Effective theory for heavy nuclei and β decays

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EMMI

Outline

Motivation

Spherical even-even and odd-mass nuclei

- Power counting
- Energy spectra
- E2 and M1 properties

β decays from odd-odd nuclei

- Low-lying odd-odd states
- Effective Gamow-Teller operator
- Uncertainty estimates

$\beta\beta$ decays

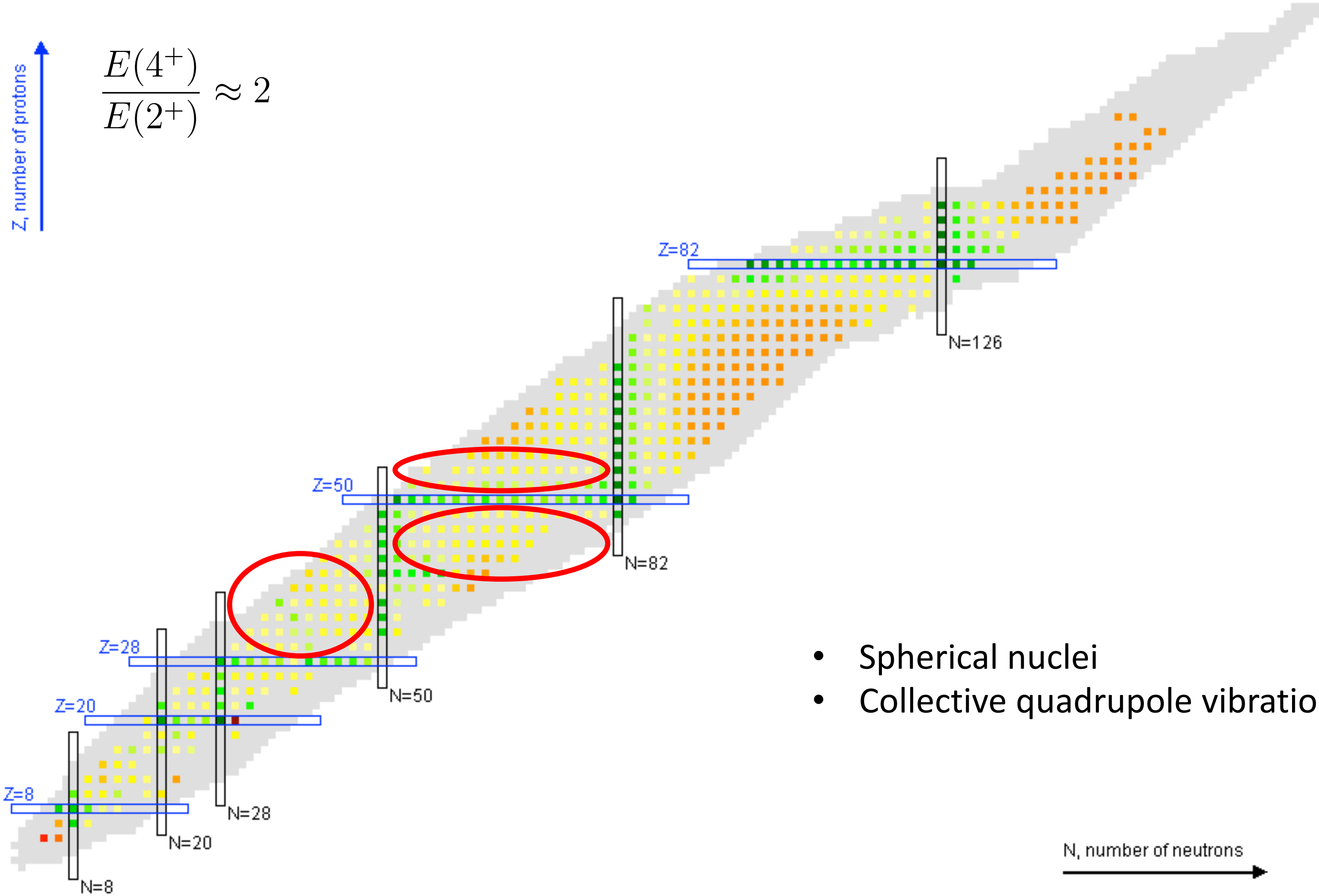
LONG TERM GOAL

- Calculate matrix elements for $0\nu\beta\beta$ decays and provide an associated uncertainty estimate

IN THIS WORK

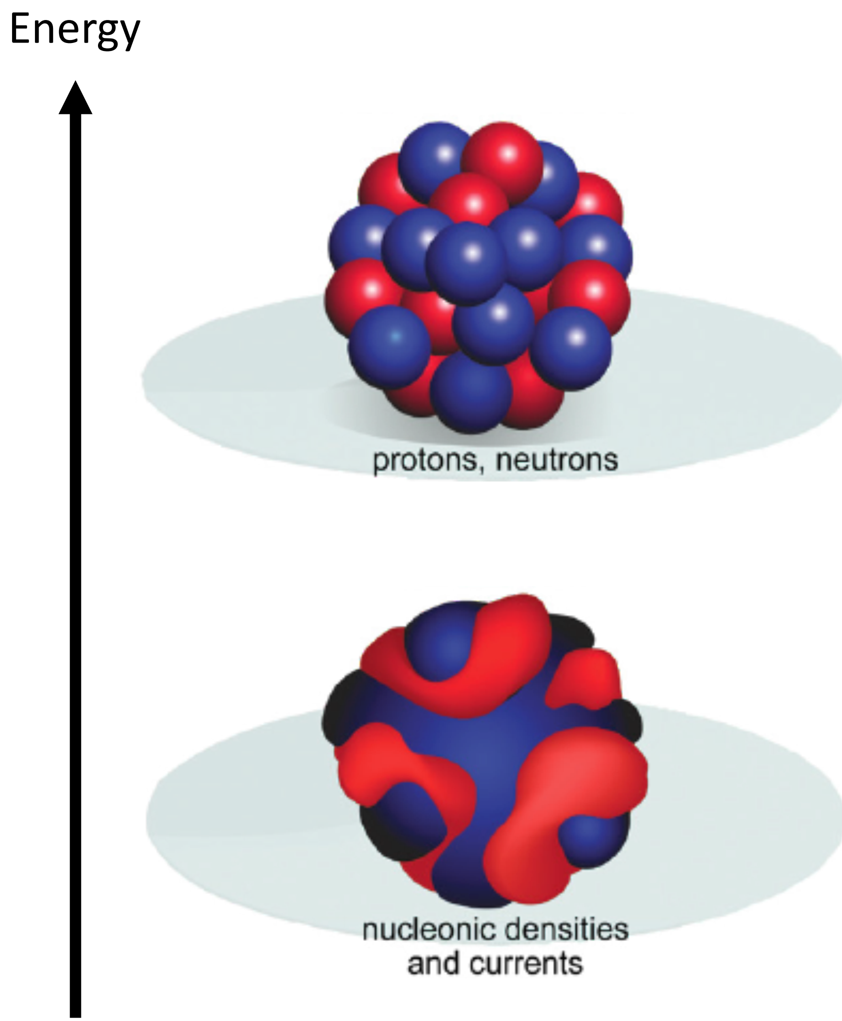
- Describe observed β and $2\nu\beta\beta$ decays in order to establish whether our ET is capable to describe them consistently

$$\frac{E(4^+)}{E(2^+)} \approx 2$$



- Spherical nuclei
- Collective quadrupole vibrations

Effective theory for collective nuclei



Chiral EFT
- Nucleon and pion fields

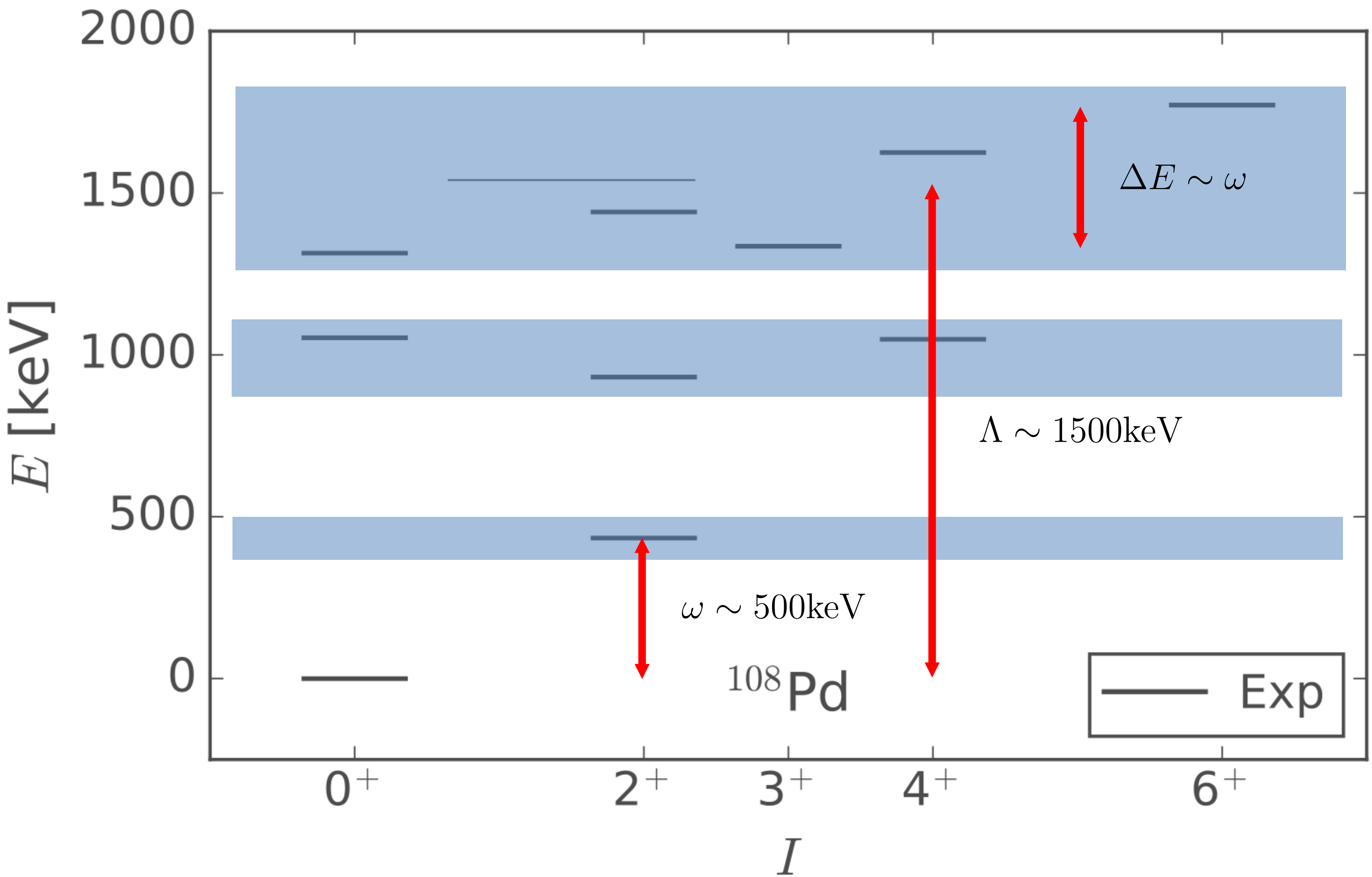
BREAKDOWN SCALE $\Lambda \sim 1500\text{keV}$

Collective ET
- Phonons
- Few fermions

$$\frac{\omega}{\Lambda} \ll 1$$

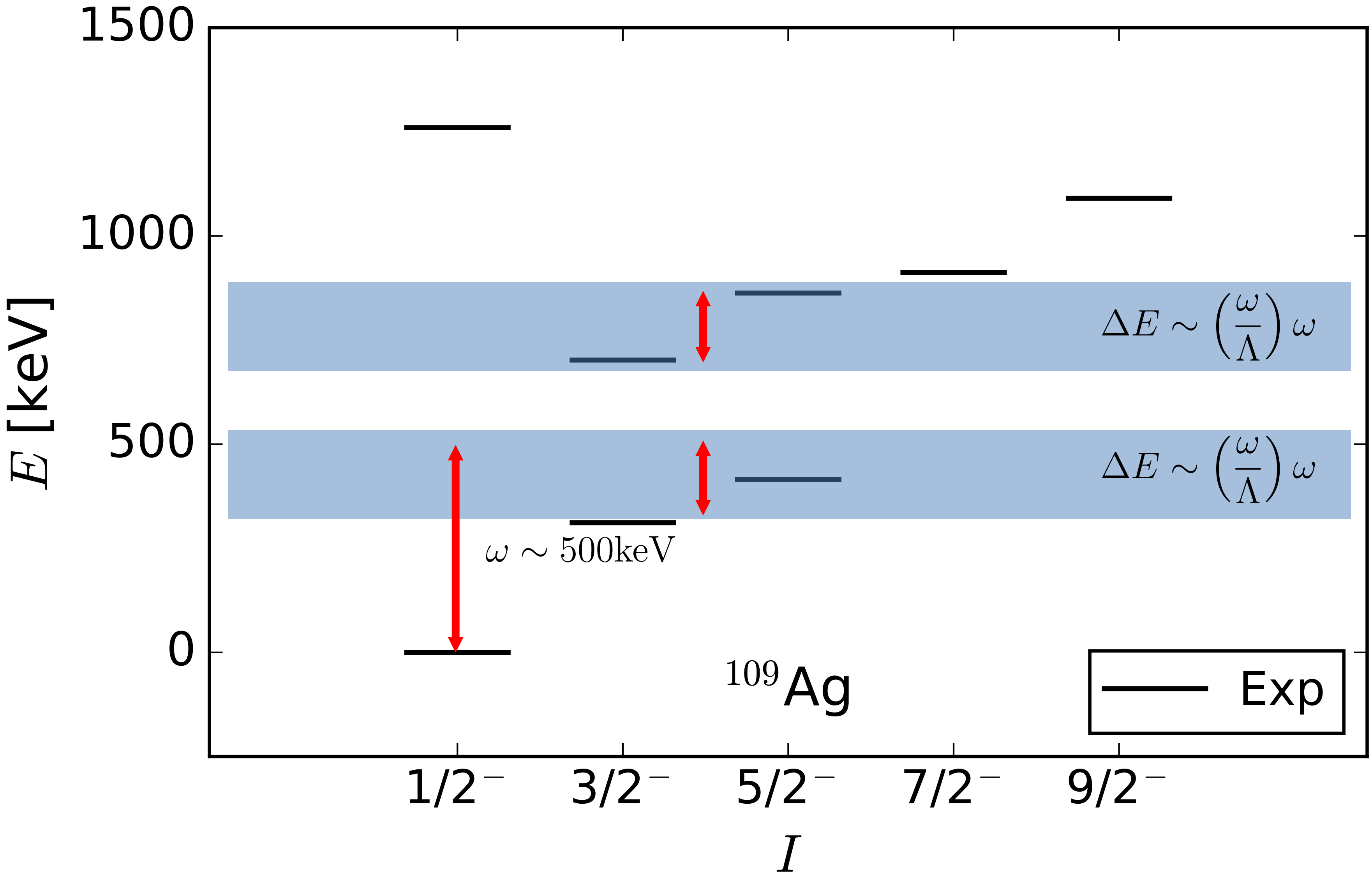
$$\omega \sim 500\text{keV}$$

Bertsch, Dean, Nazarewicz, SciDAC Rev. **6**, 42 (2007)



2000
1500
1000
500
0
E [keV]
0^+
2^+
3^+
4^+
6^+
I
$\Delta E \sim \omega$
$\Lambda \sim 1500\text{keV}$
$\omega \sim 500\text{keV}$
^{108}Pd
Exp

Relevant energy scales in odd-mass nuclei



Hamiltonian in terms of boson creation and annihilation operators (collective excitations) and fermion creation and annihilation operators (for the description of odd-mass systems)

$$\left[d_\mu, d_\nu^\dagger\right] = \delta_{\mu\nu} \qquad \left\{a_\mu, a_\nu^\dagger\right\} = \delta_{\mu\nu}$$

LO: Bohr and Mottelson's harmonic vibrator model

$$H_{\rm LO} \equiv \omega_1 \hat{N} \qquad \hat{N} \equiv d^\dagger \cdot \tilde{d}$$

NLO: Interactions between collective core and the odd fermion

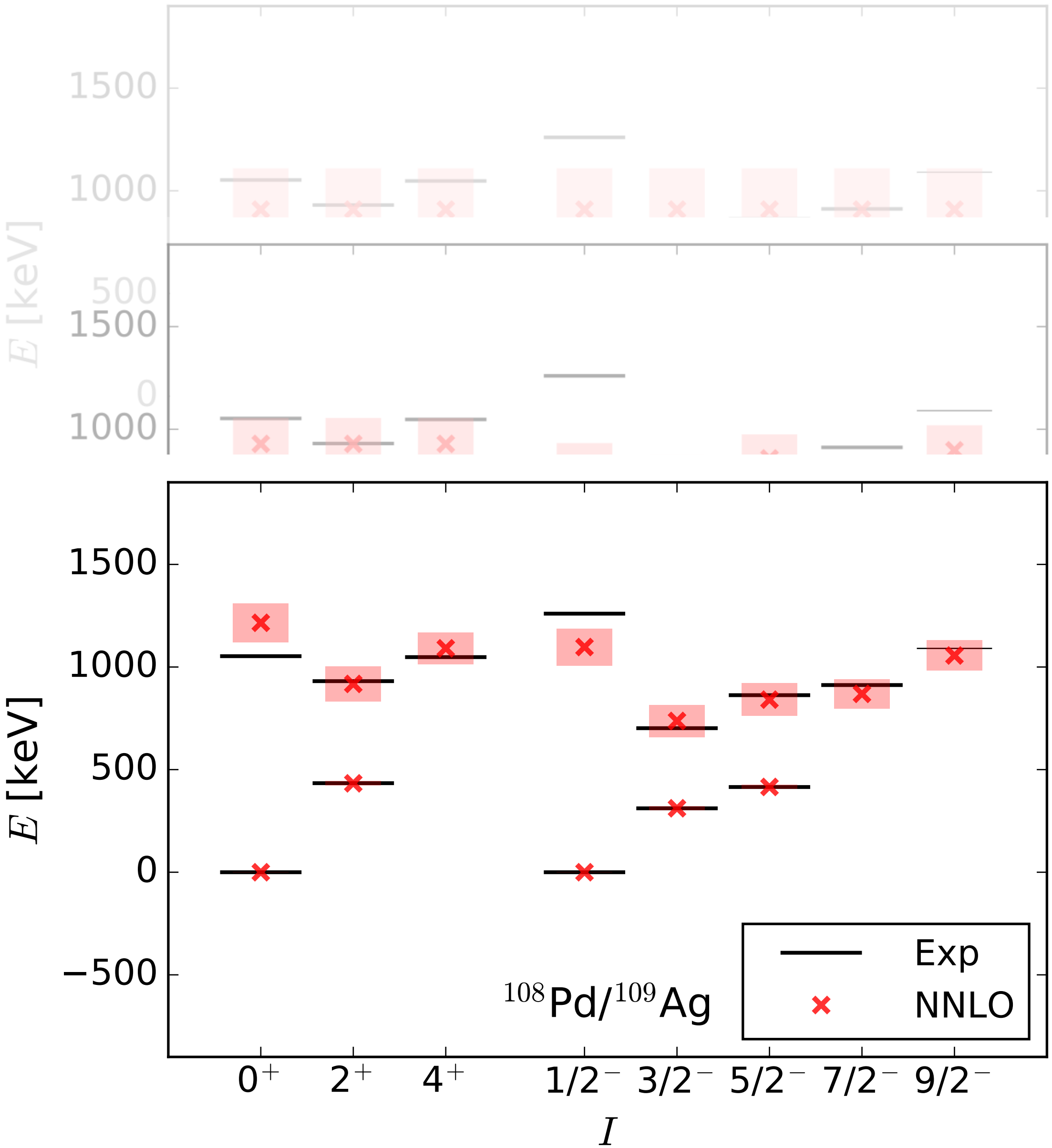
$$H_{\rm NLO} \equiv g_{Jj} \hat{\mathbf{J}} \cdot \hat{\mathbf{j}} + \omega_2 \hat{N} \hat{n} \qquad \hat{\mathbf{J}} = \sqrt{10}\left(d^\dagger \otimes \tilde{d}\right)^{(1)} \qquad \hat{\mathbf{j}} = \frac{1}{\sqrt{2}}\left(a^\dagger \otimes \tilde{a}\right)^{(1)} \qquad \hat{n} \equiv a^\dagger \cdot \tilde{a}$$

NNLO: Anharmonicities

$$H_{\rm NNLO} \equiv g_N \hat{N}^2 + g_v \hat{\Lambda}^2 + g_J \hat{J}^2 \qquad \hat{\Lambda}^2 \equiv -\left(d^\dagger \cdot d^\dagger\right)\left(\tilde{d} \cdot \tilde{d}\right) + \hat{N}^2 - 3\hat{N}$$

Coello Pérez, Papenbrock; Phys. Rev. C **92**, 064309 (2015)
Coello Pérez, Papenbrock; Phys. Rev. C **94**, 054316 (2016)

Order-by-order improvement



LO:
- One LEC
- Harmonic behavior

NLO:
- Two additional LECs
- Particle-core interactions

NNLO:
- Three additional LECs
- Anharmonic corrections

Accuracy and precision show an order-by-order increase of precision at the expense of reduced predictive power

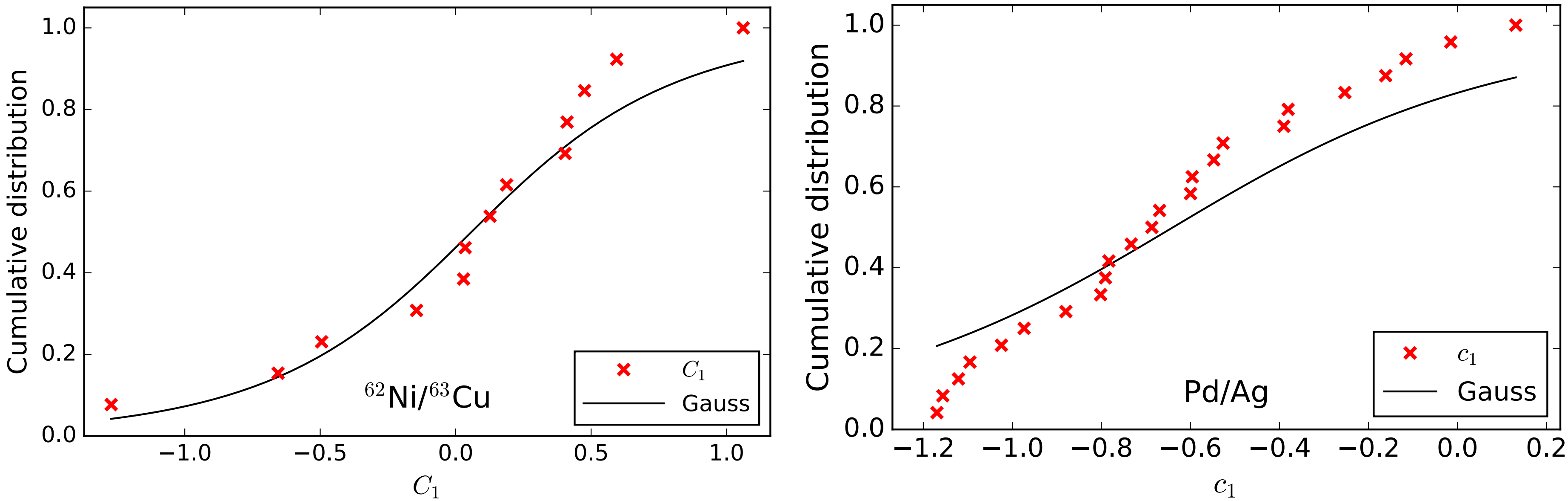
Testing the power counting

Observables $E = \omega \sum_{n}^{\infty} c_n \varepsilon^n, \quad \varepsilon \equiv \frac{N\omega}{\Lambda}$

LECs assumed to be of order one

$$\mathrm{pr}^{(\mathrm{G})}(\tilde{c}_i|c) = \frac{1}{\sqrt{2\pi} s c} e^{-\frac{\tilde{c}_i^2}{2s^2c^2}}$$

$$\mathrm{pr}(c) = \frac{1}{\sqrt{2\pi}\sigma c} e^{-\frac{\log^2 c}{2\sigma^2}}$$



E2 operator

Most general positive-parity rank-two tensor

$$\hat{Q} = Q_0 \left(d^\dagger + \tilde{d}\right) + Q_1 \left(d^\dagger \otimes \tilde{d}\right)^{(2)}$$

All terms scale similarly at breakdown

$$Q_1 \sim \sqrt{\frac{\omega}{\Lambda}} Q_0$$

Natural scaling

$$B \sim A \quad \Rightarrow \quad B \in \left[A\sqrt{\frac{\omega}{\Lambda}}, A\sqrt{\frac{\Lambda}{\omega}} \right]$$

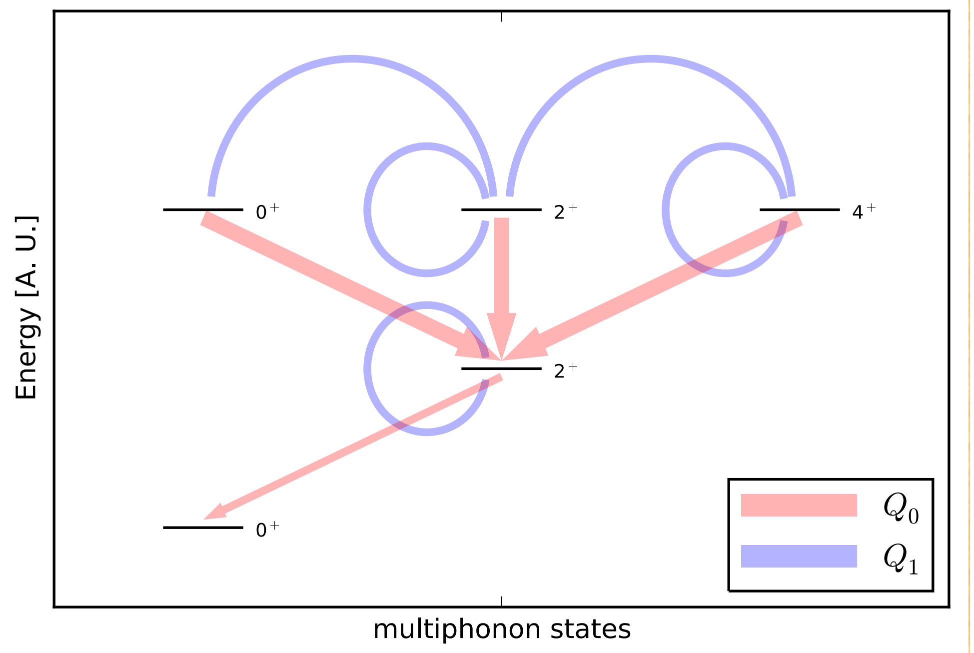
LO

•Phonon-annihilating transitions

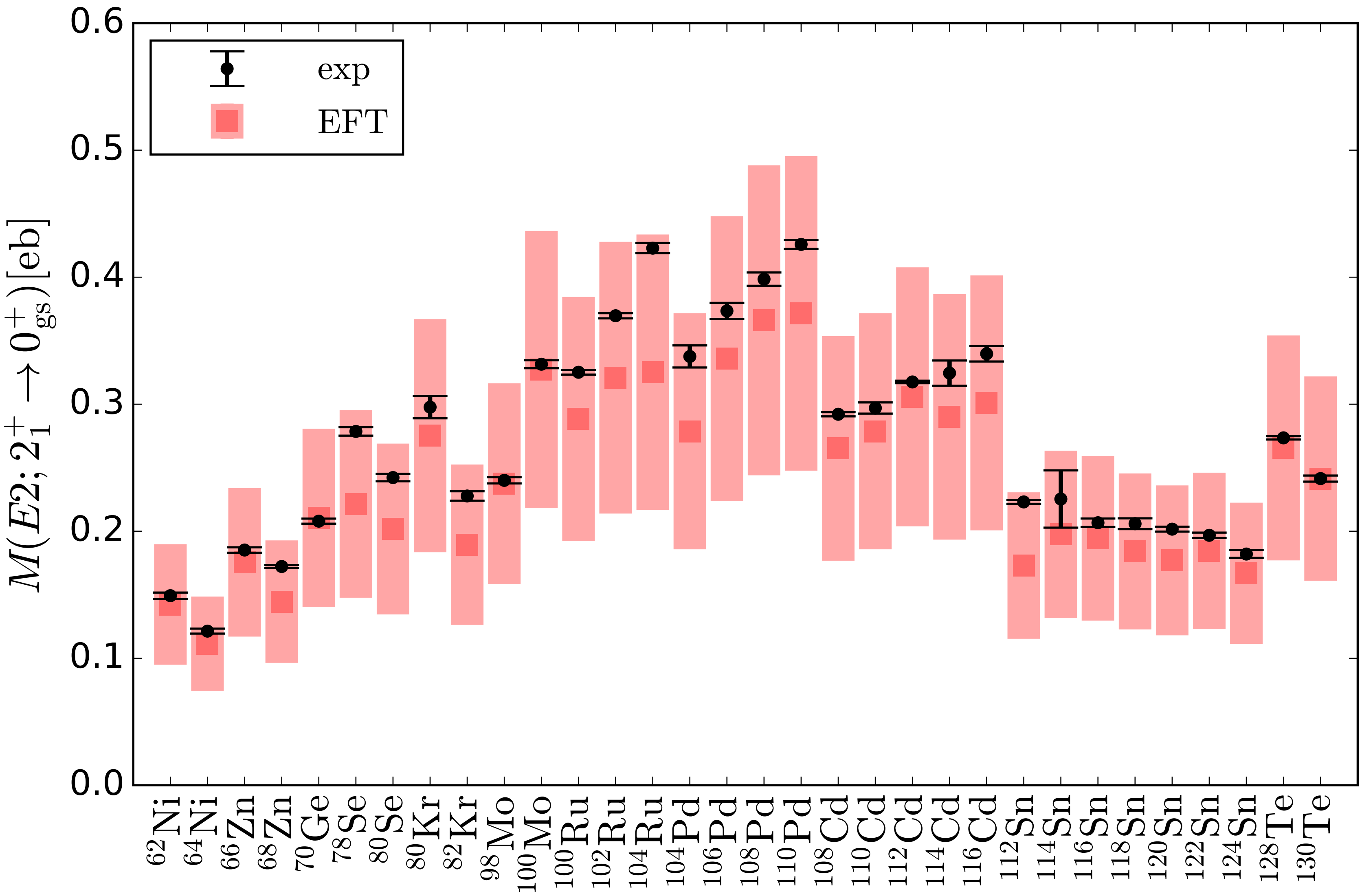
NLO

•Phonon-conserving transitions

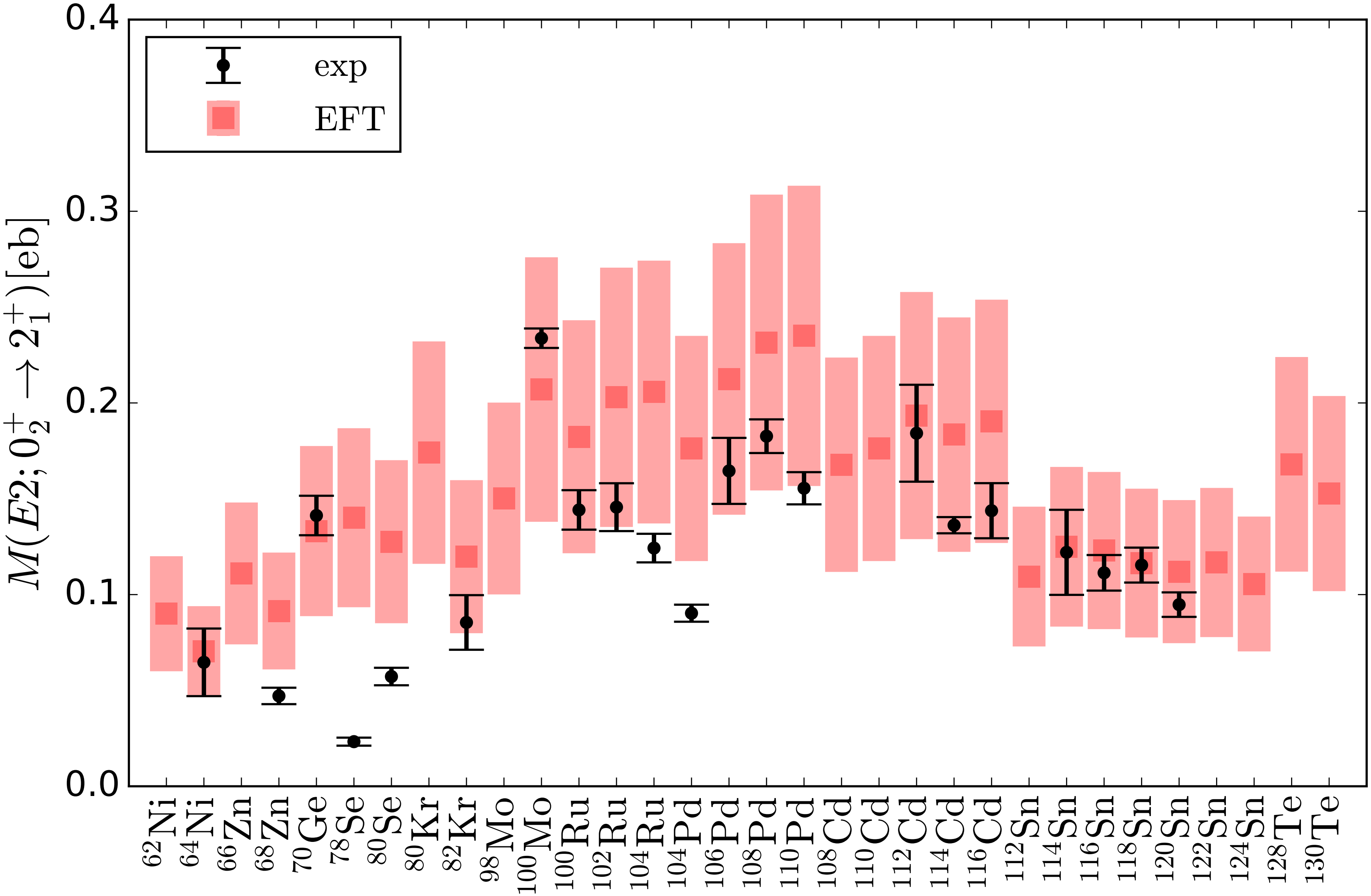
•Static E2 moments



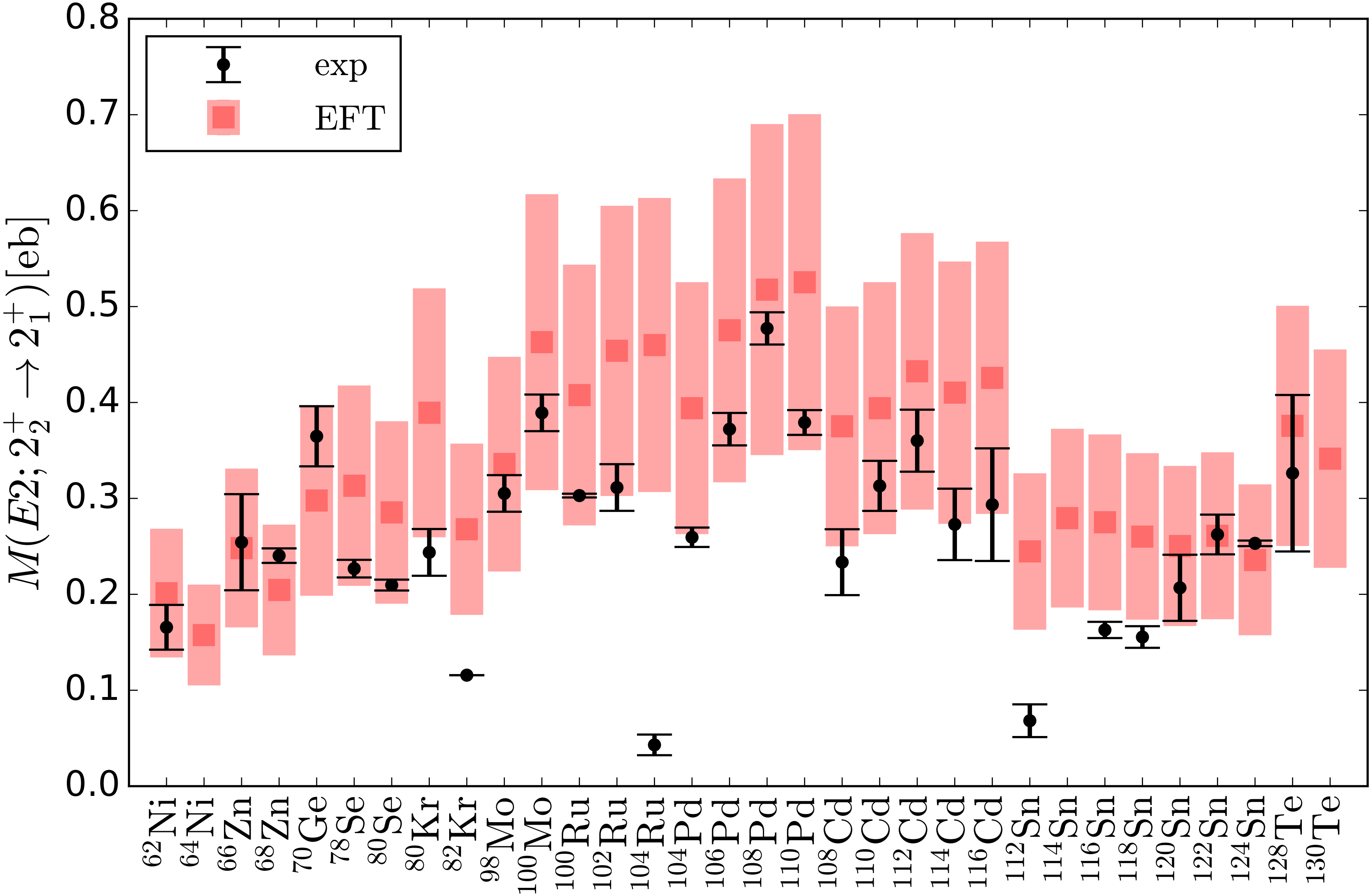
E2 transition matrix elements



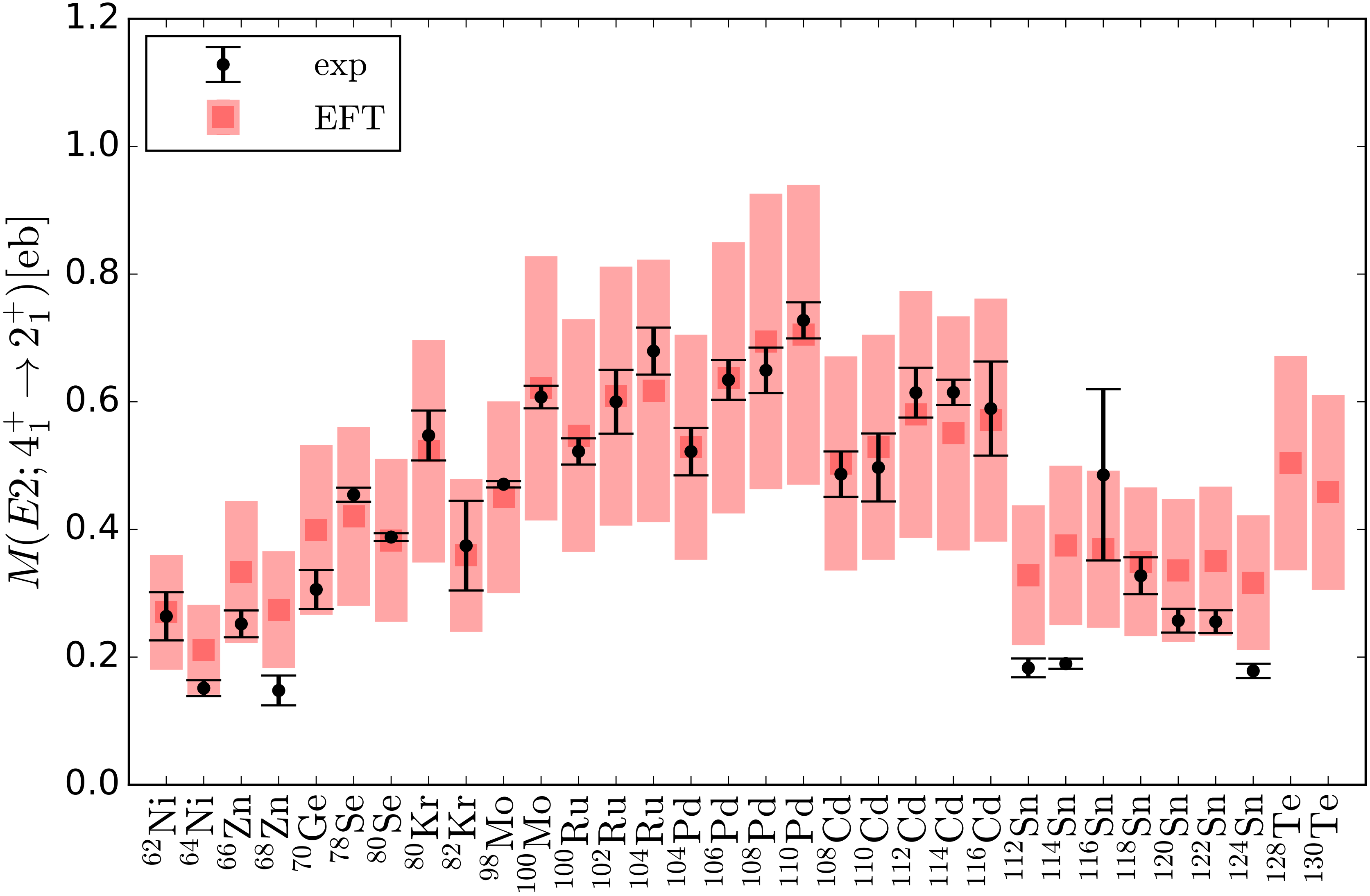
E2 transition matrix elements



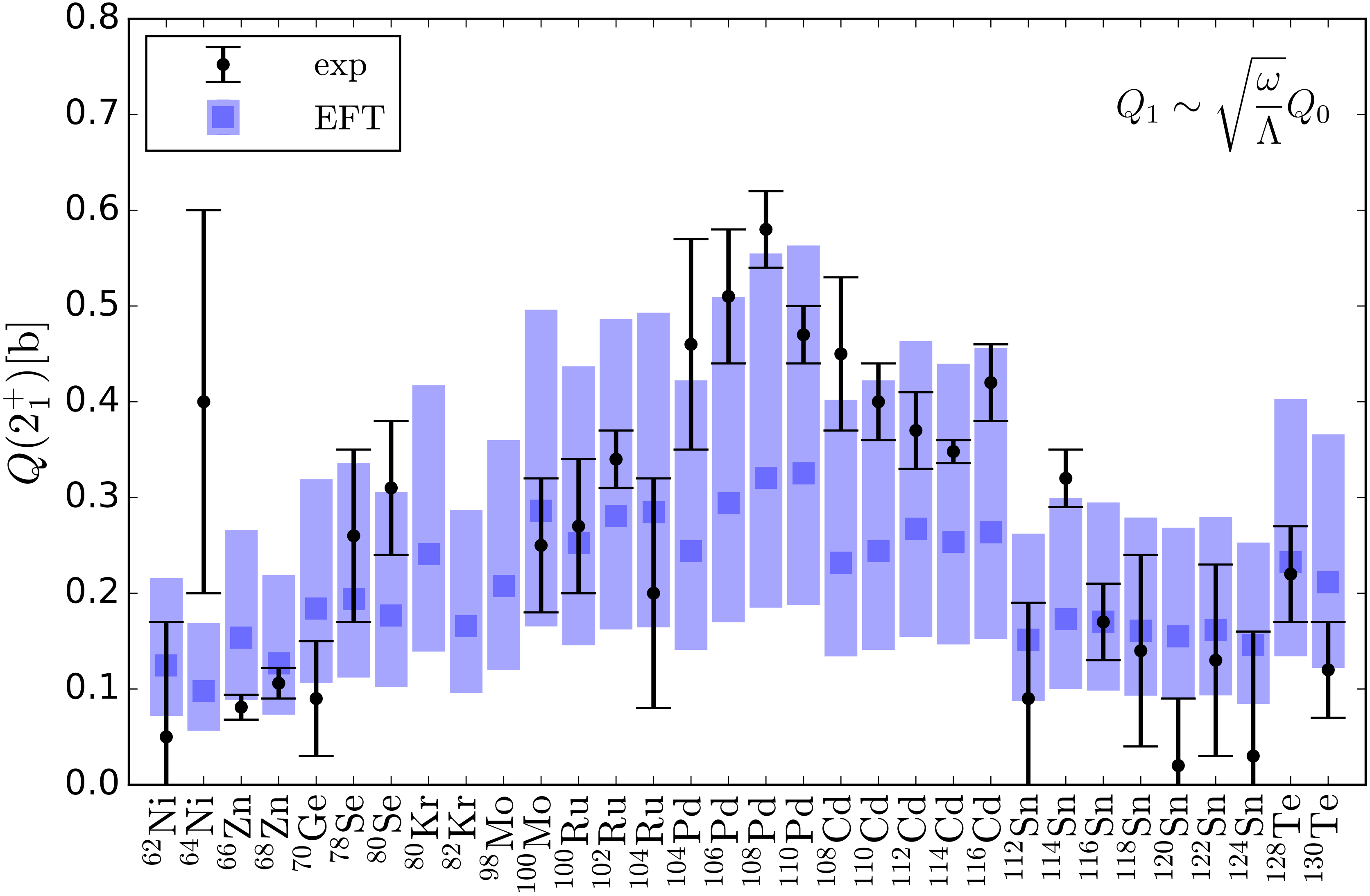
E2 transition matrix elements



E2 transition matrix elements



Static E2 moments



Most general operator of rank one

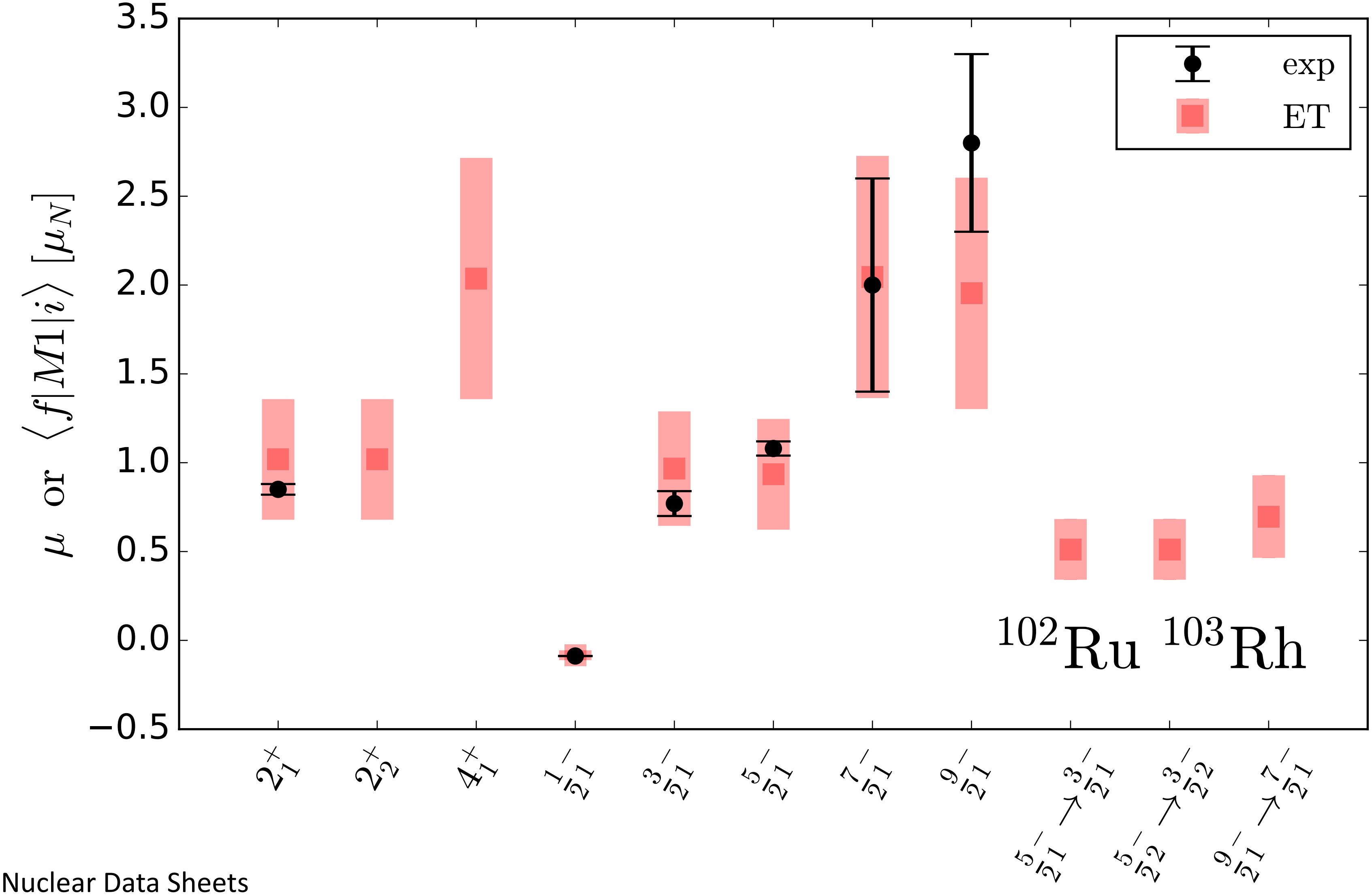
$$\hat{\mu} = \mu_d \hat{\mathbf{J}} + \mu_a \hat{\mathbf{j}} + \left[\left(d^\dagger + \tilde{d} \right) \otimes \left(\mu_{d1} \hat{\mathbf{J}} + \mu_{a1} \hat{\mathbf{j}} \right) \right]^{(1)}$$

LO term:
- Two LECs
- Phonon-conserving transition
- Static M1 moments

NLO term:
- Two LECs
- Phonon-annihilating transition

Static M1 static moments and M1 transition matrix elements

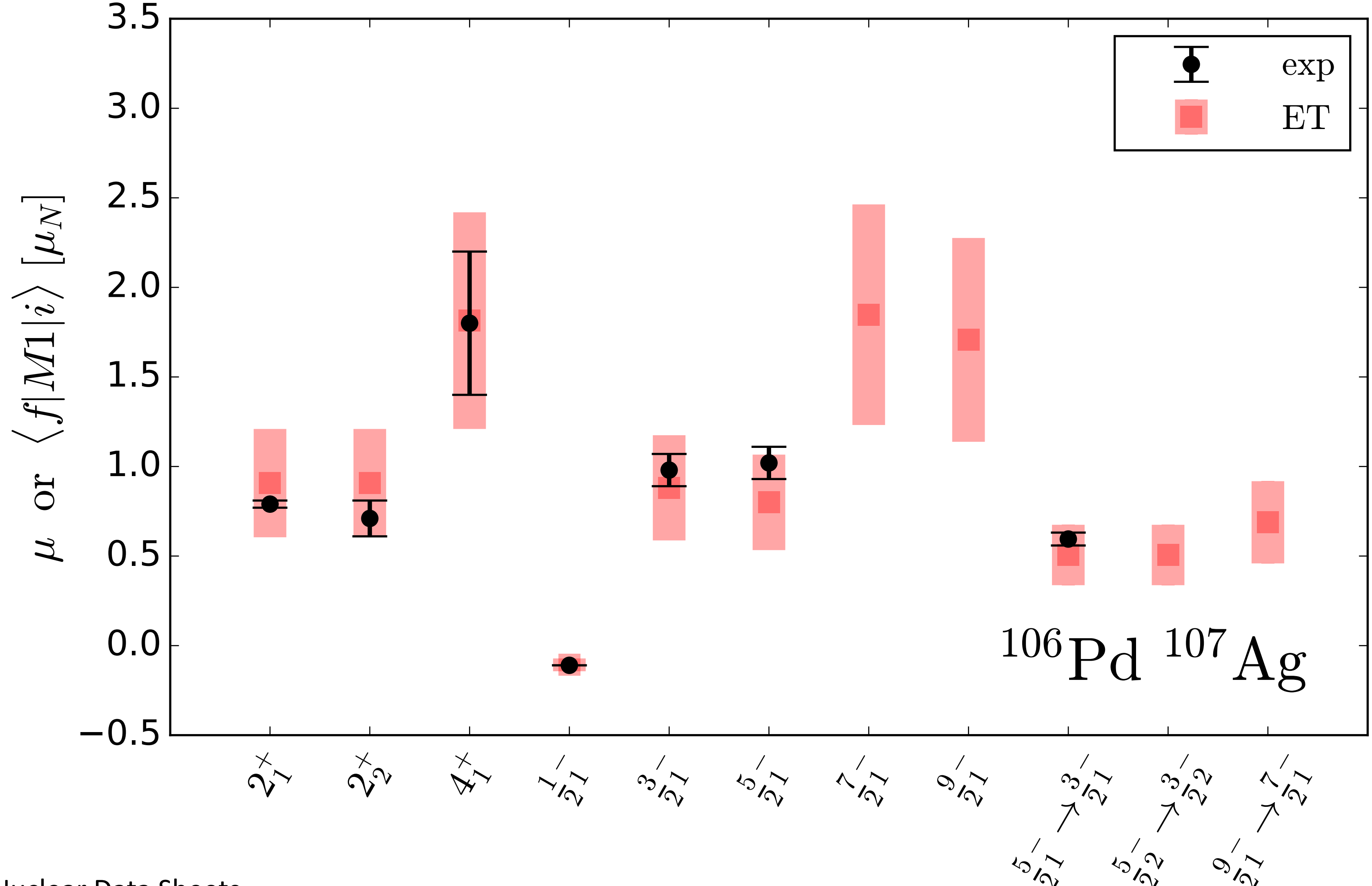
$$\hat{\mu} = \mu_d \hat{\mathbf{J}} + \mu_a \hat{\mathbf{j}} + \left[\left(d^\dagger + \tilde{d} \right) \otimes \left(\mu_{d1} \hat{\mathbf{J}} + \mu_{a1} \hat{\mathbf{j}} \right) \right]^{(1)}$$



Data: Nuclear Data Sheets

Static M1 static moments and M1 transition matrix elements

$$\hat{\mu} = \mu_d \hat{\mathbf{J}} + \mu_a \hat{\mathbf{j}} + \left[\left(d^\dagger + \tilde{d} \right) \otimes \left(\mu_{d1} \hat{\mathbf{J}} + \mu_{a1} \hat{\mathbf{j}} \right) \right]^{(1)}$$



Data: Nuclear Data Sheets

Low-lying positive-parity odd-odd states are constructed as

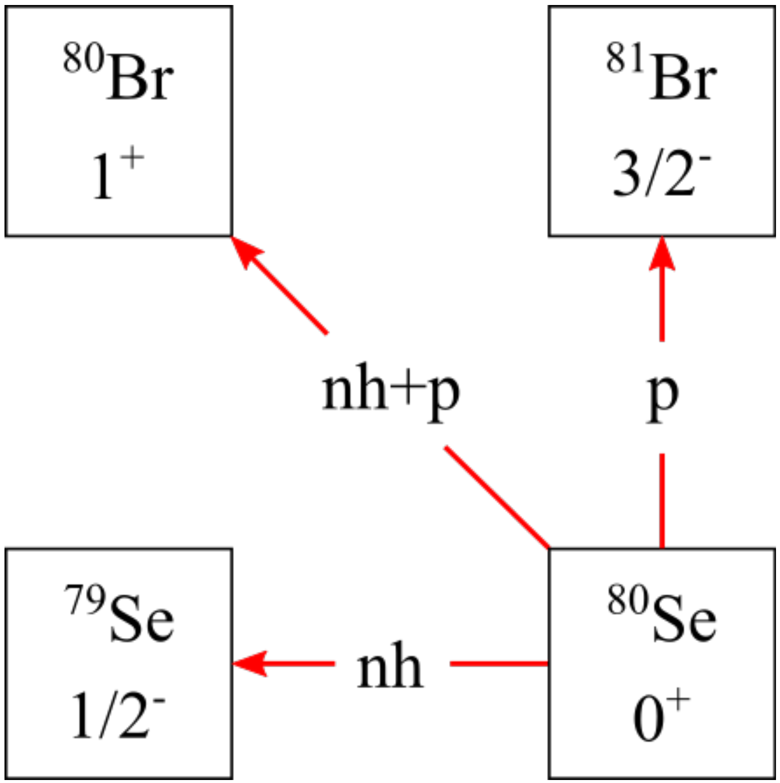
$$|IM; j_p; j_n\rangle = \sum_{\mu\nu} C^{IM}_{j_n \mu j_p \nu} n^\dagger_\mu p^\dagger_\nu |0\rangle$$

where

$$|j_n - j_p| \leq I \leq j_n + j_p$$

and

$$\pi_n \pi_p = 1$$



Effective Gamow-Teller operator

Most general rank-one operator coupling odd-odd and even-even states

$$\hat{O}_\beta = C_\beta \, (\tilde{p} \otimes \tilde{n})^{(1)} + \sum_\ell C_{\beta\ell} \left[\left(d^\dagger + \tilde{d} \right) \otimes (\tilde{p} \otimes \tilde{n})^{(\ell)} \right]^{(1)} + \sum_{L\ell} C_{\beta L\ell} \left[\left(d^\dagger \otimes d^\dagger + \tilde{d} \otimes \tilde{d} \right)^{(L)} \otimes (\tilde{p} \otimes \tilde{n})^{(\ell)} \right]^{(1)}$$

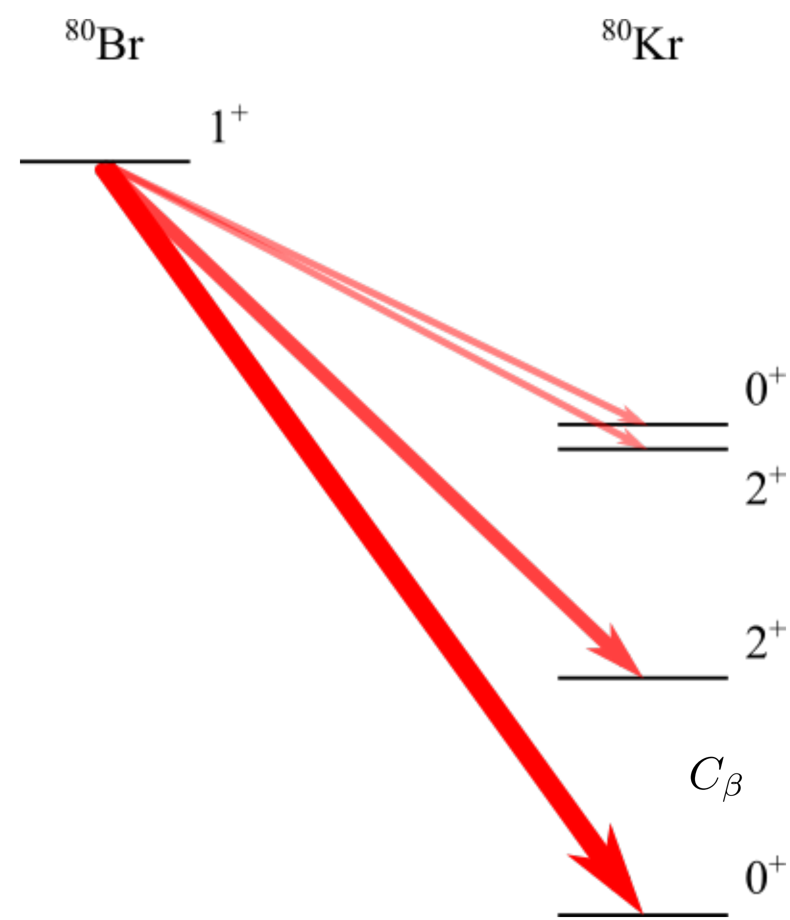
LO term:
- Couples states with $\Delta\mathcal{N} = 0$

NLO term:
- Couples states with $\Delta\mathcal{N} = 1$

NNLO term:
- Couples states with $\Delta\mathcal{N} = 2$

From the power counting

$$\frac{C_{\beta\ell}}{C_\beta} \overset{\text{EFT}}{\sim} 0.58(^{+42}_{-25}) \quad \text{and} \quad \frac{C_{\beta L\ell}}{C_\beta} \overset{\text{EFT}}{\sim} 0.33(^{+25}_{-14})$$



Corrections to odd-odd states

$$\langle 0^+_{\rm gs}|\hat{O}_{\rm GT}\Delta|I_i^+\rangle \sim \frac{\omega}{\Lambda} M_{\rm GT}\left(I_i^+ \to 0_1^+\right)$$

Corrections to the effective GT operator

$$\langle 0^+_{\rm gs}|\Delta\hat{O}_{\rm GT}|I_i^+\rangle \sim \frac{\omega}{\Lambda} M_{\rm GT}\left(I_i^+ \to 0_1^+\right)$$

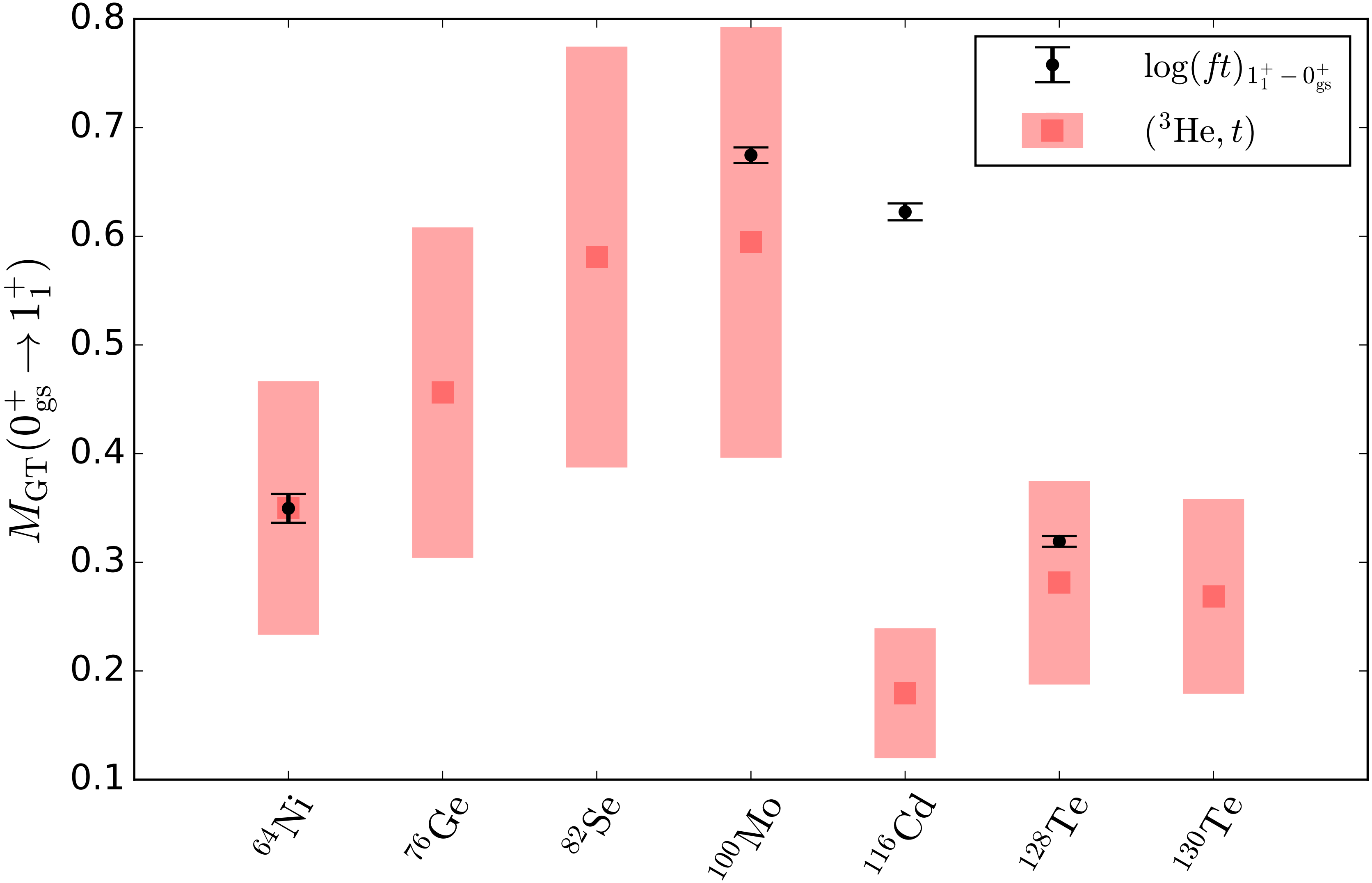
Uncertainty estimate

$$\Delta M_{\rm GT}\left(I_i^+ \to 0_1^+\right) \sim \frac{\omega}{\Lambda} M_{\rm GT}\left(I_i^+ \to 0_1^+\right)$$

or

$$\Delta \log(ft)_{if} \sim \frac{\omega}{\Lambda}\frac{2}{\ln 10} = 0.29$$

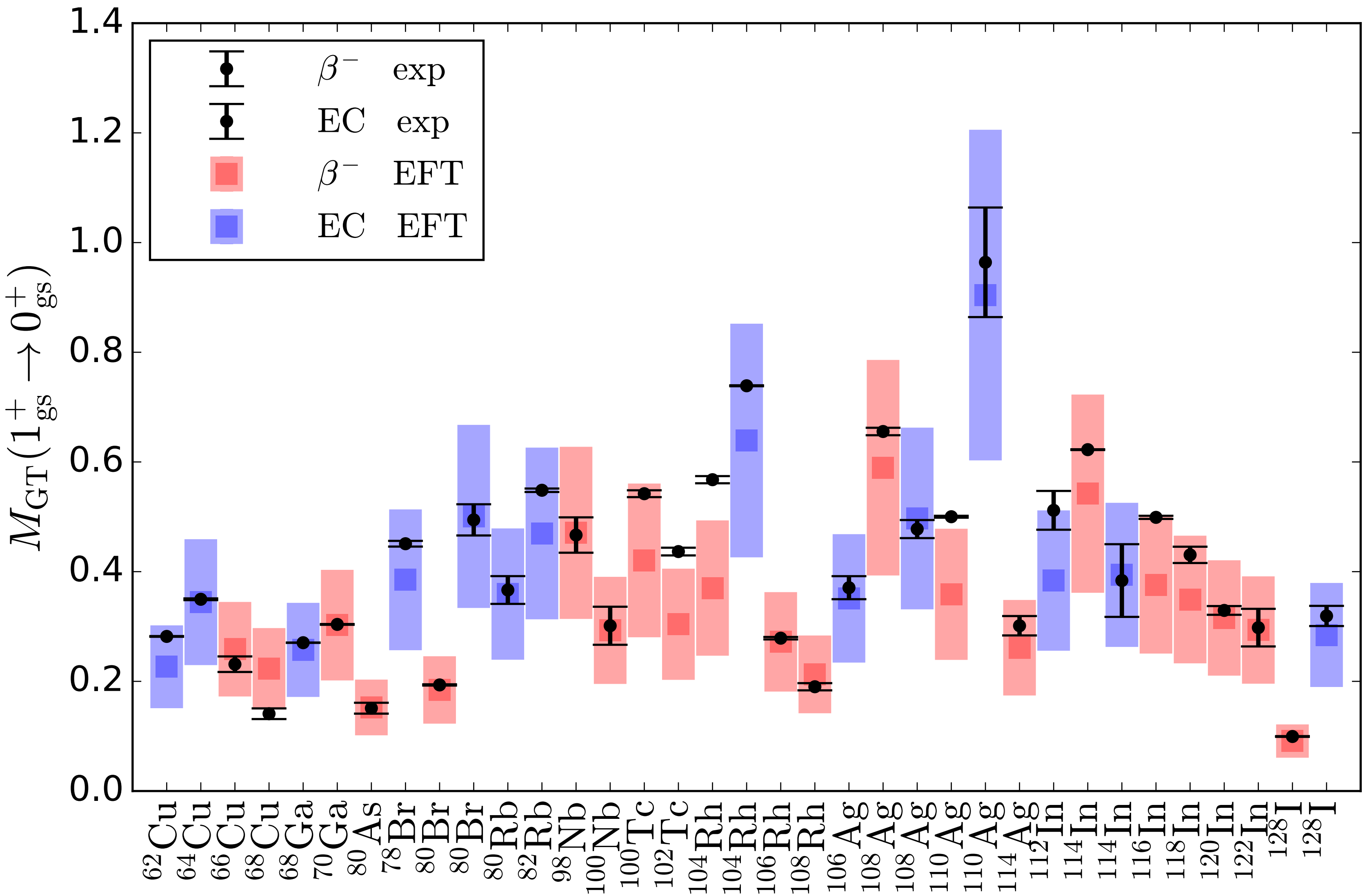
LEC from charge-exchange reaction experiments



Popescu et al., Phys. Rev. C **79**, 064312 (2009)
Thies et al., Phsy. Rev. C **86**, 014304 (2012)
Frekers et al., Phys. Rev. C **94**, 014614 (2016)

Thies et al., Phys. Rev. C **86**, 044309 (2012)
Akimune et al., Phys. Lett. B **394**, 23 (1997)
Puppe et al., Phys. Rev. C **86**, 044603 (2012)

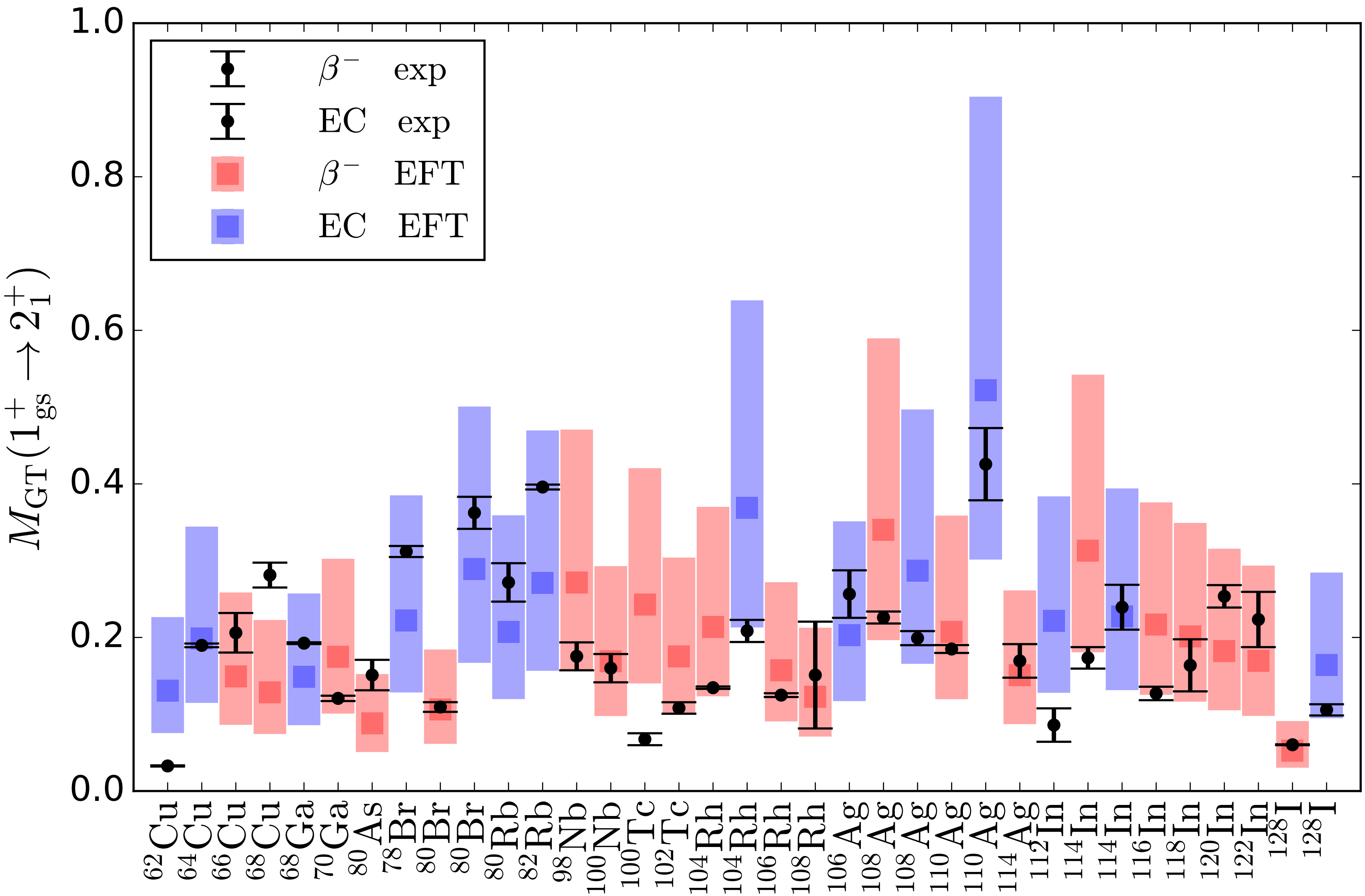
Matrix elements for single-β decays



Coello Pérez, Menéndez, Schwenk; arXiv:1708.06140

Data: Nuclear Data Sheets

Matrix elements for single-β



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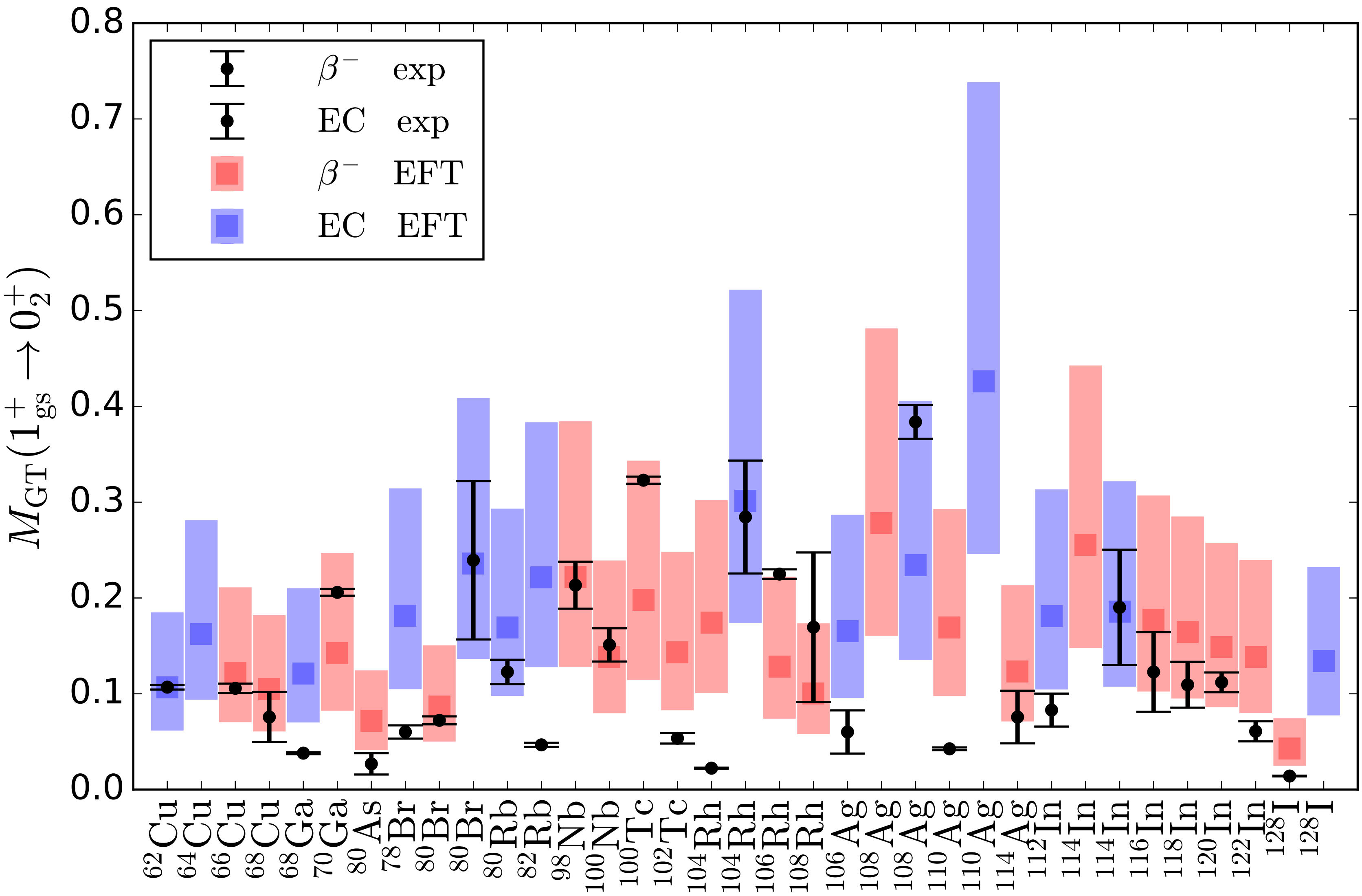
Matrix elements for single-β



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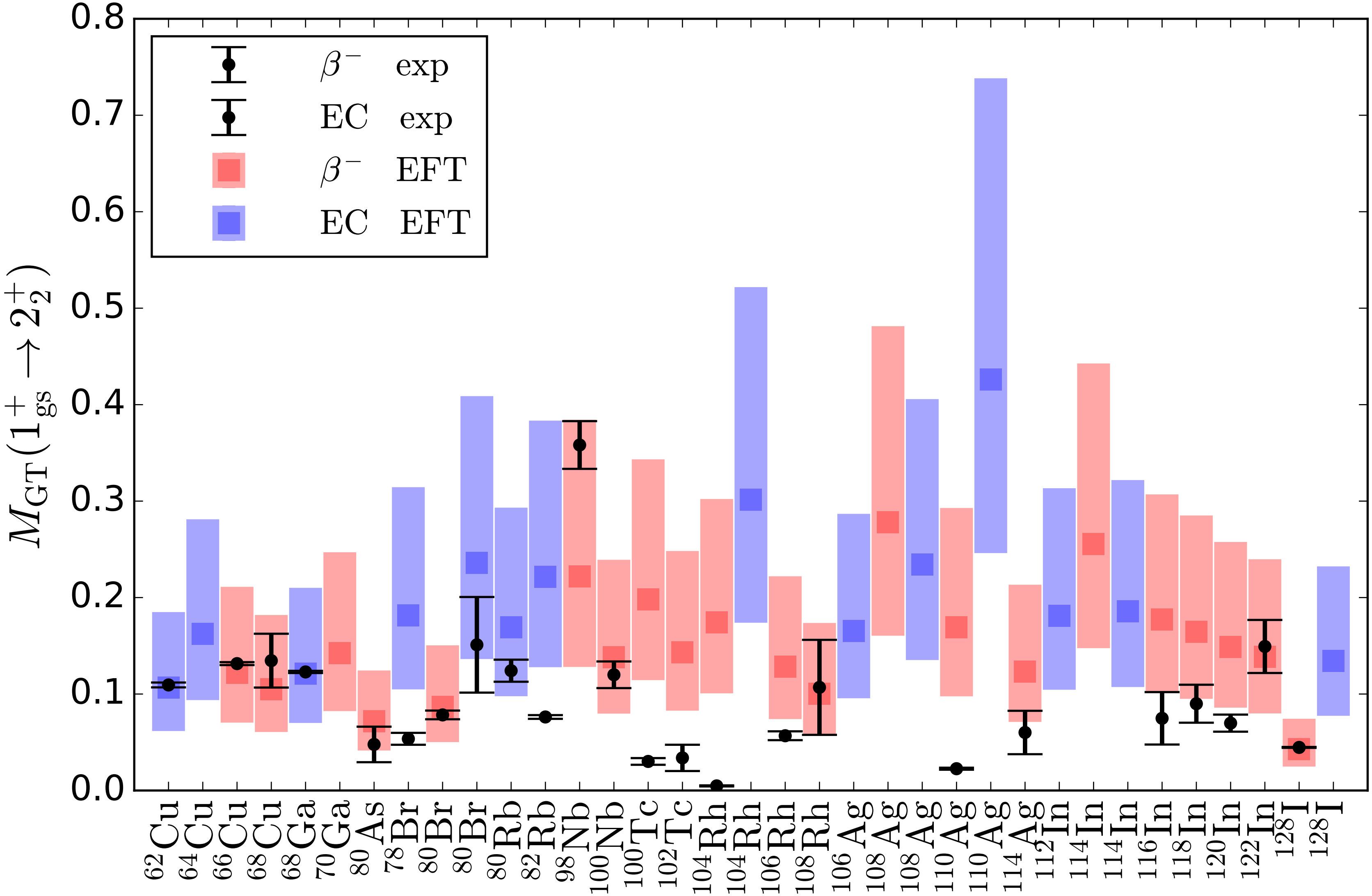


Coello Pérez, Menéndez, Schwenk; arXiv:1708.06140

Data: Nuclear Data Sheets

GT matrix elements for $2\nu\beta\beta$ decay

$$M_{\rm GT}^{2\nu} = \sum_n \frac{\langle f || \sum_a \boldsymbol{\sigma}_a \tau_a^+ || 1_n^+ \rangle \langle 1_n^+ || \sum_b \boldsymbol{\sigma}_b \tau_b^+ || i \rangle}{D_{nf}/m_e}$$

SSD approximation

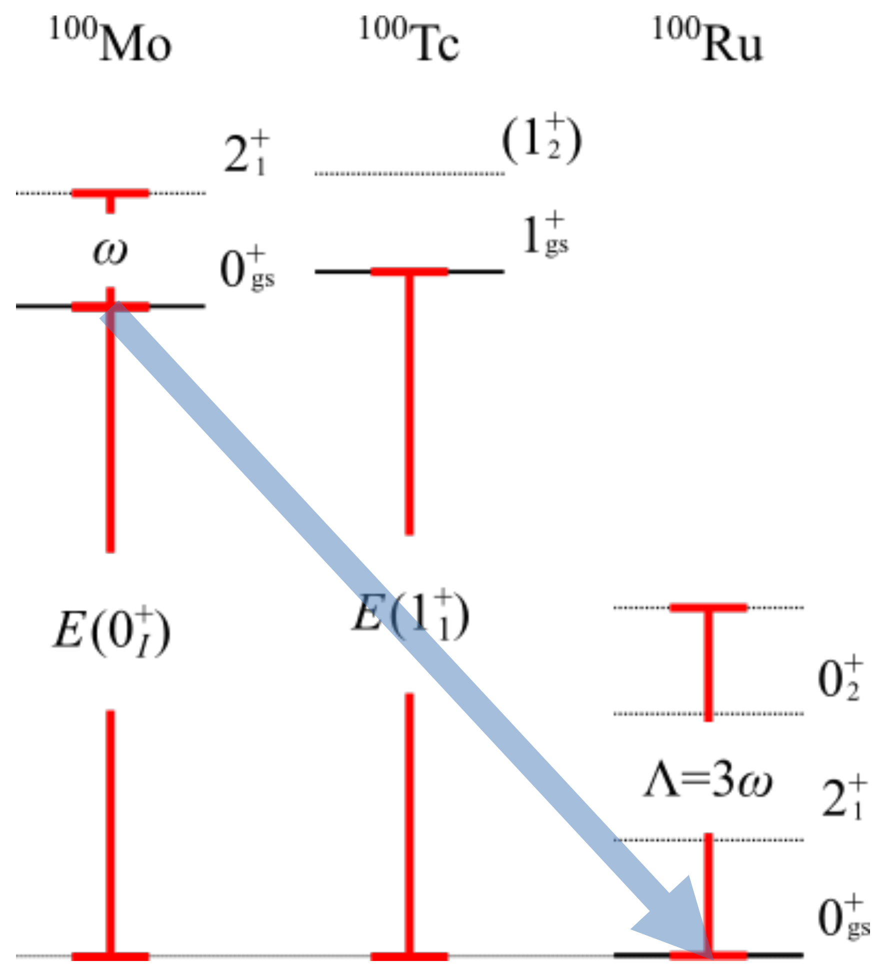
$$M_{\rm GT}^{2\nu}(i \to f) \approx \frac{M_{\rm GT}(1_1^+ \to 0_f^+) M_{\rm GT}(0_i^+ \to 1_1^+)}{D_{1f}/m_e c^2}$$

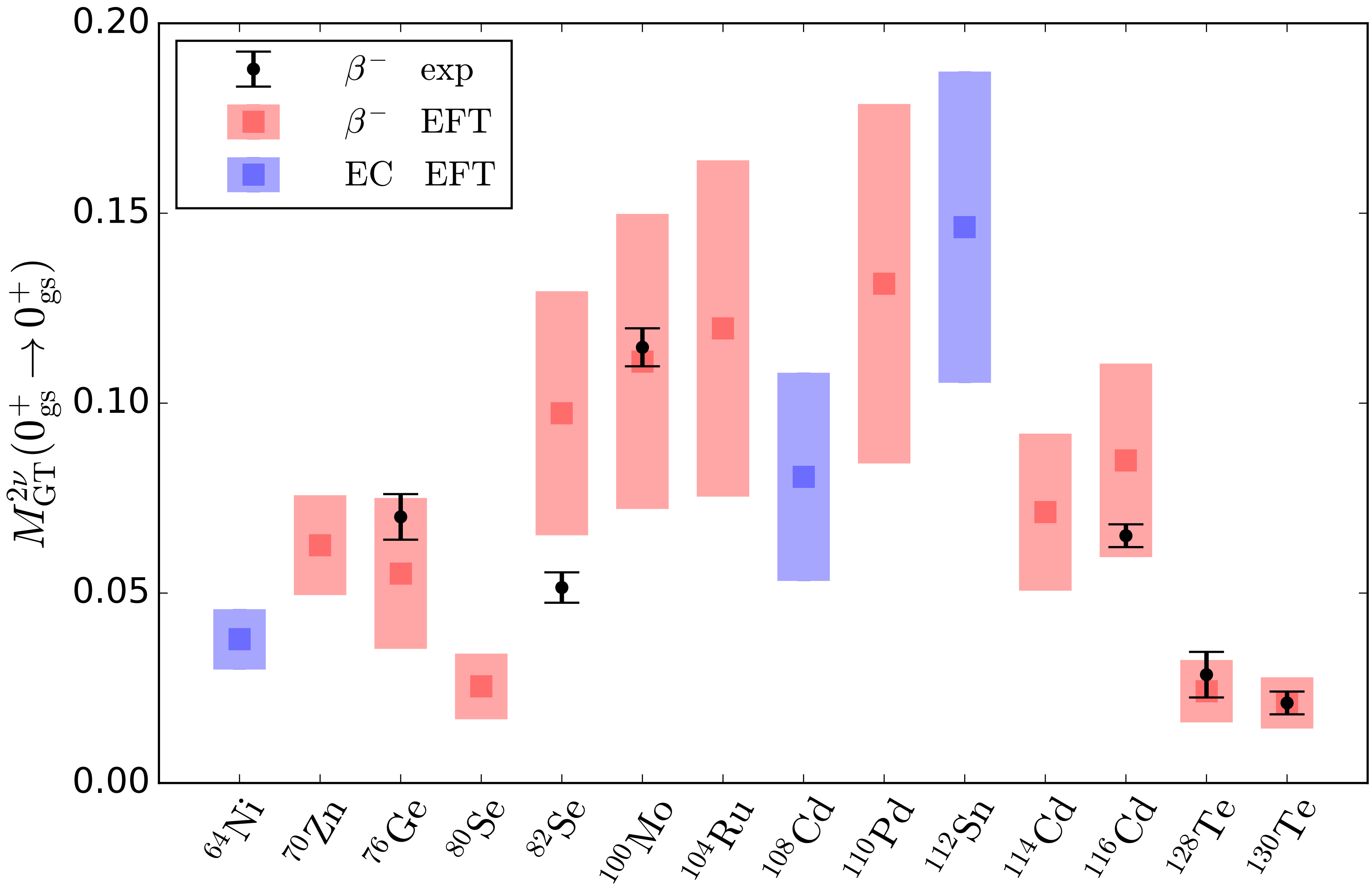
Percentual uncertainty estimate

$$\delta(\text{gs} \to \text{gs}) = \frac{D_{11}}{\Lambda} \Phi\left(\frac{\omega}{\Lambda}, 1, \frac{D_{11}+\omega}{\omega}\right)$$

where

$$\Phi(z,s,a) \equiv \sum_{n=0}^{\infty} \frac{z^n}{(a+n)^s}$$





Coello Pérez, Menéndez, Schwenk; arXiv:1708.06140

Data: Barabash, Nucl. Phys. A **935**, 52 (2015)

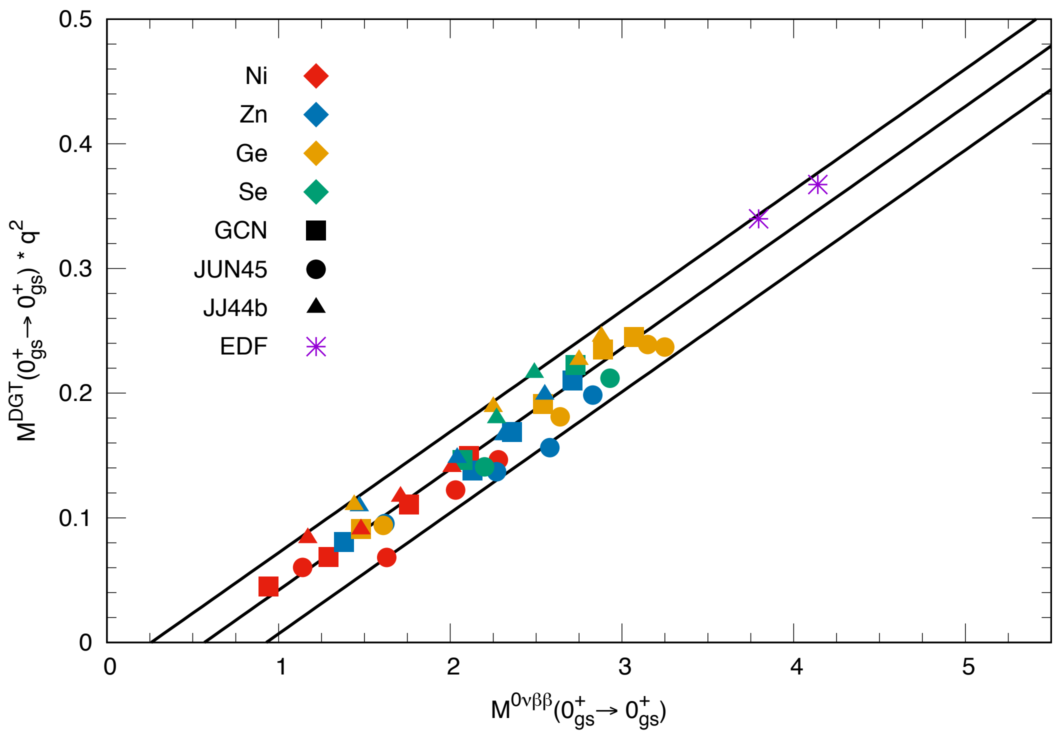
SUMMARY

Matrix elements for β and $2\nu\beta\beta$ decays from spherical nuclei can be consistently described within our simple ET when theoretical uncertainties are taken into account

OUTLOOK

NLO corrections to the Hamiltonian for the odd-odd system and the effective GT operator are expected to decrease the uncertainty by a factor of 1/3. These corrections are feasible

The correlation between the matrix element of the double Gamow-Teller operator and the matrix element for $0\nu\beta\beta$ decay can be employed to calculate the later matrix element within the ET, thus providing a matrix element for this decay with uncertainty estimate .



Thanks