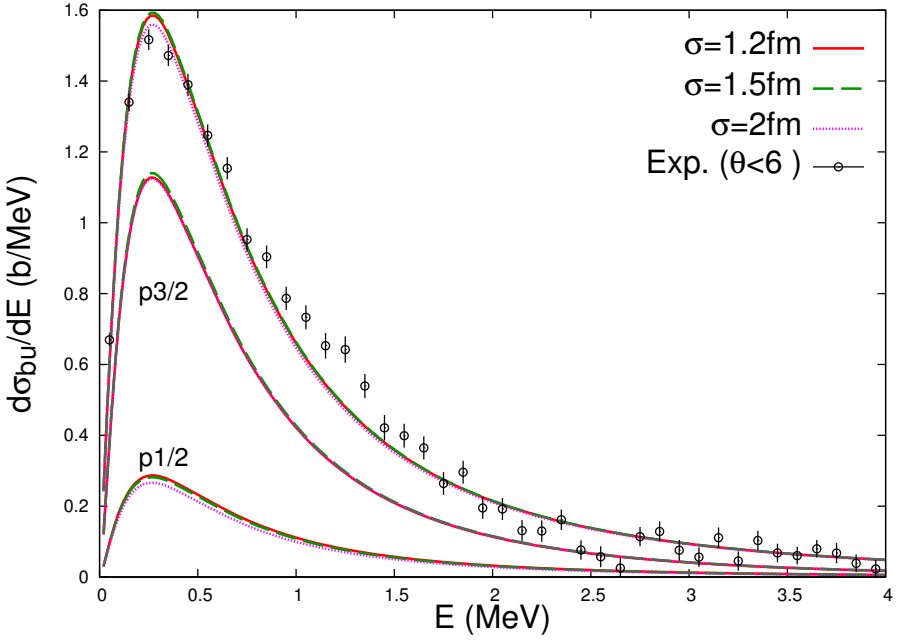
Total breakup cross section
and p contributions



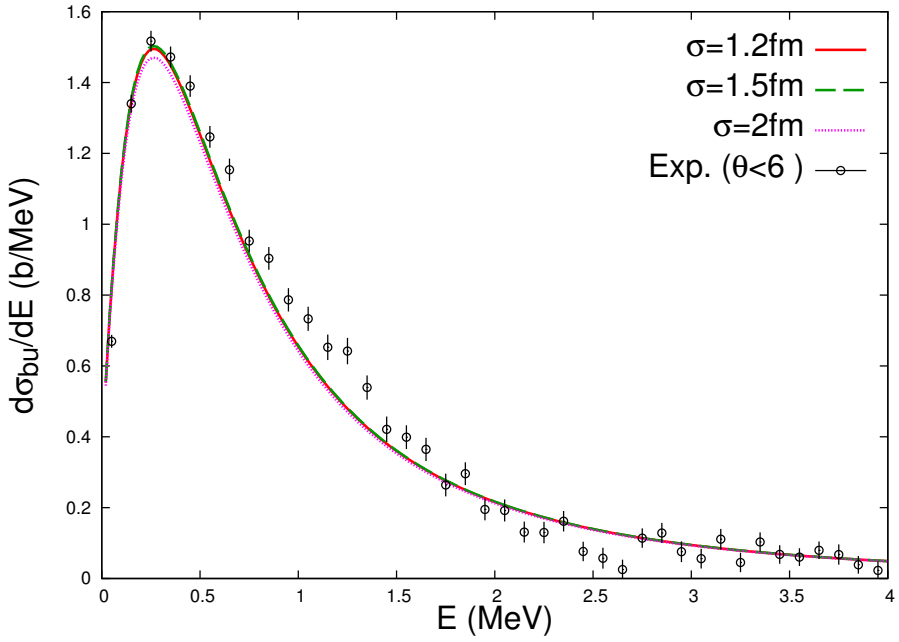
- All calculations provide very similar results,
  for all σ, despite the difference in the internal part of the wave function
  $\Rightarrow$ reaction is peripheral

NLO analysis of ^{11}Be+Pb$\rightarrow^{10}$Be+n+Pb @ 69AMeV

Total breakup cross section and p contributions

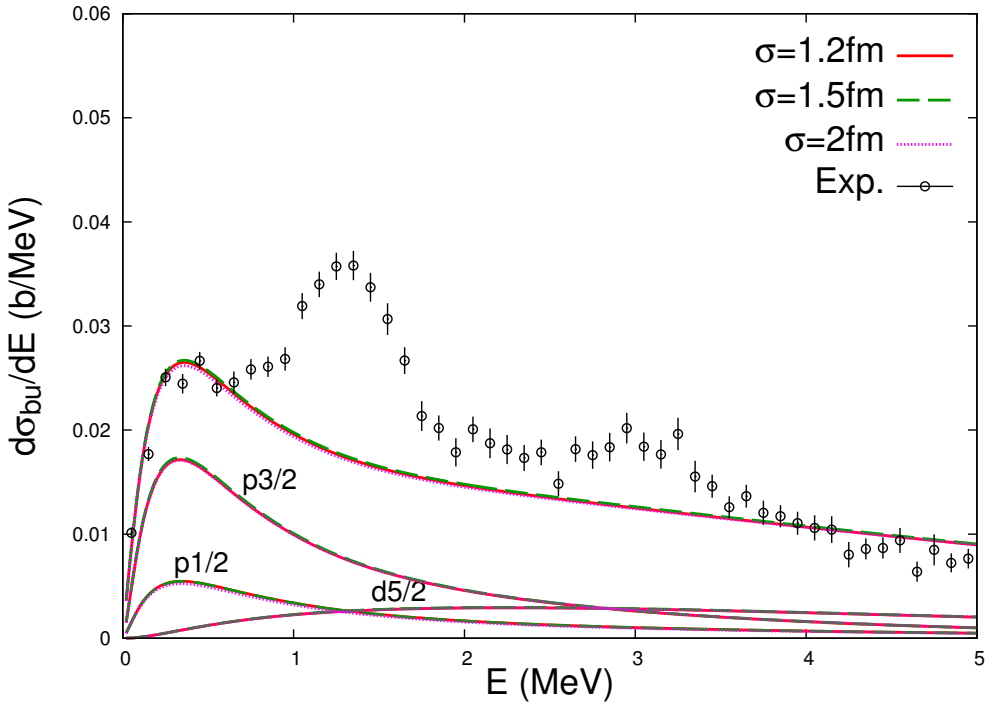


Folded with experimental resolution



- All calculations provide very similar results,
  for all σ, despite the difference in the internal part of the wave function
  $\Rightarrow$ reaction is peripheral
- Excellent agreement with data [Fukuda *et al.* PRC 70, 054606 (2004)]
  $\Rightarrow$ ab initio results used to constrain ^{11}Be EFT description are correct

NLO analysis of ^{11}Be+C$\rightarrow^{10}$Be+n+C @ $67A$MeV



Exp. [Fukuda *et al.* PRC 70, 054606 (2004)]

- All potentials produce very similar breakup cross sections
  $\Rightarrow$ still peripheral (even if nuclear dominated)
- Order of magnitude of experiment well reproduced
- Breakup strength missing at the $5/2^+$ and $3/2^+$ resonances

$\Rightarrow$ for this observable, the continuum must be better described

Ab initio description of ^{10}Be-n continuum

Provides the most accurate calculation for the ^{10}Be-n continuum

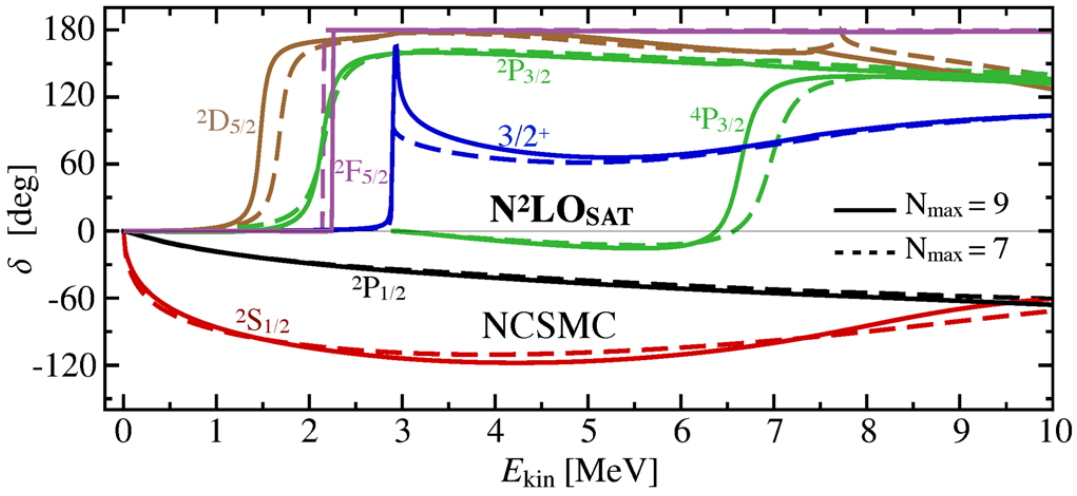
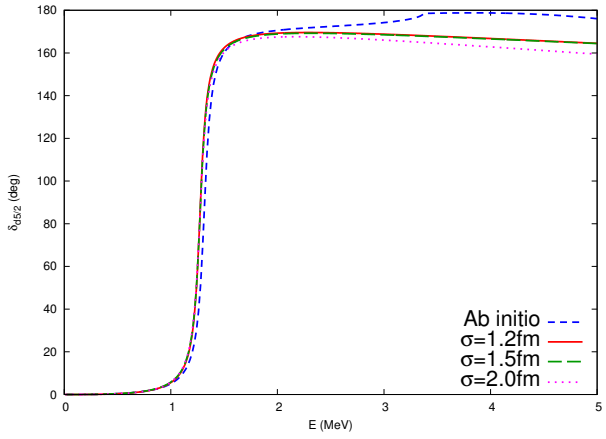


FIG. 3. The $n + {}^{10}$Be phase shifts as a function of the kinetic energy in the center-of-mass frame. NCSMC phase shifts for the N^2LO_{SAT} interaction are compared for two model spaces indicated by N_{max}.

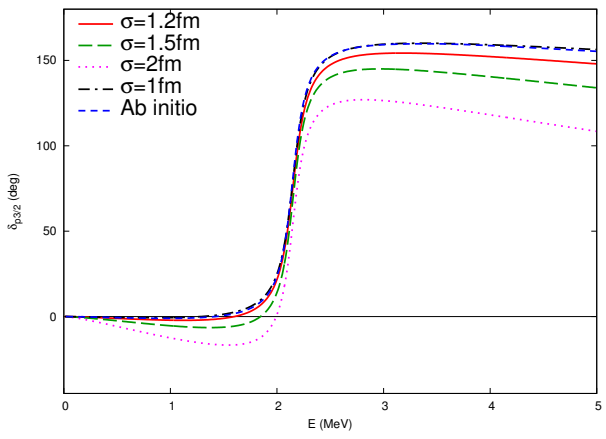
Idea : constrain the ^{10}Be-n potential in the reaction code beyond NLO to reproduce ab initio δ_{lj},
i.e. fit V_0 and V_2 to reproduce ϵ_{lj} & Γ_{lj} (in $d_{\frac{5}{2}}$, $p_{\frac{3}{2}}$, and $d_{\frac{3}{2}}$)

$d\frac{5}{2}$, $p\frac{3}{2}$ and $d\frac{3}{2}$: potentials fitted to ϵ^{res} and Γ

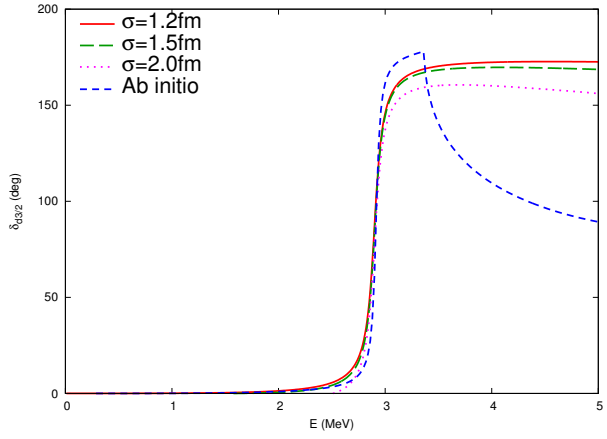
$d\frac{5}{2}$



$p\frac{3}{2}$



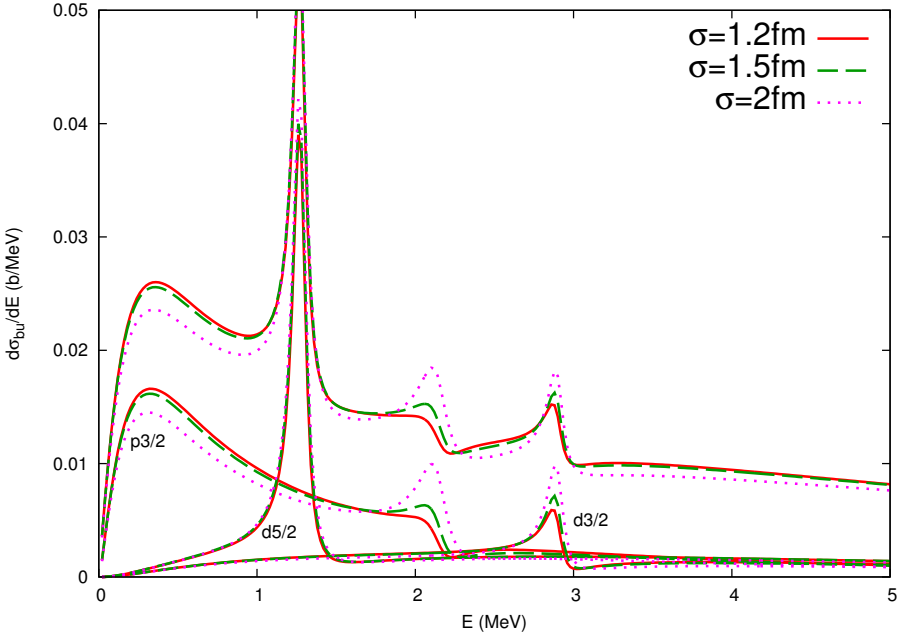
$d\frac{3}{2}$



- Identical $\delta_{d\frac{5}{2}}$ up to 1.5 MeV
  Excellent agreement with ab initio results up to 2 MeV
- Large variation in $\delta_{p\frac{3}{2}}$ and $\delta_{d\frac{3}{2}}$ obtained by effective potentials
  Broad potential ($\sigma = 2$ fm) cannot reproduce correct behaviour

^{11}Be+C$\rightarrow^{10}$Be+n+C @ $67A$MeV (beyond NLO)

Total breakup cross section
and dominant contributions



- All potentials produce similar breakup cross sections (but $\sigma = 2$ fm)
  Differences in $p\frac{3}{2}$ and $d\frac{3}{2}$ contributions due to differences in δ_{lj}
- In nuclear breakup, resonances play significant role

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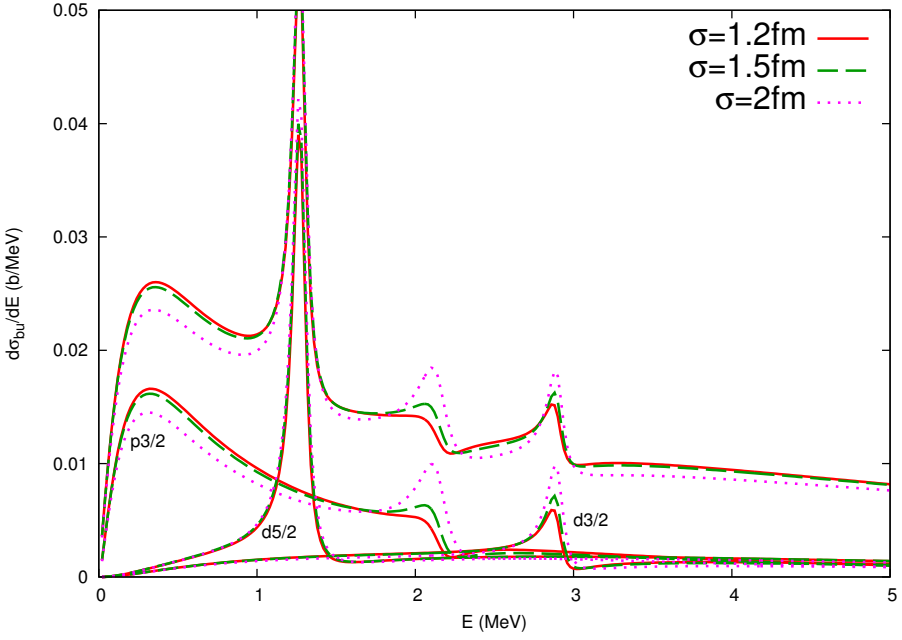
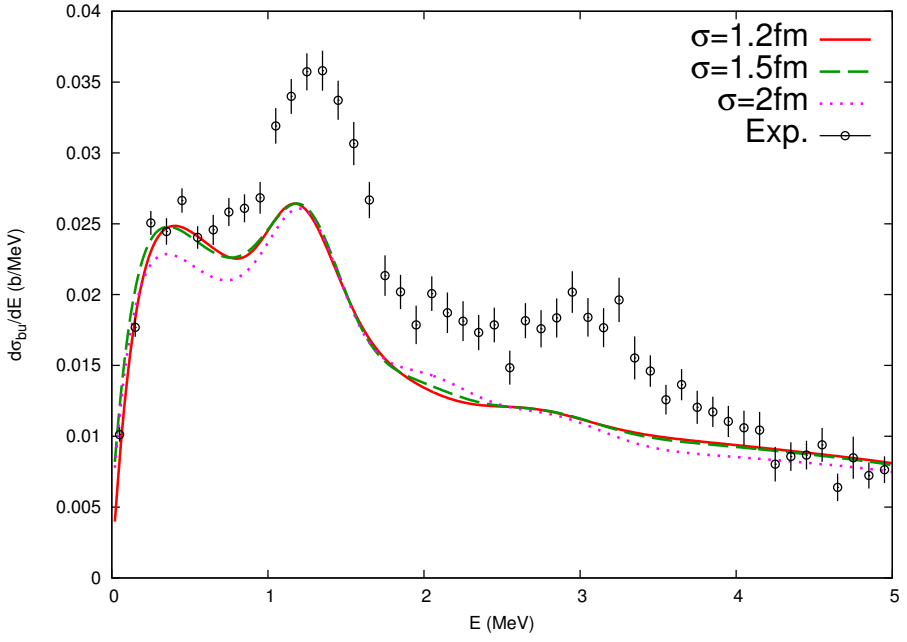


Folded with experimental resolution [Fukuda *et al.* PRC 70, 054606 (2004)]



- All potentials produce similar breakup cross sections (but $\sigma = 2$ fm)
  Differences in $p\frac{3}{2}$ and $d\frac{3}{2}$ contributions due to differences in δ_{lj}
- In nuclear breakup, resonances play significant role
- But resonant breakup not correctly described
  due to missing degrees of freedom in the effective model [^{10}Be(2^+)]

SF vs ANC

Calci *et al.* predict $\mathcal{S}_{1s\frac{1}{2}} = 0.90$, but we use $\mathcal{S}_{1s\frac{1}{2}} = 1\ldots$

$\Rightarrow$ repeat calculations with $\mathcal{S}_{1s\frac{1}{2}} = 0.90$ (keeping $C_{\frac{1}{2}^+} = 0.786$ fm$^{-1/2}$)

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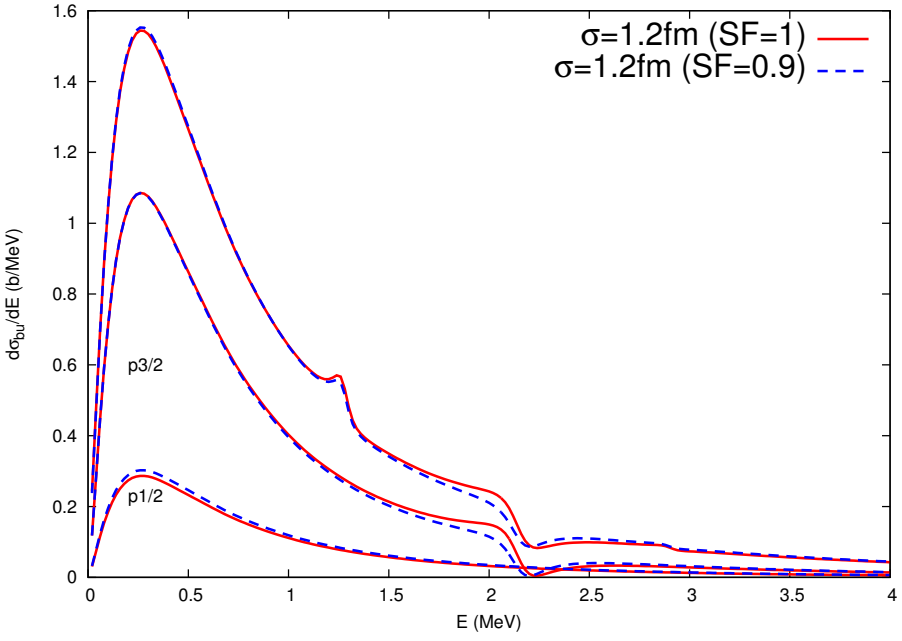

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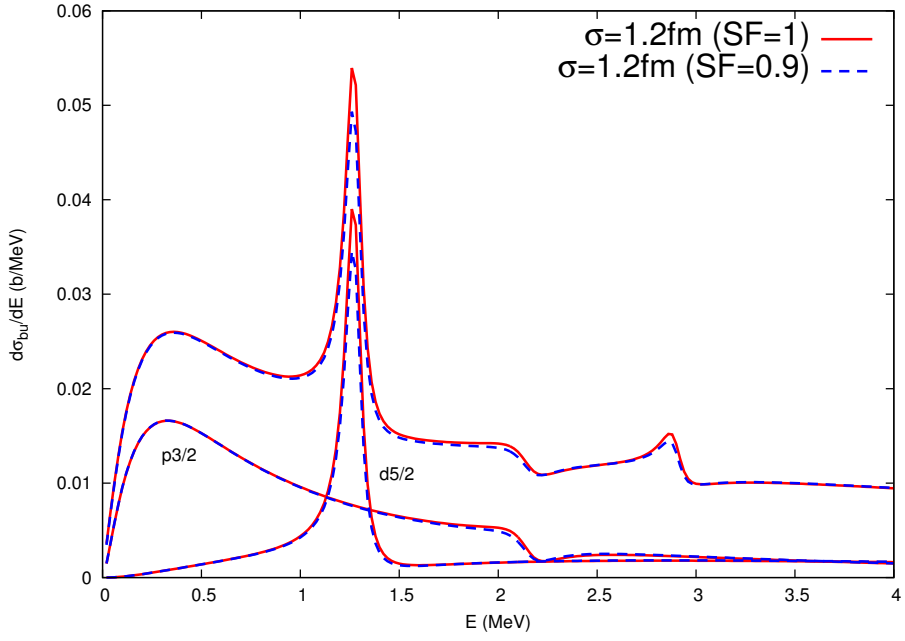

No difference $\Rightarrow$ SF cannot be extracted from these measurements

One exception : resonant breakup, where SF plays a role

$\Rightarrow$ influence of the short-range details (?)

1. Reaction model

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   - EFT description of ^{11}Be @ NLO
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3. Optical potentials
   - Double-folding potential from χEFT NN interactions
   - ^{16}O-^{16}O calculations

4. Summary

Nucleus-nucleus interaction

The reaction model require nucleus-nucleus interaction

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

Problem : the core is usually radioactive
it is difficult to find V_{cT} in the literature

Idea : using a double-folding procedure
with accurate NN interactions from χEFT

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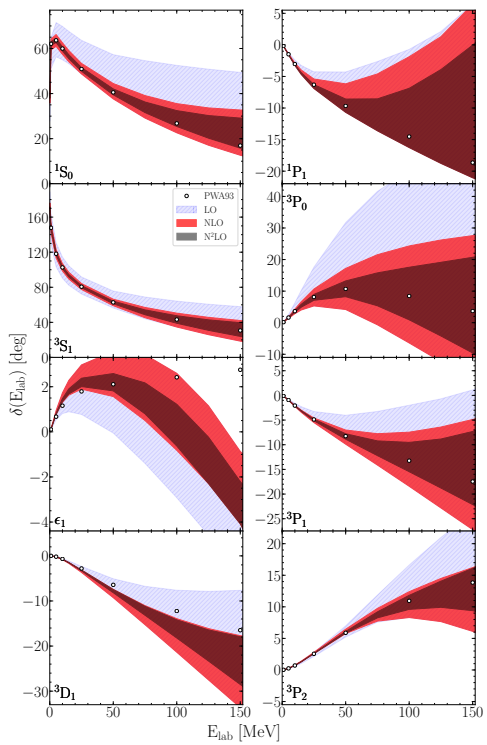
Idea : using a double-folding procedure
with accurate NN interactions from χEFT

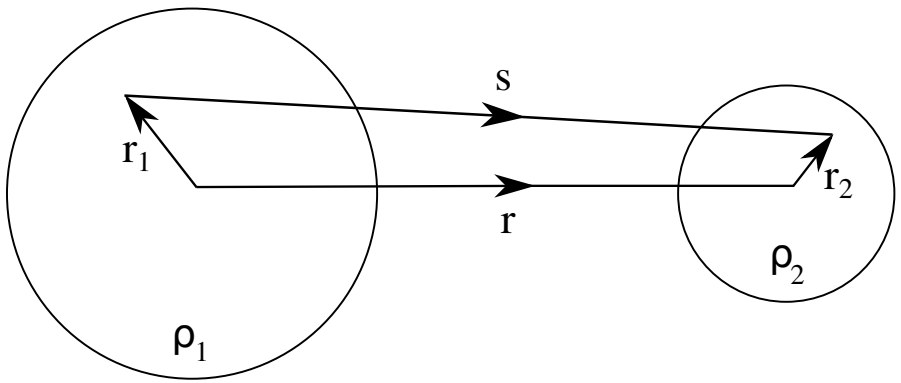
Gezerlis *et al.* have developed
local NN interactions up to N^{2}LO
[PRL 111, 032501 (2013),
PRC 90, 054323 (2014)]

Based on this formalism,
we build a double-folding potential
Calculations by L. Huth
arXiv :1708.02527



Double-folding potential

We build a double-folding potential at the Hartree-Fock level



$$V_F = \sum_{i \in A_1, j \in A_2} \left[\langle ij|v_D|ij\rangle + \langle ij|v_{EX}|ji\rangle \right]$$

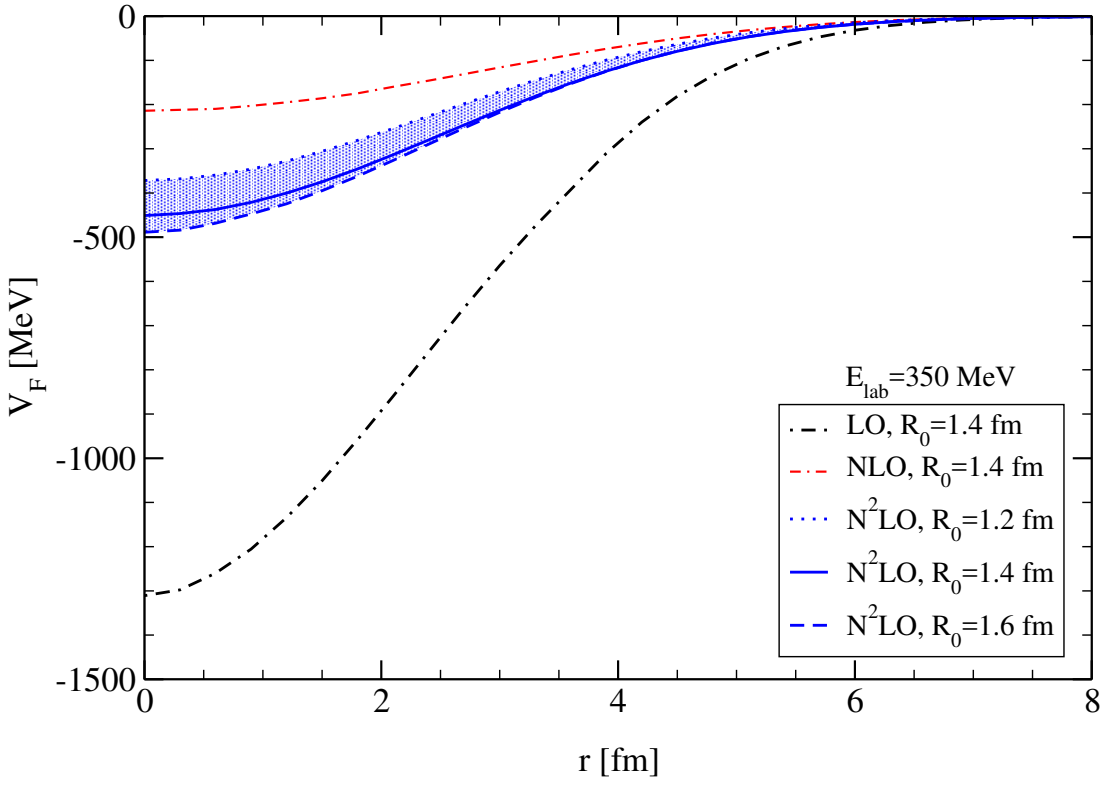
using simple Fermi densities as input for the nuclei

^{16}O-^{16}O potential

We build the potential

- at different orders
- for different cutoffs

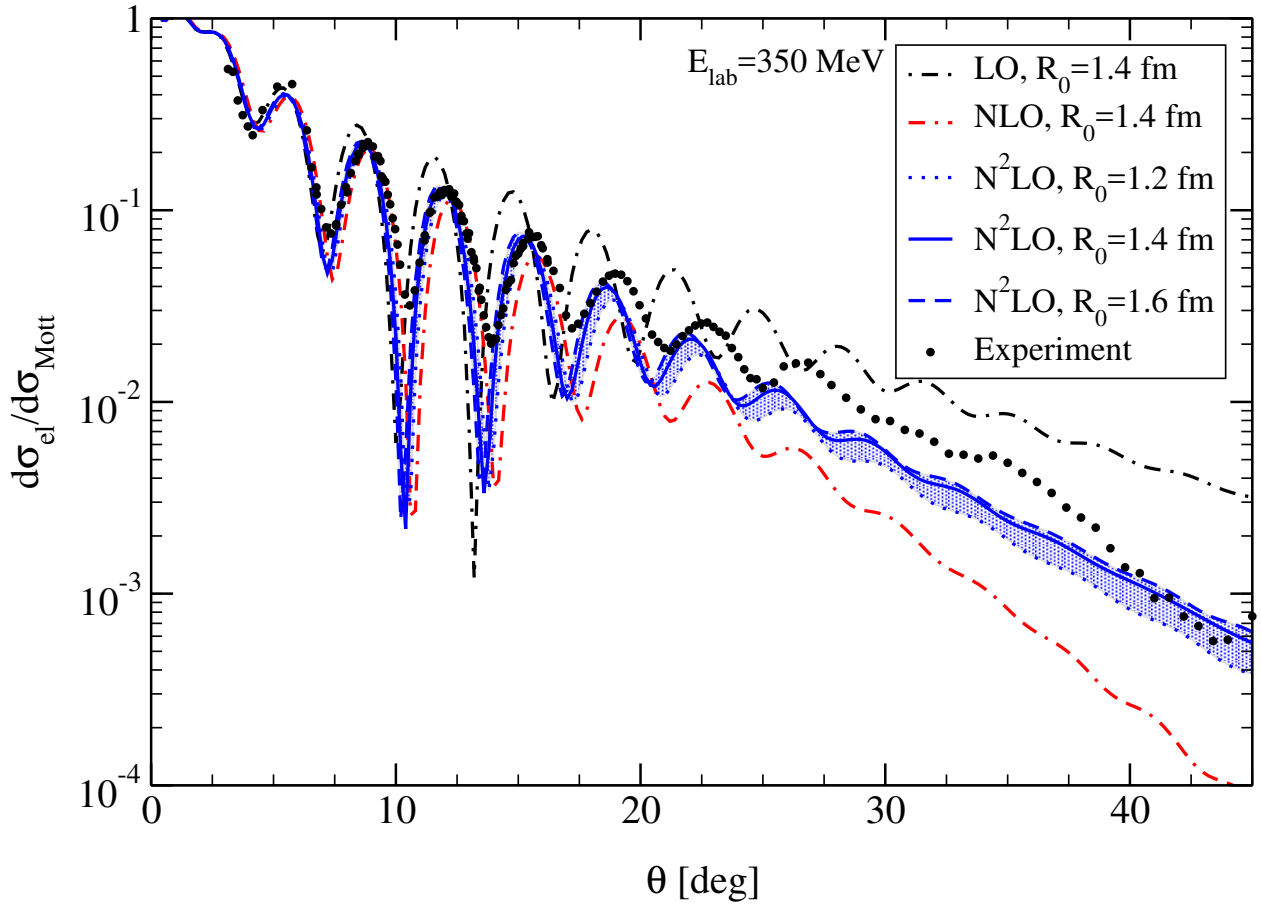
Calculations by V. Durant
arXiv :1708.02527



The imaginary part is assumed proportional to V_F

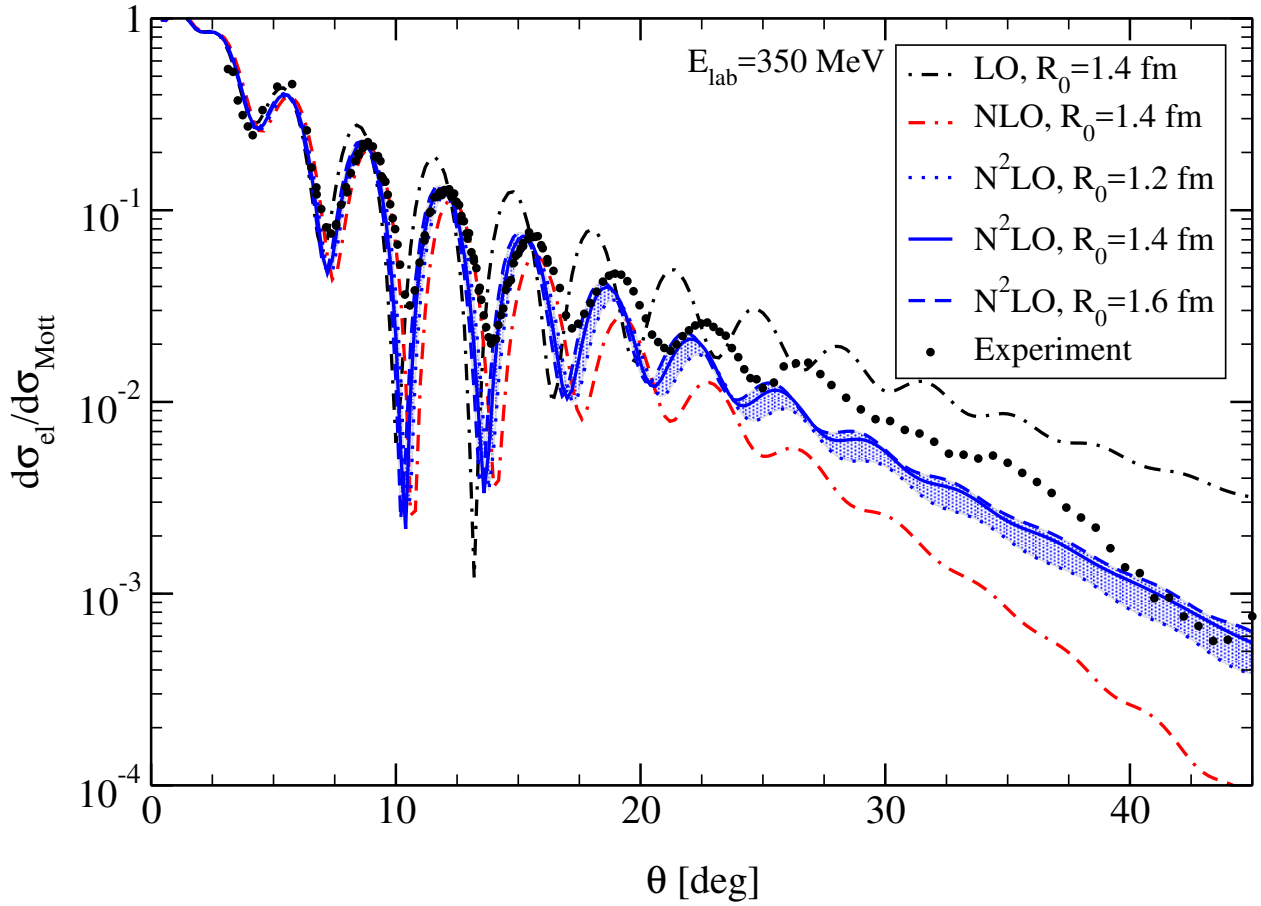
$$U_F(r) = (1 + N_W\ i)\ V_F(r) \qquad \text{with } N_W = 0.6 - 0.8$$

^{16}O-^{16}O elastic scattering @350 MeV



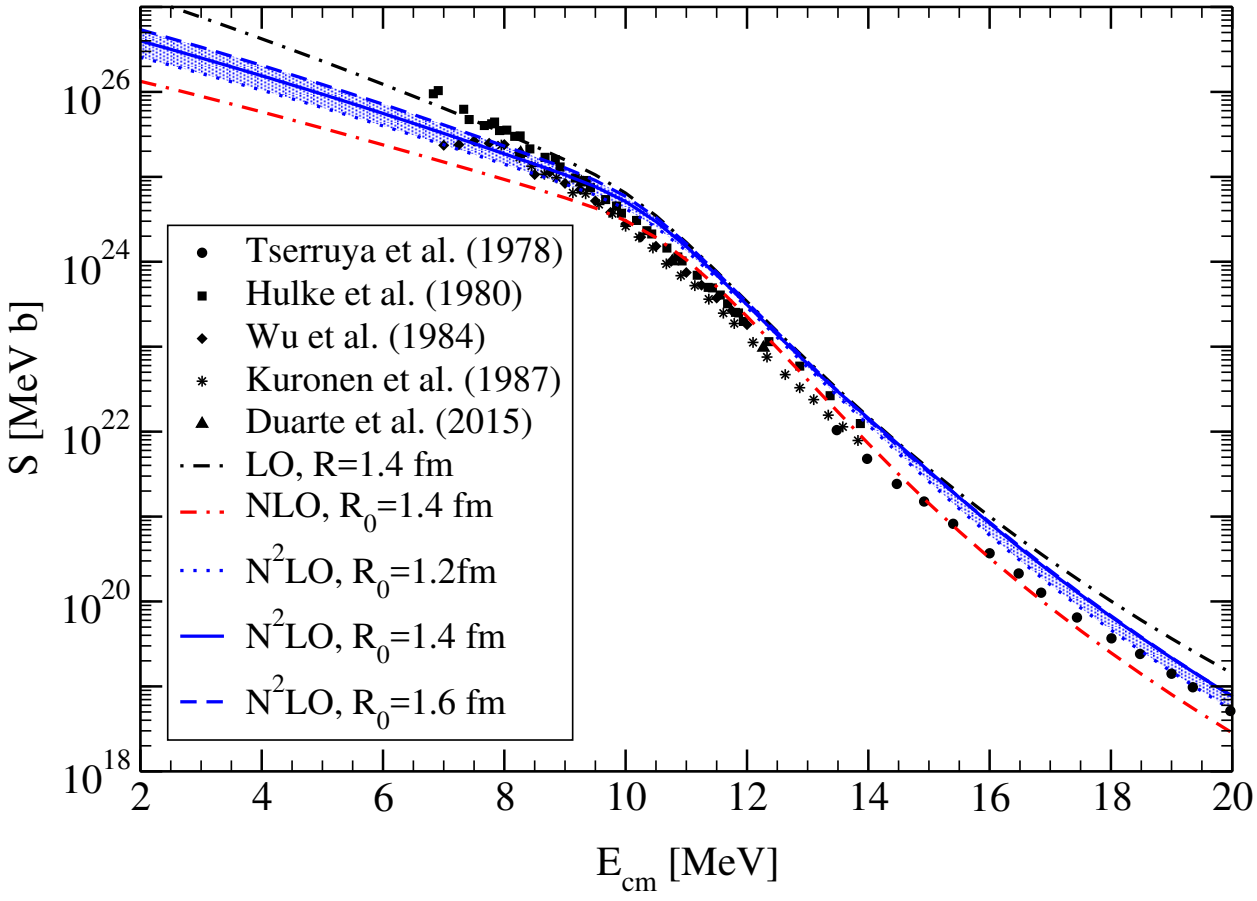
- Good agreement with experiment (no fitting parameter)
- Systematic order-by-order behaviour
- Small uncertainty related to the cutoff

^{16}O-^{16}O elastic scattering @350 MeV



- Good agreement with experiment (no fitting parameter)
- Systematic order-by-order behaviour
- Small uncertainty related to the cutoff
- Larger uncertainty to N_W

^{16}O-^{16}O low-energy fusion



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- Systematic order-by-order behaviour
- Small uncertainty related to the cutoff

Summary and prospect

- Exotic nuclei studied mostly through reactions
- Mechanism of reactions with halo nuclei understood
  Can we understand what reactions probe using halo EFT ? Yes
- Using Gaussian potentials, we reproduce the ANC
  and phase shifts predicted by ab initio calculations
- Our study shows
  - peripherality of breakup reactions
  - ab initio results (ANC & δ_{lj}) lead to agreement with data
- Optical potentials can be built by double-folding
  - Using χEFT NN interactions
  - Good agreement with experiment (no fitting parameter)
- EFT provides various ways to improve reaction modelling
  In the future :
  - Include missing degrees of freedom in ^{11}Be description
  - Study the sensitivity of the folding method to the inputs

Thanks. . .

to you for your attention

and to my collaborators

Victoria Durant
Lukas Huth
Hans-Werner Hammer
Achim Schwenk

Daniel Phillips

Daniel Baye
Gerald Goldstein

Dynamical eikonal approximation (DEA)

Three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\,\Psi(\boldsymbol{r},\boldsymbol{R})$$

with condition $\Psi \underset{Z\to-\infty}{\longrightarrow} e^{iKZ}\Phi_0$

Eikonal approximation : factorise $\Psi = e^{iKZ}\,\widehat{\Psi}$

$$T_R\Psi = e^{iKZ}[T_R + vP_Z + \frac{\mu_{PT}}{2}v^2]\,\widehat{\Psi}$$

Neglecting T_R vs P_Z and using $E_T = \frac{1}{2}\mu_{PT}v^2 + \epsilon_0$

$$i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - \epsilon_0 + V_{cT} + V_{fT}]\,\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

solved for each $\boldsymbol{b}$ with condition $\widehat{\Psi} \underset{Z\to-\infty}{\longrightarrow} \Phi_0(\boldsymbol{r})$

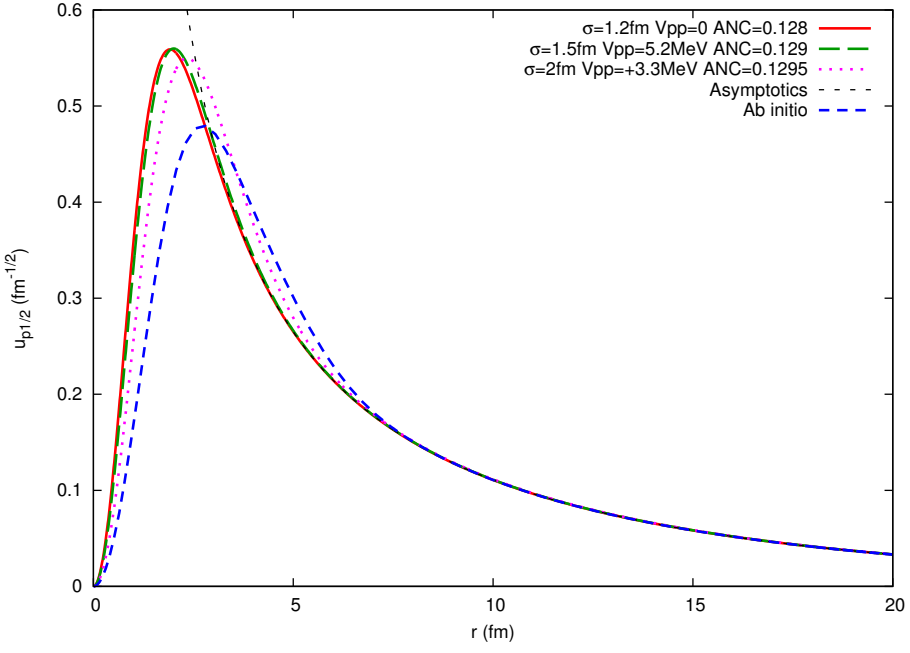
This is the dynamical eikonal approximation (DEA)

[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

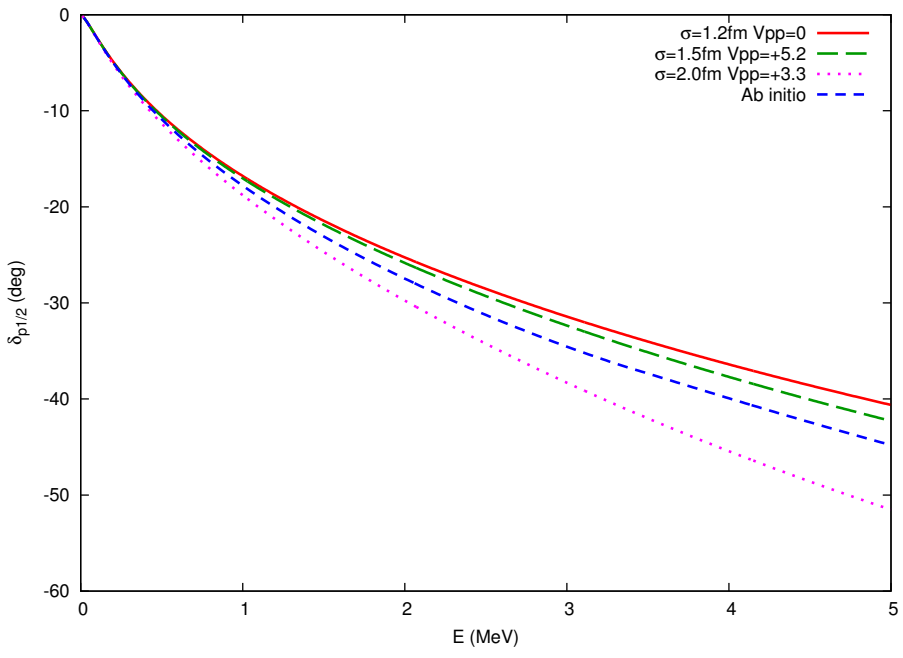
$p\frac{1}{2}$: @ NLO potentials fitted to $\epsilon_{\frac{1}{2}^-}$ and $C_{\frac{1}{2}^-}$

Potentials fitted to $\epsilon_{0p\frac{1}{2}} = -0.184$ MeV and $C_{0p\frac{1}{2}} = 0.129$ fm$^{-1/2}$

Excited-state wave function



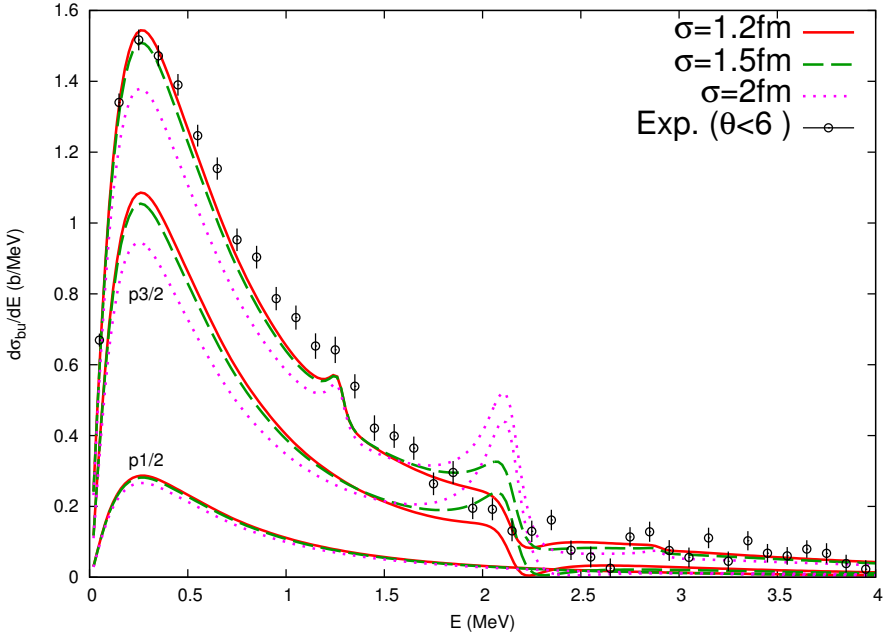
$p_{1/2}$ phaseshifts



- Wave functions : same asymptotics but different interior
- Larger variation in $\delta_{p\frac{1}{2}}$ obtained by effective potentials
  Fair agreement with ab initio results up to 0.5 MeV

^{11}Be+Pb$\rightarrow^{10}$Be+n+Pb @ $69A$MeV (beyond NLO)

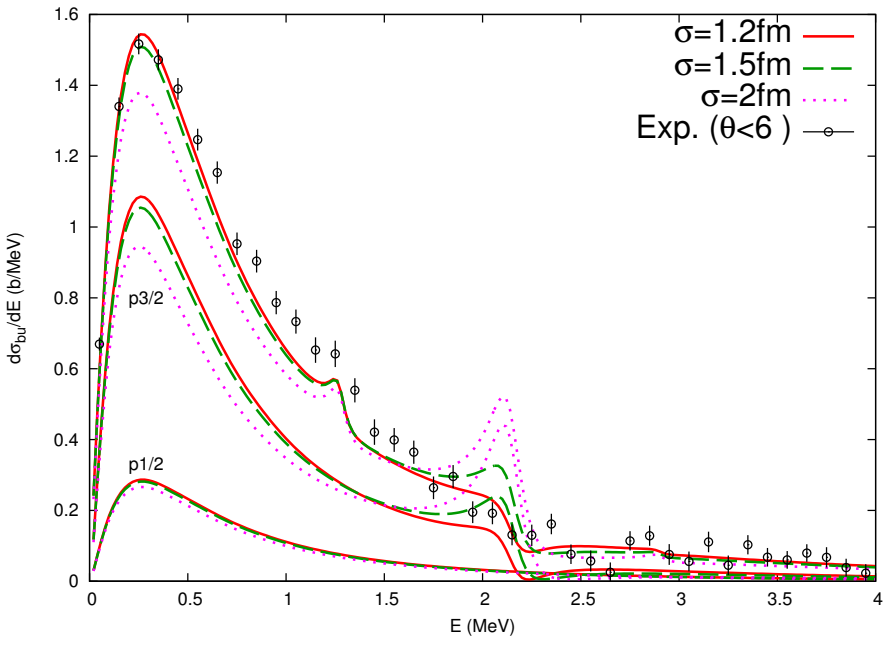
Total breakup cross section and p contributions



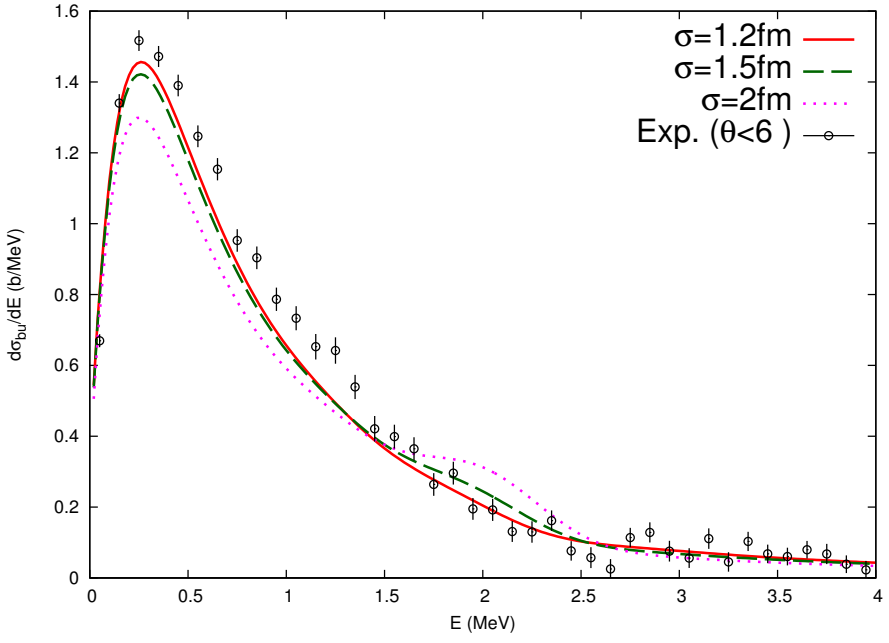
- Major differences in $p_{3/2}$ partial wave ; due to differences in $\delta_{p_{3/2}}$
- Broad potential ($\sigma = 2$ fm) produces unrealistic $p_{3/2}$ contribution
- Tiny peak at 1.27 MeV due to $d_{\frac{5}{2}}$ resonance

^{11}Be+Pb$\rightarrow^{10}$Be+n+Pb @ 69AMeV (beyond NLO)

Total breakup cross section and p contributions



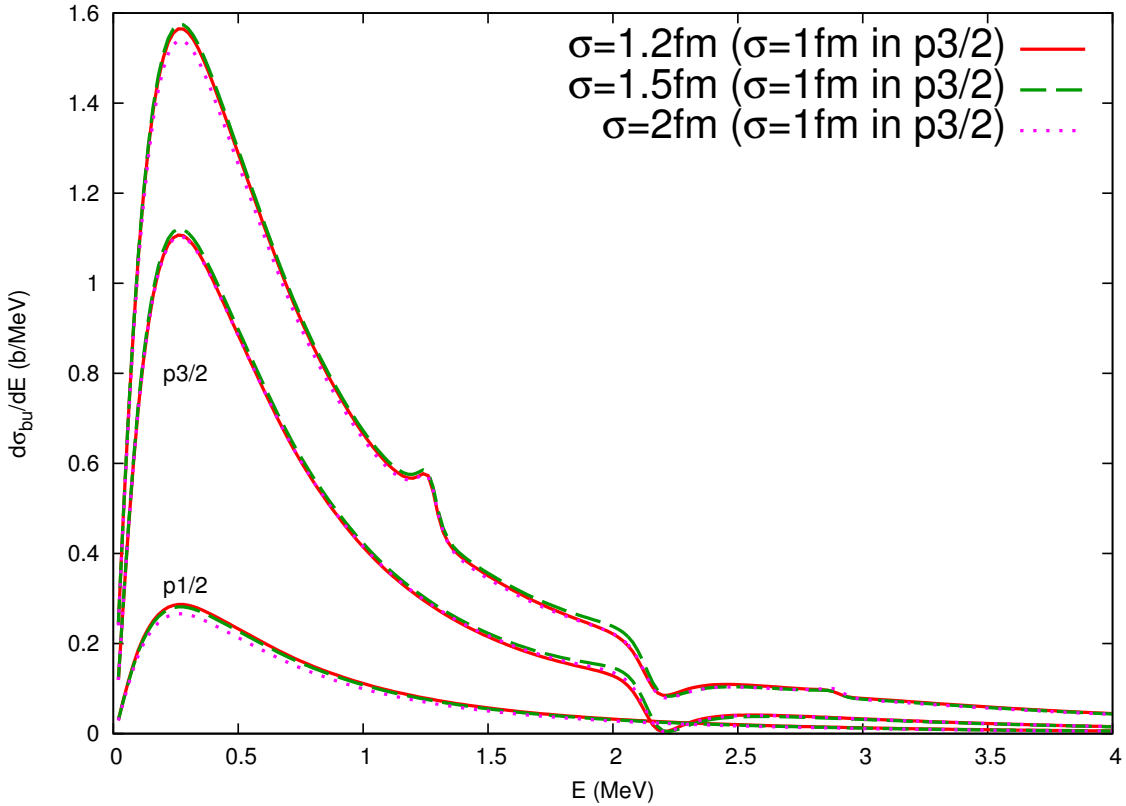
Folded with experimental resolution



- Major differences in $p_{3/2}$ partial wave ; due to differences in $\delta_{p_{3/2}}$
- Broad potential ($\sigma = 2$ fm) produces unrealistic $p_{3/2}$ contribution
- Tiny peak at 1.27 MeV due to $d_{\frac{5}{2}}$ resonance not enough to match data
- Good agreement with data [Fukuda *et al.* PRC 70, 054606 (2004)]
  Best agreement with $\sigma = 1.2$ and 1.5 fm, whose $\delta_{p3/2} \sim \delta^{\text{ab initio}}_{3/2^-}$

Role of $\delta_{p3/2}$

Calculations repeated with different potentials (σ = 1.2, 1.5 or 2 fm) but in $p_{3/2}$, where $\sigma = 1$ fm (perfect agreement with ab initio)

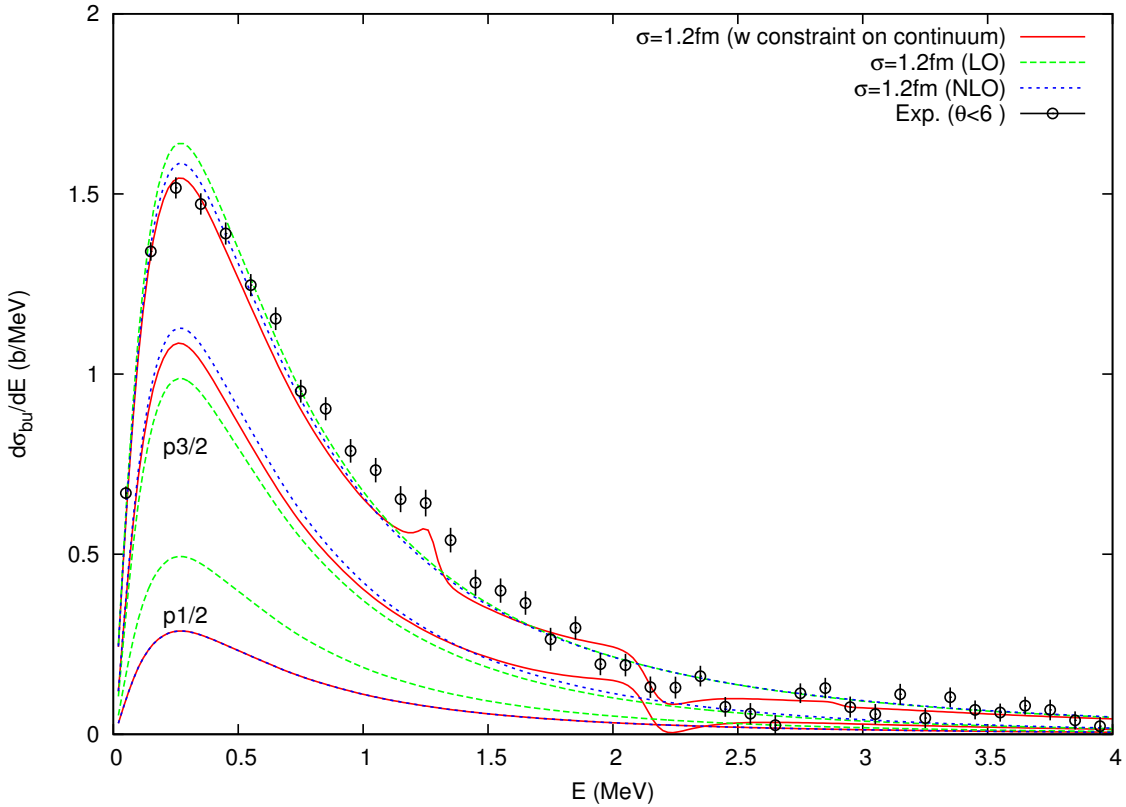


All potentials provide the same $p_{3/2}$ contribution

- confirms the peripherality of reaction (no influence of the internal part)
- shows the significant role of phaseshifts

LO, NLO and beyond

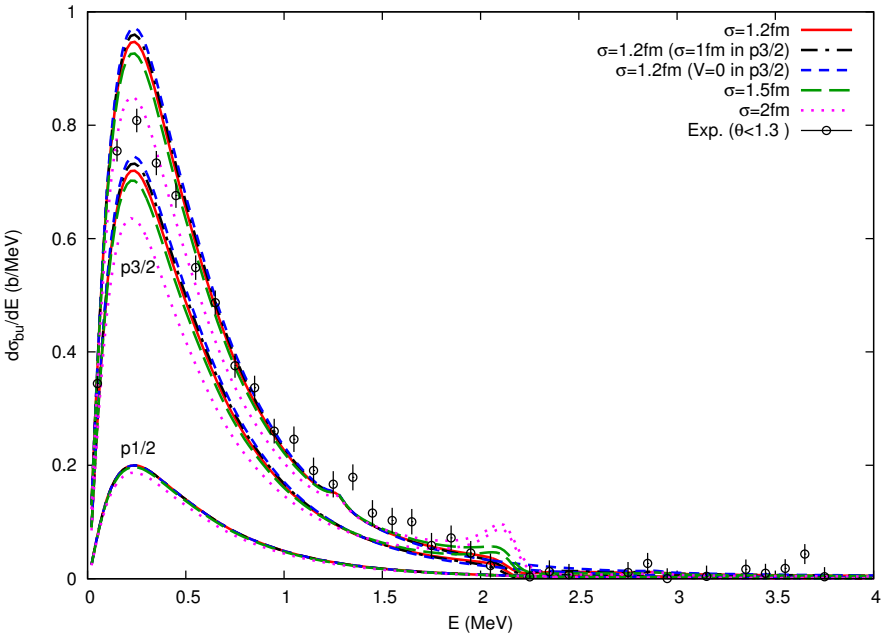
Calculations repeated with $\sigma = 1.2$ fm @ LO, NLO and beyond



- Similar $p_{3/2}$ contributions, consistent with $\delta_{p3/2} = 0$
- Significant change in $p_{1/2}$ contribution due to excited bound state

^{11}Be+Pb$\rightarrow^{10}$Be+n+Pb @ 69AMeV

Total breakup cross section and p contributions



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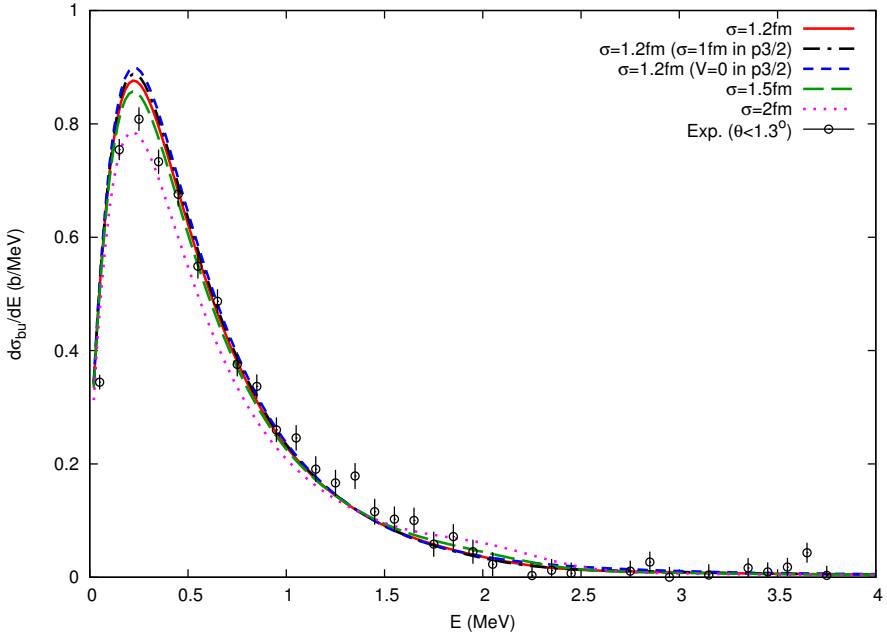
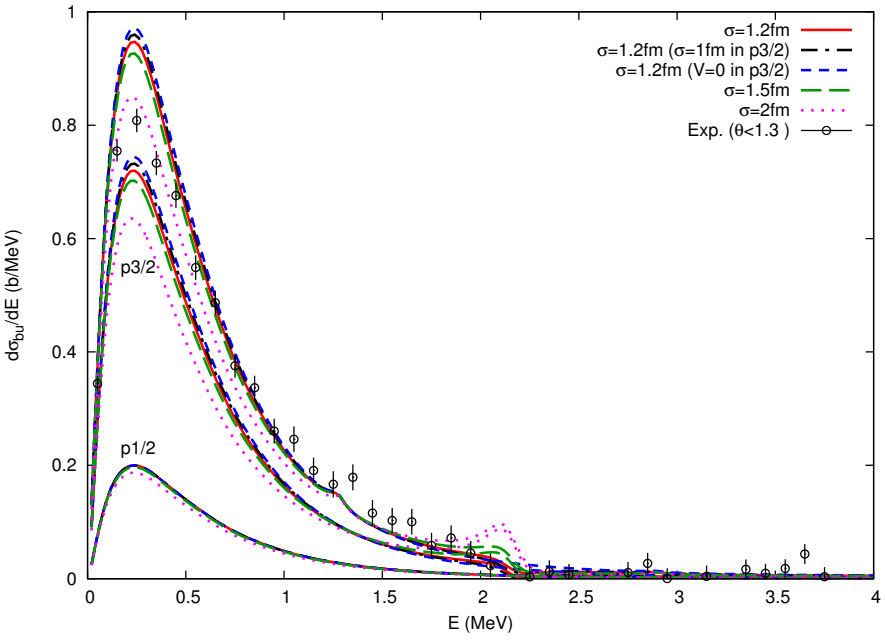


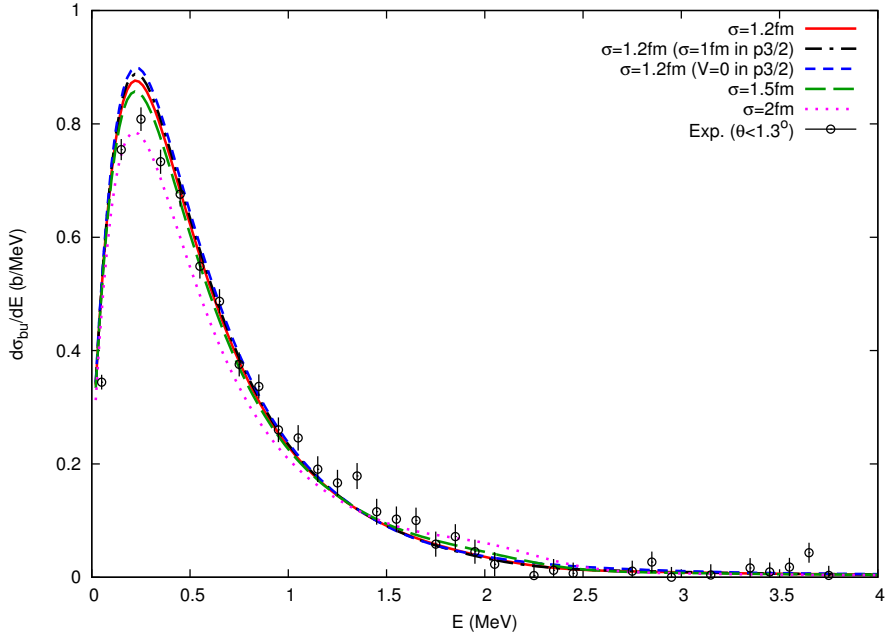
- Major differences in $p_{3/2}$ partial wave ; due to differences in $\delta_{p_{3/2}}$
- Broad potential ($\sigma = 2$ fm) produces unrealistic $p_{3/2}$ contribution

^{11}Be+Pb$\rightarrow^{10}$Be+n+Pb @ 69AMeV

Total breakup cross section and p contributions



Folded with experimental resolution



- Major differences in $p_{3/2}$ partial wave ; due to differences in $\delta_{p_{3/2}}$
- Broad potential ($\sigma = 2$ fm) produces unrealistic $p_{3/2}$ contribution
- Excellent agreement with experiment