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Abstract. The fusion of trust relevant information provided by multiple sources is one of the major challenges of trust establishment, which in turn is a key research topic in the growing field of cloud computing. We present a novel fusion operator for combining information from different sources, representing propositions under uncertainty. The operator especially extend the state-of-the-art by explicitly considering weights and the handling of conflicting dependent opinions. We provide a use case that demonstrates the applicability of our approach and shows the capability of the novel operator to a more reliable and transparent assessment of the trustworthiness of cloud providers.

1 Introduction

Trust establishment is considered to be a major enabler for unfolding the potential of cloud computing. Currently, potential users (e.g., enterprises, governments, individuals) of cloud services often feel that they lose the control of their data and they are not sure whether they can trust the providers. A recent survey [1] shows the growing concerns of the users about cloud providers regarding their outsourced data. These concerns of the users represent considerable obstacles for the acceptance and market success of cloud services.

Cloud providers provide assurances about the services and security measures in terms of service level agreements (SLAs). SLAs, written with legal jargon, are meant to protect the providers and not the cloud users [2]. In a recent survey [3], 46.6% of cloud users quote the legal contents of the SLAs as unclear, while only 29.3% users quote the opposite. Although cloud providers are using SLAs to advertise their competence and capabilities, potential customers still hesitate to consider them a basis for identifying dependable and trustworthy providers.

To overcome this lack of trust, a couple of initiatives have been launched, for example, *(i)* CloudCommons provides a marketplace where users provide detailed information on the competencies of the cloud providers and *(ii)* the Consensus Assessment Initiative (CAI) questionnaire [4] by the Cloud Security Alliance (CSA) asks the cloud providers for a detailed self-assessment of their security controls.

Finally, there are other possible ways to assess the trustworthiness of cloud providers, e.g., *(i)* using property-based attestation to assess the trustworthiness of subsystems and components underlying the offered services, *(ii)* taking users' feedback into account to assess the overall reputation of a cloud provider, and *(iii)* asking for expert assessments.

We conclude that when assessing trustworthiness of a cloud provider, the customers are supported best, if they can consider multiple attributes (e.g., security, availability, and functionality depending on their requirements) and take into account information related to attributes from multiple sources. To this end, a metric is required and in particular, operators that provide means for the fusion of the available information. The operators should hold even under uncertainty (in the sense of incomplete or unreliable information) and conflict (in the sense of contradictory information).

In recent publications [5, 6], the authors have already provided a formal approach for modelling and assessing the trustworthiness of complex systems. The formal approach is applicable for combining opinions – from now onwards, we refer to the information provided by a source as an opinion – that are considered to be independent (like on the availability of the service and on the quality of the customer support). In this paper, we extend the approach with operators providing a way for aggregating dependent opinions. Dependent opinions are based on observations of identical events by multiple sources. These observations regard a specific attribute of a cloud service, or a combination of attributes in the form of logical propositions. In particular, we extend the state-of-the-art by providing means for taking into account *(i)* the preferences of the customers regarding which opinions should be given a higher weight as well as *(ii)* modelling and expressing the degree of conflict of a set of opinions. Finally, we discuss the applicability and capabilities of the fusion operators in the use case of assessing the trustworthiness of cloud providers. However, the operators themselves are not restricted to this field by any means.

The rest of the paper is organized as follows: Section 2 presents the related work, Section 3 discusses modelling trustworthiness in cloud computing with a cloud marketplace use case, Section 4 presents the definitions of the fusion operators, Section 5 provides the rationale behind the definitions of the operators, Section 6 exemplify the impact of the operators on opinions. Finally, we evaluate the use case in Section 7 and draw our conclusion in Section 8.

2 Related Work

There are several approaches and trends for establishing trust on service providers in cloud computing marketplaces (or service marketplaces in general). We discuss these approaches in two subsequent sections: 1) Applied trends and 2) Research trends. The applied trends especially shows that there are plenty of sources which should be considered when evaluating the trustworthiness of a cloud provider.

Applied Trends

SLAs: In practice, one way to establish trust for cloud providers is the fulfilment of SLAs. SLA validation [7] and monitoring [8] schemes are used to quantify what exactly a cloud provider is offering and which assurances are actually met. These schemes are complimentary when SLAs are considered as one of the sources of trust information for establishing trust on cloud providers.

Audits: Cloud providers use different audit standards (e.g., SAS 70 II, FISMA, ISO 27001) to assure users about their offered services and platforms. These audit standards are used as one of the trust indicators by the cloud providers to ensure consumers about security and privacy measures.

Ratings & Measurements: There are numerous commercial portals with integrated trust and reputation systems (e.g., eBay, Epinions, RateMDs) that provide means for identifying reliable and trustworthy products and services. Most of these systems rely on user feedback and recommendations to evaluate a particular entity and do not consider technical details or the composition of the service. Recently, a cloud marketplace (CloudCommons)[1] was launched to support the users in identifying reliable cloud providers. Here, cloud providers are rated based on a questionnaire that needs to be filled in by current cloud users. In the future, CloudCommons aims to combine user feedback with technical measurements for assessing and comparing the trustworthiness of cloud providers. Hence, measurement tools and recommendation platforms are important sources for extracting trust information about the cloud providers.

Self-assessment Questionnaire: The Cloud Security Alliance (CSA) proposed a detailed questionnaire for providing security control transparency – called the Consensus Assessment Initiative (CAI) questionnaire [4]. This questionnaire provides means for assessing the capabilities and competencies of cloud providers in terms of different attributes (e.g., compliance, information security, governance). One can extract trust information by assessing the completed questionnaire and consider that information for evaluating trustworthiness of cloud providers.

Research Trends

Commercial platform providers become more and more aware that trust establishment is an important issue. They are also aware that trust is not only related to the technical enforcement for security mechanisms but also involves taking into account user ratings and providing transparency. The scientific research community is already a big step ahead, especially with regard to formal models and metrics of trust.

Trust Models and Uncertainty: In the field of trust modelling, there are a number of approaches modelling trust and especially the (un-)certainty of a trust value, well-known approaches are given in [9, 10, 10–16]. However, these approaches do not tackle the issue of deriving the trustworthiness of a service

[1] http://beta-www.cloudcommons.com/web/cc/about-smi

provider based on the different attributes of a service. Instead, the challenge of these approaches is to find good models for deriving trust from direct experience of a user, recommendations from third parties, and also additional information, e.g., social relationships. Especially, those models aim on providing robustness to attacks, e.g., misleading recommendations, re-entry, Sybil attacks, etc. For these tasks, they usually provide operators for combining evidences from different sources about the same target (also called consensus or aggregation) and for weighing recommendations based on the trustworthiness of the source (also called discounting or concatenation). However, the goal of these existing approaches is not to provide operators for the evaluation of propositions associated with opinions.

Trust Operators for Evaluating Propositions: In the field of trust, there are researchers who proposed operators for combining different properties (or more precisely opinions on different propositions) under (un-)certainty [6,14,17]. They proposed a set of operators (i.e., AND, OR, NOT) for evaluating propositions associated with opinions. These operators are only able to evaluate and combine opinions on independent propositions. Moreover, *subjective logic* provides a set of operators [18, 19] that are able to aggregate dependent opinions; particularly, the *averaging fusion* operator and the *consensus operator for dependent opinions*. These kind of operators are commonly used as an aggregation function for group decision making (e.g., group of n experts provide n opinions) for constructing a final score (e.g., trust score) [20]. Both of those operators, proposed in [18,19], have the limitation that it is not possible to address conflict among opinions (which leads to a high degree of ambiguity after aggregation).

To overcome the limitations, we introduce the operator for *conflict-aware fusion* based on a previously established representation of trust, named *CertainTrust* [14]. In [14], it has been shown that there exists a bijective mapping between the *CertainTrust*'s representation of an opinion and *subjective logic*'s representation of a binomial opinion; thus, we say the representations are equivalent. Both models provide three degrees of freedom related to an initial expectation, the quality of past observations and the associated (un-)certainty. We choose *CertainTrust*, as this representation is built on independent parameters reflecting the (relative) quality of past observations (average rating) and the associated (un-)certainty; in particular those parameters can be independently assessed and interpreted[2]. Furthermore, *CertainTrust* provides a simple graphical representation (i.e., HTI). The parameters of binomial opinions in *subjective logic* (belief b, disbelief d, and uncertainty u) are interrelated by $b + d + u = 1$. This has as a consequence that the range of possible values for each parameter depends on the actual values of the other parameters, e.g., from $u = 0.8$ it follows b (or d) can only be chosen in the range of $[0, 0.2]$. Furthermore, binomial opinions can also be visualized in a quantitative way using the opinion triangle.

In this paper, we benefit from the equivalence between the both representations as it provides the mathematical foundation for the average fusion operator, that we choose as a starting point..

[2]Only in the case of $c = 0$, we defined $t = 0.5$ (cf. [14])

3 Assessing the Trustworthiness in Cloud Computing

Assessing the trustworthiness of a cloud provider requires statements on the expected behaviour of the offered services or systems. The expectation of a customer can be stated in the form of different attributes a service should have. On an abstract level, those attributes can come, for instance, from the fields of security, privacy, performance, customer support, and so on. More precisely, examples for attributes can be stated as follows:

– Latency: "System A to respond within $100ms$."
– Security: "Service provider B ensures that my data is kept confidential."
– Availability: "Cloud A provides 99.99% uptime in a year."
– Customer support: "Cloud B's customer support is competent".

When modelling the trustworthiness of a cloud-based service, one can logically model the relevant attributes in the form of propositions and combine them using propositional logic. More specifically, the opinions on the fulfilment of those propositions are combined. As long as the propositions are considered to be independent, the operators for AND ($\wedge$) and OR ($\vee$) are sufficient (see [6, 21]). However, when the independence cannot be assumed (i.e., dependent propositions) those operators are no longer sufficient. For instance, this is the case when one has to combine two opinions based on the same observation made by different sources regarding a cloud provider's attributes.

The dependency among propositions as well as opinions needs further discussion. For example, if a cloud consumer wants to know whether a cloud provider is trustworthy with respect to the above mentioned attributes, the consumer can derive opinions from different sources. If these sources (e.g., providers, consumers, accreditators, experts) observe the same attributes using similar methods and their estimates are equal, it is enough to take only one of the estimates into account. However, the sources may have missed or misinterpreted certain events of the same observation processes, which can produce varying resulting opinions. Thus, while the individual opinions about the propositions (i.e., attributes) vary from source to source, they are still dependent.

In the following, we provide a use case that shows why the consideration of different sources is important for trust assessment. The use case is a cloud marketplace where cloud providers act as sellers and cloud users act as buyers.

3.1 Use Case – Cloud Marketplace

In our use case (cf. Figure 1), the main objective of cloud marketplace is to offer cloud services to the users as well as to support them in selecting trustworthy cloud providers. The cloud marketplace aims to identify the trustworthy cloud providers by using a reliable and transparent mechanism for assessing their trustworthiness (e.g., of Cloud A). To keep the scenario simple, we will deal with one cloud provider (Cloud A), the cloud users, and four sources of opinions.

When joining the marketplace, Cloud A has to fill in a questionnaire on its competencies (i.e., CAIQ), as designed, verified and published by CSA in

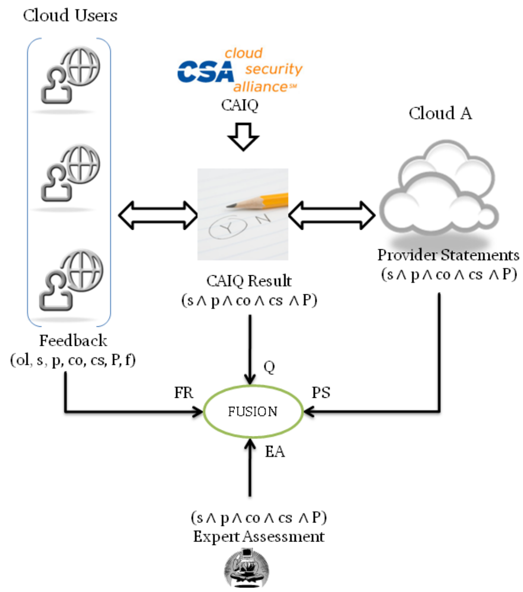


Fig. 1. Cloud Marketplace – Trust Assessment with Multiple Sources

STAR[3], to be able to act as a seller in the cloud marketplace. Cloud A also publishes service level agreements (SLAs) as a part of its "provider statements". To ensure a reliable assessment of trustworthiness of the cloud provider (Cloud A), the users incorporate opinions from multiple sources, e.g., collecting expert assessments (EA), and feedback and recommendations by other users (FR) in addition to the provider statements (PS) and the questionnaire Q.

We assume that all these opinions about Cloud A's overall trustworthiness (modelled as propositions) are extracted from different parties. The propositions are modelled in terms of the previously introduced attributes: quality of the customer support (cs), security (s) and privacy (p) measures, performance (P), compliance (co) and functionality (f). Alternatively, they can also be given as an overall statement on the trustworthiness of the cloud provider (ol).

In our example, the opinions (derived from expert assessment, provider statements and questionnaire) on the fulfilment of those propositions are combined using CertainLogic AND operator (i.e., ($s \wedge p \wedge co \wedge cs \wedge P$)). Users' opinions on the above mentioned attributes can be an overall rating (ol) or individual feedback on each of the attributes. A number of users feedback on different attributes are assumed to be combined using consensus operator [14] and we denote the construction as (ol, s, p, co, cs, P, f) in Figure 1.

Finally, when combining the opinions (on the fulfilment of the propositions) from those different sources, the users may prefer one source over the other. In

[3] https://cloudsecurityalliance.org/research/initiatives/star-registry/

our use case, we assume that users put higher weights on ER, FR and Q than PS based on their preferences.

The aggregation (in the following called *fusion*) of opinions from different sources is especially challenging, as those opinions from the different sources may be conflicting, it may based on incomplete information or unreliable sources, and thus, it is subject to uncertainty. Therefore, the evaluation mechanism (i.e., fusion operation) should reflect the preferences, degree of conflict (DoC) and the uncertainty when combining multiple opinions (on propositions) to calculate the overall trustworthiness of Cloud A.

4 A New Model for the Fusion of Opinions

When modelling trust we consider that the trust-relevant information is subject to uncertainty. Therefore, we model trust as a subjective probability, which goes along with the definition of trust provided in [22]. Particularly, we use the representation that has been proposed with *CertainTrust* [23] and *CertainLogic* [6]. In these models, the truth of a proposition is expressed by a construct called an *opinion*[4]. An opinion o is defined as a triple of values, $o = (t, c, f) \in \{[0, 1] \times [0, 1] \times [0, 1]\}$, where t denotes the average rating, c the certainty associated with the average rating, and f denotes the initial expectation assigned to the truth of the statement[5]. As shown in [5,23], the assessment of the parameters can be based on evidence from past experience, based on expert assessments, derived from opinions in subjective logic [17], or derived from a Bayesian probability distribution.

Each opinion $o = (t, c, f)$ is also associated with a expectation value, i.e., a point estimate, taking into account the initial expectation f, the average rating t, and the certainty c as follows:

$$E(t, c, f) = t * c + (1 - c) * f \tag{1}$$

Thus, the expectation value shifts from the initial expectation value f to the average rating t with increasing certainty c.

Beyond providing means for explicitly modelling uncertainty, the metric also provides a graphical representation (named the Human Trust Interface (HTI)), which supports an intuitive access for users (see Section 6).

In the following, we define the novel operators for the fusion of dependent opinions.

4.1 Definition of the Fusion Operators

We provide three types of fusion operators, i.e., operators that are suitable for aggregating dependent opinions on a single proposition. At first, we introduce

[4] Thus, the informal notion of an opinion is similar to the way the term opinion was used before.

[5] For a detailed introduction of this representation please have a look at [5,23].

Table 1. Definition of the Average Fusion Operator

$$t_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)} = \begin{cases} \dfrac{\sum_{i=1}^{n} t_{A_i}}{n} & \text{if } c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 1\ , \\ 0.5 & \text{if } c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 0\ , \\ \dfrac{\sum_{i=1}^{n} (c_{A_i} t_{A_i} \prod_{j=1,\ j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n} (c_{A_i} \prod_{j=1,\ j\neq i}^{n} (1 - c_{A_j}))} & \text{if } \{c_{A_i}, c_{A_j}\} \neq 1\ . \end{cases}$$

$$c_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)} = \begin{cases} 1 & \text{if } c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 1\ , \\ \dfrac{\sum_{i=1}^{n} (c_{A_i} \prod_{j=1,\ j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n} (\prod_{j=1,\ j\neq i}^{n} (1 - c_{A_j}))} & \text{if } \{c_{A_i}, c_{A_j}\} \neq 1\ . \end{cases}$$

$$f_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)} = \frac{\sum_{i=1}^{n} f_{A_i}}{n}$$

the *average fusion* operator. This operator is equivalent[6] to the *averaging fusion* operator [18] and *consensus operator for dependent opinions* [19] defined in Jøsang's subjective logic. The equivalence serves as an argument for the mathematical validity of our *average fusion* operator that we use as a starting point for introducing a novel fusion operator. This operator (i.e., conflict-aware fusion) is capable of dealing with conflict as well as preferences (as weights). Note that the *weighted fusion*[7] operator is an intermediate step towards defining the novel *conflict-aware fusion* operator.

Definition 4.1 (*A.FUSION*)
*Let A be a proposition and let $o_{A_1} = (t_{A_1}, c_{A_1}, f_{A_1})$, $o_{A_2} = (t_{A_2}, c_{A_2}, f_{A_2})$, ..., $o_{A_n} = (t_{A_n}, c_{A_n}, f_{A_n})$ be n opinions associated to A. The **average fusion** is denoted as $o_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)} = (t_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)}, c_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)}, f_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)})$ where $t_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)}$, $c_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)}$, $f_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)}$ are defined in Table 1. We use the symbol ($\widehat{\oplus}$) to designate the operator $A.FUSION$ and we define $o_{\widehat{\oplus}(A_1,A_2,\ldots,A_n)} \equiv \widehat{\oplus}((o_{A_1}), (o_{A_2}), \ldots, (o_{A_n}))$.*

Definition 4.2 (*W.FUSION*)
*Let A be a proposition and let $o_{A_1} = (t_{A_1}, c_{A_1}, f_{A_1})$, $o_{A_2} = (t_{A_2}, c_{A_2}, f_{A_2})$, ..., $o_{A_n} = (t_{A_n}, c_{A_n}, f_{A_n})$ be n opinions associated to A. Furthermore, the weights w_1, w_2, ..., w_n (with $w_1, w_2, \ldots, w_n \in \mathbb{R}_0^+$ and $w_1 + w_2 + \ldots + w_n \neq 0$) are assigned to the opinions o_{A_1}, o_{A_2}, ..., o_{A_n}, respectively. The **weighted fusion** is denoted as $o_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)} = (t_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)}, c_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)}, f_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)})$*

[6] A sketch of the proof is given in Appendix I. The proof is based on the bijective mapping between the both representations; note that [18] only defines binary operators.

[7] This weighted fusion differs from the fusion operator that was recently proposed in [24], as they consider two weights in their definition: one weight from the agent who provide the opinion and other weight from the agent who fuse the weighted opinions.

Table 2. Definition of the Weighted Fusion Operator

$$t_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)} = \begin{cases} \dfrac{\sum_{i=1}^{n} w_i t_{A_i}}{\sum_{i=1}^{n} w_i} & \text{if } c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 1 \;, \\ 0.5 & \text{if } c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 0 \;, \\ \dfrac{\sum_{i=1}^{n} (c_{A_i} t_{A_i} w_i \prod_{j=1,\, j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n} (c_{A_i} w_i \prod_{j=1,\, j\neq i}^{n} (1 - c_{A_j}))} & \text{if } \{c_{A_i}, c_{A_j}\} \neq 1 \;. \end{cases}$$

$$c_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)} = \begin{cases} 1 & \text{if } c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 1 \;, \\ \dfrac{\sum_{i=1}^{n} (c_{A_i} w_i \prod_{j=1,\, j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n} (w_i \prod_{j=1,\, j\neq i}^{n} (1 - c_{A_j}))} & \text{if } \{c_{A_i}, c_{A_j}\} \neq 1 \;. \end{cases}$$

$$f_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)} = \frac{\sum_{i=1}^{n} w_i f_{A_i}}{\sum_{i=1}^{n} w_i}$$

where $t_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)}$, $c_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)}$, $f_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)}$ *are defined in Table 2. We use the symbol* ($\widehat{\oplus}_w$) *to designate the operator W.FUSION and we define* $o_{\widehat{\oplus}_w(A_1,A_2,\ldots,A_n)} \equiv \widehat{\oplus}_w((o_{A_1}, w_1), (o_{A_2}, w_2), \ldots, (o_{A_n}, w_n))$.

Definition 4.3 (*C.FUSION*)
Let A be a proposition and let $o_{A_1} = (t_{A_1}, c_{A_1}, f_{A_1})$, $o_{A_2} = (t_{A_2}, c_{A_2}, f_{A_2})$, ..., $o_{A_n} = (t_{A_n}, c_{A_n}, f_{A_n})$ *be n opinions associated to A. Furthermore, the weights* w_1, w_2, ..., w_n *(with* $w_1, w_2, \ldots, w_n \in \mathbb{R}_0^+$ *and* $w_1+w_2+\ldots+w_n \neq 0$*) are assigned to the opinions* o_{A_1}, o_{A_2}, ..., o_{A_n}, *respectively. The* ***conflict-aware fusion*** *is denoted as*

$$o_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)} = ((t_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}, c_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}, f_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}), DoC)$$

where $t_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}$, $c_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}$, $f_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}$, *the degree of conflict DoC are defined in Table 3. We use the symbol* ($\widehat{\oplus}_c$) *to designate the operator C.FUSION and we define* $o_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)} \equiv \widehat{\oplus}_c((o_{A_1}, w_1), (o_{A_2}, w_2), \ldots, (o_{A_n}, w_n))$.

In Table 1, 2 and 3, for all opinions if it holds $c_{A_i} = 0$ (complete uncertainty), the expectation values (cf. 1) depends only on f. However, for soundness we define $t_{A_i} = 0.5$ in this case.

5 Properties and Rationale for the Operators

The goal of this paper is to extend the functionality of the *consensus operator for dependent opinions* and *averaging fusion* operators presented in [18, 19]

Table 3. Definition of the Conflict-aware Fusion Operator

$$t_{\hat{\oplus}_c(A_1,A_2,\ldots,A_n)} = \begin{cases} \dfrac{\sum_{i=1}^{n} w_i t_{A_i}}{\sum_{i=1}^{n} w_i} & \text{if } c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 1 \ , \\ 0.5 & \text{if } c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 0 \ , \\ \dfrac{\sum_{i=1}^{n} (c_{A_i} t_{A_i} w_i \prod_{j=1,\ j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n} (c_{A_i} w_i \prod_{j=1,\ j\neq i}^{n} (1 - c_{A_j}))} & \text{if } \{c_{A_i}, c_{A_j}\} \neq 1 \ . \end{cases}$$

$$c_{\hat{\oplus}_c(A_1,A_2,\ldots,A_n)} = \begin{cases} 1 * (1 - DoC) & \text{if } c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 1 \ , \\ \dfrac{\sum_{i=1}^{n} (c_{A_i} w_i \prod_{j=1,\ j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n} (w_i \prod_{j=1,\ j\neq i}^{n} (1 - c_{A_j}))} * (1 - DoC) & \text{if } \{c_{A_i}, c_{A_j}\} \neq 1 \ . \end{cases}$$

$$f_{\hat{\oplus}_c(A_1,A_2,\ldots,A_n)} = \frac{\sum_{i=1}^{n} w_i f_{A_i}}{\sum_{i=1}^{n} w_i}$$

$$DoC = \frac{\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} DoC_{A_i,A_j}}{\frac{n(n-1)}{2}}$$

$$DoC_{A_i,A_j} = \left| t_{A_i} - t_{A_j} \right| * c_{A_i} * c_{A_j} * \left(1 - \left| \frac{w_i - w_j}{w_i + w_j} \right| \right)$$

with regard to preferential weighting and conflict awareness. Furthermore, the operators are designed to be compatible with CertainTrust [14] representation.

At first, we outline the necessary and desirable mathematical properties regarding our designed operators. Afterwards, we provide the rationale behind the definition of the *conflict-aware* fusion operator. Note that this operator can also handle preferential weights.

5.1 Properties of the Operators

We characterize the desirable properties in two groups: i) Fusion-specific and ii) Weight-specific. The *Fusion-specific* properties are the ones which are shown desirable and necessary for the state-of-the-art fusion operators [18, 19]. The *Weight-specific* properties are useful to show the relationship among the average, weighted and conflict-aware fusion, that also extend to easier computation of the expectation value E (cf. equation 1) of fused opinions. Moreover, these properties are aligned with the desirable properties for arithmetic mean-based averaging operations [20]. As fusion operation belongs to the family of arithmetic mean-based averaging operations [20], those particular properties are also desirable for our extended fusion operators.The properties that hold for our defined operators are outlined as follows:

1. Fusion-specific Properties: *Idempotency*, *Commutativity* & *Permutability* belong to this group.
2. Weight-specific Properties: *Weight Partitioning*, *Invariance to Weight Scaling* and three properties regarding *Weighted average of expectation value for common weight and/or certainty* belong to this particular group.

The formal theorems regarding the properties are discussed in the following. The proofs for the theorems are provided in Appendix A–H.

Fusion-specific Properties

Idempotence: When aggregating the same opinion twice, no additional information is gained for the resulting fused opinion. This should be reflected in the fusion operation by designing it to be idempotent. Formally, the following theorem 5.1 thus represents a desirable property of the fusion operator that holds for average, weighted and conflict-aware fusion.

Theorem 5.1 (Idempotence)
It holds $\widehat{\oplus}(o_{A_1}, o_{A_1}, \cdots, o_{A_1}) = o_{A_1}$ *and* $\widehat{\oplus}_w(o_{(A_1,w_1)}, o_{(A_1,w_2)}, \cdots, o_{(A_1,w_n)}) = o_{A_1}$ *and* $\widehat{\oplus}_c(o_{(A_1,w_1)}, o_{(A_1,w_2)}, \cdots, o_{(A_1,w_n)}) = o_{A_1}$.

Commutativity and Permutability: In averaging operations, the order of the operands should not affect the final outcome of the calculation. Therefore, the extended fusion operators are designed to be commutative as well as indifferent to a permutation of the operands. This makes them compliant with the following two theorems 5.2 and 5.3.

Theorem 5.2 (Commutativity)
For two opinions, it holds
$\widehat{\oplus}(o_{A_1}, o_{A_2}) = \widehat{\oplus}(o_{A_2} o_{A_1})$
$\widehat{\oplus}_w((o_{A_1}, w_1), (o_{A_2}, w_2)) = \widehat{\oplus}_w((o_{A_2}, w_2), (o_{A_1}, w_1))$
$\widehat{\oplus}_c((o_{A_1}, w_1), (o_{A_2}, w_2)) = \widehat{\oplus}_c((o_{A_2}, w_2), (o_{A_1}, w_1))$.

Theorem 5.3 (Permutability of n opinions)
Let $\pi : [1, \ldots, n] \mapsto [1, \ldots, n]$ *denote a permutation such that* $o_{A_{\pi(i)}} = o_{A_i}$, *then it holds*
$\widehat{\oplus}_w(o_{A_1}, ..., o_{A_n}) = \widehat{\oplus}_w(o_{A_{\pi(1)}}, ..., o_{A_{\pi(n)}})$
$\widehat{\oplus}_c(o_{A_1}, ..., o_{A_n}) = \widehat{\oplus}_c(o_{A_{\pi(1)}}, ..., o_{A_{\pi(n)}})$

In this regard, one can argue that the associativity property is also desirable. But, it is not desirable as the defined operations for the fusion operators belong to the family of arithmetic mean based averaging operation. Note that for general arithmetic mean based averaging operations, associativity is not a desirable property [20].

5.2 Weight-specific Properties

The fusion operators fulfil a number of useful properties regarding the relationship between average and weighted fusion, that also extend to easier computation of the expectation value E (cf. equation 1) of fused opinions. In the following, we consider primarily *weighted fusion* with weight $w_i \in \mathbb{R}_0^+$, $0 < i \leq n$, where $i, n \in \mathbb{N}$ and $\sum_{i=1}^{n} w_i \neq 0$.

Weighted Fusion Partitioning to Average Fusion: For rational weights, the weighted fusion operator ($W.FUSION$) and ($C.FUSION$) is isomorphic to the average fusion operator ($A.FUSION$).

Theorem 5.4 (Weight Partitioning)
For $w_1 = \frac{a_1}{b_1}, w_2 = \frac{a_2}{b_2}, \cdots, w_n = \frac{a_n}{b_n} \in \mathbb{Q}_0^+$ *it holds*

1. $\widehat{\oplus}_w((o_{A_1}, w_1), (o_{A_2}, w_2), \cdots, (o_{A_n}, w_n)) =$
$\widehat{\oplus}_w(\underbrace{(o_{A_1}, 1), \cdots, (o_{A_1}, 1)}_{a_1 \cdot \prod_{j\neq 1} b_j \textit{ times}}, \underbrace{(o_{A_2}, 1), \cdots, (o_{A_2}, 1)}_{a_2 \cdot \prod_{j\neq 2} b_j \textit{ times}}, \cdots, \underbrace{(o_{A_n}, 1), \cdots, (o_{A_n}, 1)}_{a_n \cdot \prod_{j\neq n} b_j \textit{ times}}) =$
$\widehat{\oplus}(\underbrace{o_{A_1}, \cdots, o_{A_1}}_{a_1 \cdot \prod_{j\neq 1} b_j \textit{ times}}, \underbrace{o_{A_2}, \cdots, o_{A_2}}_{a_2 \cdot \prod_{j\neq 2} b_j \textit{ times}}, \cdots, \underbrace{o_{A_n}, \cdots, o_{A_n}}_{a_n \cdot \prod_{j\neq n} b_j \textit{ times}})$
2. $\widehat{\oplus}_c((o_{A_1}, w_1), (o_{A_2}, w_2), \cdots, (o_{A_n}, w_n)) =$
$\widehat{\oplus}_c(\underbrace{(o_{A_1}, 1), \cdots, (o_{A_1}, 1)}_{a_1 \cdot \prod_{j\neq 1} b_j \textit{ times}}, \underbrace{(o_{A_2}, 1), \cdots, (o_{A_2}, 1)}_{a_2 \cdot \prod_{j\neq 2} b_j \textit{ times}}, \cdots, \underbrace{(o_{A_n}, 1), \cdots, (o_{A_n}, 1)}_{a_n \cdot \prod_{j\neq n} b_j \textit{ times}}) =$
$\widehat{\oplus}(\underbrace{o_{A_1}, \cdots, o_{A_1}}_{a_1 \cdot \prod_{j\neq 1} b_j \textit{ times}}, \underbrace{o_{A_2}, \cdots, o_{A_2}}_{a_2 \cdot \prod_{j\neq 2} b_j \textit{ times}}, \cdots, \underbrace{o_{A_n}, \cdots, o_{A_n}}_{a_n \cdot \prod_{j\neq n} b_j \textit{ times}})$

Invariance to Weight Scaling: As a weighted aggregation function, the weighted fusion operation ($W.FUSION$) and conflict-aware fusion operation ($C.FUSION$) is invariant to scaling of its weight terms by a constant.

Theorem 5.5 (Invariance to Weight Scaling)
$\forall k \neq 0$ *it holds*

1. $\widehat{\oplus}_w((o_{A_1}, w_1{*}k), (o_{A_2}, w_2{*}k), \cdots, (o_{A_n}, w_n{*}k)) = \widehat{\oplus}_w((o_{A_1}, w_1), (o_{A_2}, w_2), \cdots, (o_{A_n}, w_n))$
2. $\widehat{\oplus}_c((o_{A_1}, w_1{*}k), (o_{A_2}, w_2{*}k), \cdots, (o_{A_n}, w_n{*}k)) = \widehat{\oplus}_c((o_{A_1}, w_1), (o_{A_2}, w_2), \cdots, (o_{A_n}, w_n))$

Weighted average of expectation value for common weight or common certainty: The primary decision criterion in CertainTrust [14] is the expected trust value $E(t, c, f) = t{*}c + (1-c){*}f$ (cf. equation 1)associated with an opinion $o = (t, c, f)$. Thus, in many cases, the computation of this expectation value is the final objective subsequent to applying the fusion operators. As $E \in \mathbb{R}_0^+$, averaging operations conducted on the expectation values using arithmetic operations, as opposed to the opinions using fusion, are computationally preferable. The following theorems 5.6, 5.7, 5.8 outline under which conditions this is possible.

Theorem 5.6 (Weighted average of expectation value for common weight w)

For $w_1 = w_2 = \cdots = w_n = w$ it holds
$E(\widehat{\oplus}_w((o_{A_1}, w), (o_{A_2}, w), \cdots, (o_{A_n}, w))) = E(\widehat{\oplus}(o_{A_1}, o_{A_2}, ..., o_{A_n})) = \frac{\sum_{i=1}^{n} E(o_{A_i})}{n}$.

Theorem 5.7 (Weighted average of expectation value for common certainty c)

For $c_{A_1} = c_{A_2} = \cdots = c_{A_n} = c$, it holds
$E(\widehat{\oplus}_w((o_{A_1}, w_1), (o_{A_2}, w_2), ..., (o_{A_n}, w_n))) = \frac{\sum_{i=1}^{n} w_i E(o_{A_i})}{\sum_{i=1}^{n} w_i}$.

Weighted average of expectation value for common certainty and common weights: When defining the $W.FUSION$ and $C.FUSION$ we made sure that whenever one uses identical certainty values (i.e., $c_{A_1} = c_{A_2}$ or $c_{A_1} = c_{A_2} = \cdots = c_{A_n}$) and weights (i.e., $w_1 = w_2$ or $w_1 = w_2 = \cdots = w_n$), the result is the same as the result of the average fusion (cf. Thm. 5.6).

Theorem 5.8 (Weighted average of expectation value for common c and w)

For $w_1 = w_2 = \cdots = w_n = w$ and $c_{A_1} = c_{A_2} = \cdots = c_{A_n} = c$, it holds

1. $\frac{\sum_{i=1}^{n} E(o_{A_i})}{n} = E(\widehat{\oplus}_w((o_{A_1}, w), (o_{A_2}, w), \cdots, (o_{A_n}, w)))$
2. $\frac{\sum_{i=1}^{n} E(o_{A_i})}{n} = E(\widehat{\oplus}_c((o_{A_1}, w), (o_{A_2}, w), \cdots, (o_{A_n}, w)))$

5.3 Rationale for the Conflict-aware Fusion Operator

The rationale behind the definition of the *conflict-aware* fusion needs extensive discussion. The basic concept of this operator is as follows: the operator extends the *weighted fusion* by calculating the degree of conflict (DoC) between two input opinions. Then, the value of $(1 - DoC)$ is multiplied with the certainty (c) that would be calculated by the weighted fusion (the parameters for t and f are the same as in the weighted fusion).

Now, we discuss the calculation of the DoC for two opinions. For the parameter, it holds $DoC \in [0, 1]$. This parameter depends on the average ratings (t), the certainty values (c), and the weights (w). The weights are assumed to be selected by the users and the purpose of the weights is to model the preferences of the user when aggregating opinions from different sources. We assume that the compliance of their preferences are ensured under a policy negotiation phase. For example, users might have given three choices: High (2), Low (1) and No preference (0) (opinion from a particular source is not considered), to express their preference on the sources from which the opinions are extracted. Note that the weights are not introduced to model the reliability of sources. In this case, it would be appropriate to use the discounting operator [14, 17] to explicitly consider reliability of sources and apply the fusion operator on the results to influence users' preferences. The values of DoC can be interpreted as follows:

- **No conflict ($DoC = 0$):** For $DoC = 0$, it holds that there is *no conflict* between the two opinions. This is true if both opinions agree on the average rating, i.e., $t_{A_1} = t_{A_2}$ or in case that at least one opinion has a certainty $c = 0$ (for completeness we have to state that it is also true if one of the weights is equal to 0, which means the opinion is not considered).
- **Total conflict ($DoC = 1$):** For $DoC = 1$, it holds that the two opinions are weighted equally ($w_1 = w_2$) and contradicts each other to a maximum. This means, that both opinions have a maximum certainty ($c_{A_1} = c_{A_2} = 1$) and maximum divergence in the average ratings, i.e., $t_{A_1} = 0$ and $t_{A_2} = 1$ (or $t_{A_1} = 1$ and $t_{A_2} = 0$).
- **Conflict ($DoC \in]0, 1[$):** For $DoC \in]0, 1[$, it holds that there are two opinions contradict each other to a certain degree. This means that the both opinions does not agree on the average ratings, i.e., $t_{A_1} \neq t_{A_2}$, having certainty values other than 0 and 1. The weights can be any real number other than 0.

Next, we argue for integrating the degree of conflict (DoC) into the resulting opinion by multiplying the certainty with $(1 - DoC)$. The argument is, in case that there are two (equally weighted) conflicting opinions, then this indicates that the information which these opinions are based on is not representative for the outcome of the assessment or experiment. Thus, for the sake of representativeness, in case of total conflict (i.e., $DoC = 1$), we reduce the certainty ($c_{(o_{A_1}, w_1)\widehat{\oplus}(o_{A_2}, w_2)}$) of the resulting opinion by a multiplicative factor, $(1 - DoC)$ (i.e., the certainty is 0).

For n opinions, degree of conflict (i.e., DoC_{A_i,A_j}) in Table 3 is calculated for each opinion pairs. The challenge is how to calculate the DoC among n opinions to adjust the certainty ($c_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}$) parameter of the resulting opinion. There are three possible ways that we have considered when calculating the DoC. These are as follows:

- One of the ways is to calculate the average of all possible DoC_{A_i,A_j} values of all pairs. For instance, if there are n opinions there can be at most $\frac{n(n-1)}{2}$ pairs and degree of conflict is calculated for each of those pairs individually. Finally, all the pair-wise DoC values are averaged (i.e., averaging $\frac{n(n-1)}{2}$ pairs of DoC_{A_i,A_j}) to adjust the certainty (i.e., $c_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}$) parameter of the resulting opinion (cf. Table 3).
- Another way is to calculate the degree of conflict (DoC) for each pair of opinions and adjust the certainty ($c_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}$) $\frac{n(n-1)}{2}$ times if there are n opinions. In this case, we get $\frac{n(n-1)}{2}$ certainty values which are then averaged to calculate the final certainty value.
- The other way is to calculate the degree of conflict (DoC) pair-wise and multiply all pair-wise values at once with the certainty ($c_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}$) of the resulting opinion. This approach has two drawbacks: i) it suffers from a multiplicative effect which means that the certainty is affected heavily with the increasing number of opinions, ii) it also heavily affect the certainty in case a single opinion radically conflict with others.

The first two approaches are equally capable of detecting conflicting opinions as the conflict analysis is done pair-wise. Either of these approaches performs better (in detecting conflicting information) than the third approach, especially in a complex setting where a large collection of sources are available and only one of the sources radically conflicts with the other sources when providing opinions. In this case, either of the first two approaches shifts half of the uncertainty on the outlier and others receive only $\frac{1}{2n(n-1)}$ of the extra uncertainty. Moreover, the first two approaches do not suffer from the multiplicative effect alike the third approach.

Finally, we see that the connection between the DoC and the certainty ($c_{\widehat{\oplus}_c(A_1,A_2,\ldots,A_n)}$) is linear. One can argue that this connection should be handled probabilistically rather than linearly. We choose the linear approach as it is simple, does not lead to unforeseen effects and allow good integration of weights, which is important for our cloud marketplace scenario. Moreover, due to linearity, specific *Weight-specific* properties (i.e., *Weight Partitioning*, *Invariance to Weight Scaling* and *Weighted average of expectation value for common weight and certainty*) hold for *conflict-aware* fusion operator as well. The discussion of the fusion operators is also supported by numerical and graphical (i.e., HTI) examples in the next section.

6 Examples of the Fusion Operators

In the following, we present some examples showing the impact of the newly defined operators on opinions modelled with the representation used in *CertainLogic* and *CertainTrust*.

In the left part of Table 4, all the examples show the effect of the weighted fusion ($W.FUSION$) operator in different cases. The right part of the Table 4 shows the effect of average fusion (example 1) and conflict-aware fusion (example 2 & 3). The goal is to compare the advantages and disadvantages of the operators with an intuitive graphical representation (HTI) of opinions.

In the graphical representation, the color-gradient indicates the expectation value of each point in the figure. Therefore, the color of each point in the figure is calculated as a linear combination of the RGB-vectors of red ($E = 0$), yellow ($E = 0.5$), and green ($E = 1$)[8].

Example 1: The first example in Table 4 illustrates a comparison between the $W.FUSION$ and $A.FUSION$ operators.

While for the $A.FUSION$ operator it holds that both opinions have the same impact on the results (which is equivalent to $w_1 = w_2$ in the weighted fusion), the $W.FUSION$ operator supports the customization of the weights (in the example we use, $w_1 = 1$ and $w_2 = 2$ for the weighted fusion).

In the resulting opinions, one can observe the influence of the weights. In the $A.FUSION$ (right), the resulting opinion ($(0.4, 0.75, 0.5)$) is biased to o_{A_1}

[8] We have developed a Java application for the visualization of opinions (also calculating the color-gradient of the background) and for demonstrating the *fusion* operators. The examples are screen shots from this application.

because of the high certainty (0.833) associated with the opinion o_{A_1}. However, using the $W.FUSION$ (left) and giving a higher weight ($w_2 = 2$) to o_{A_2} the resulting opinion ($(0.4717, 0.6996, 0.5)$) shows a shifted bias towards o_{A_2}. This example shows how the weighted fusion enables the customization.

Example 2: The second example in Table 4 provides an interesting comparison between the *weighted fusion* (on the left) and the *conflict-aware fusion* on the right. Both cases we combine two opinions with maximum certainty but with conflicting average ratings, i.e., $o_{A_1} = (0, 1, 0.5)$ (strong negative opinion) and $o_{A_1} = (1, 1, 0.5)$ (strong positive opinion). When apply the *weighted fusion* the resulting opinion (o_w for short) is $o_w = (0.5, 1, 0.5)$. For this opinion we have to note that the expectation value of the opinion is $E(o_w) = 0.5$, due to the average rating ($t_w = 0.5$), as the certainty value of this opinion is $c_w = 1$, which means that the average rating is representative for future outcomes[9]. This in turn means, that in a repeated series of experiments we can expect a similar number of positive outcomes as negative outcomes (given a sufficiently high number of runs).

On the other hand, we have the resulting opinion (o_c for short) is $o_c = (0.5, 0, 0.5)$ and the $DoC = 1$ (maximum) of the *conflict-aware fusion*. For this opinion, we have to note that the expectation value of the opinion is $E(o_c) = 0.5$, too. However, this is due to the fact that the initial expectation value is $f_c = 0.5$. Furthermore, we see that the certainty value of this opinion is $c_c = 0$, which means that the average rating ($t_c = 0.5$) is not necessarily representative for future outcomes, i.e., it can easily change when new information becomes available.

Now, we can ask ourselves which of the resulting opinions reflects the situation better. The expectation value that the proposition under consideration is true, e.g., that the cloud provider has a competent customer service is 0.5 in both cases. In fact, if we think what would be the outcome of first request to the customer support, the information that we have collected propose that there is a probability of 0.5 for a positive experience and of 0.5 for a negative experience.

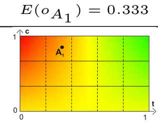
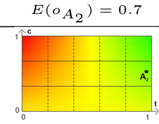
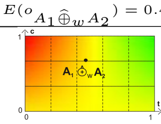
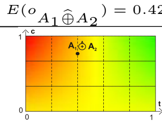
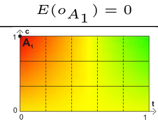
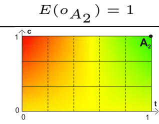
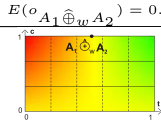
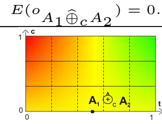
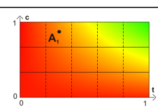
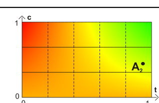
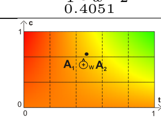
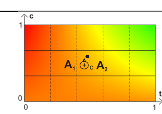
However, if we consider the case that we repeatedly run the experiment, e.g., repeated and subsequent interaction with the customer support, we should expect that the result of the second, third, ... request is as satisfying (or unsatisfying) as the first one. Therefore, we conclude that this line of argumentation leads to the statement that the *conflict-aware fusion* produces a better result than the *weighted fusion*.

Finally, we also have to mention that if one only looks at the result of the *weighted fusion*, i.e., $o_w = (0.5, 1, 0.5)$, this result is highly ambiguous and in fact, this could result from an infinite amount of opinions, e.g., $o_{A_1} = (0, 1, 0.5)$ and $o_{A_2} = (1, 1, 0.5)$. With the *conflict-aware fusion*, we address this problem by additionally providing the DoC.

Example 3: The third example in Table 4 provides another comparison between the weighted and the conflict-aware fusion. Here, the conflict between the input parameters (on the left) is not as extreme as in example 2 which is re-

[9]Recall, the expectation value is defined as $E = t * c + (1 - c) * f$.

Table 4. Examples for the Fusion Operators

Input Opinions		Resulting Opinions	
$o_{A_1} = (t_{A_1}, c_{A_1}, f_{A_1})$	$o_{A_2} = (t_{A_2}, c_{A_2}, f_{A_2})$	$o_{A_1 \widehat{\oplus} A_2}$	
Example 1			
		$W.FUSION$ $w_1 = 1; w_2 = 2$	$A.FUSION$ $w_1 = 1; w_2 = 1$
$(0.3, 0.833, 0.5)$	$(0.9, 0.5, 0.5)$	$(0.4717, 0.6996, 0.5)$	$(0.4, 0.75, 0.5)$
$E(o_{A_1}) = 0.333$	$E(o_{A_2}) = 0.7$	$E(o_{A_1 \widehat{\oplus}_w A_2}) = 0.48$	$E(o_{A_1 \widehat{\oplus} A_2}) = 0.425$
			
Example 2			
		$W.FUSION$ $w_1 = 1; w_2 = 1$	$C.FUSION$ $w_1 = 1; w_2 = 1$
$(0, 1, 0.5)$	$(1, 1, 0.5)$	$(0.5, 1, 0.5)$	$(0.5, 0, 0.5)$ $DoC = 1$
$E(o_{A_1}) = 0$	$E(o_{A_2}) = 1$	$E(o_{A_1 \widehat{\oplus}_w A_2}) = 0.5$	$E(o_{A_1 \widehat{\oplus}_c A_2}) = 0.5$
			
Example 3			
		$W.FUSION$ $w_1 = 1; w_2 = 2$	$C.FUSION$ $w_1 = 1; w_2 = 2$
$(0.3, 0.833, 0.05)$	$(0.9, 0.5, 0.35)$	$(0.4717, 0.6996, 0.25)$	$(0.4717, 0.5831, 0.25)$ $DoC = 0.166$
$E(o_{A_1}) = 0.2582$	$E(o_{A_2}) = 0.625$	$E(o_{A_1 \widehat{\oplus}_w A_2}) = 0.4051$	$E(o_{A_1 \widehat{\oplus}_c A_2}) = 0.3793$
			

flected by the $DoC = 0.166$ in the conflict-aware fusion (on the right). In this example, we also see that the reduction of the certainty (i.e., from $c_w = 0.6996$ to $c_c = 0.5831$) in the conflict-aware fusion usually leads to a lower expectation value (0.3793) than the expectation value (0.4051) in the weighted fusion. We argue that the lower expectation value in the conflict-aware fusion is justified in this example, as the average ratings of the input parameters are conflicting and thus maybe not representative. This effect comes from the reduction of the certainty (in the conflict-aware fusion) which in turn means that the expectation value is shifting closer to the initial expectation value (leading to a lower expectation value in this example).

Finally, we like to highlight the example to show how the choice of the initial expectation value (f) influence the graphical representation of the HTI.

Table 5. Opinions on Cloud Providers' Trustworthiness and User's Preferences

(a) *Opinions on **Cloud A's** Trustworthiness*

o_{FR}	$(0.05, 0.85, 0.1)$
o_{EA}	$(0.1, 0.9, 0.1)$
o_Q	$(0.9, 0.99, 0.1)$
o_{PS}	$(0.95, 0.95, 0.1)$

(b) *Opinions on **Cloud B's** Trustworthiness*

o_{FR}	$(0.85, 0.9, 0.1)$
o_{EA}	$(0.81, 0.91, 0.1)$
o_Q	$(0.9, 0.86, 0.1)$
o_{PS}	$(0.91, 0.81, 0.1)$

(c) User's Preferences (Weights) in Different Scenarios

Opinions	Scenario 1 and 3	Scenario 2
o_{FR}	$w_{FR} = 2$	$w_{FR} = 2$
o_{EA}	$w_{EA} = 2$	$w_{EA} = 2$
o_Q	$w_Q = 2$	$w_Q = 2$
o_{PS}	$w_{PS} = 1$	$w_{PS} = 2$

7 Evaluation of the Use Case

In this section, we show how the fusion operators can be applied to the cloud marketplace use case presented in Section 3.1 and how our approach supports users in selecting cloud providers. In the following, we assume that the propositions (and propositional logic terms) on the trustworthiness of Cloud A have already been evaluated (using CertainLogic AND where applicable, see [6]) as given in Figure 1. Thus, we are now in the situation where we have to combine the resulting four opinions (Q, PS, FR, and EA) on the trustworthiness of Cloud A, i.e., we have to compute $\widehat{\oplus}_c(o_Q, o_{PS}, o_{FR}, o_{EA})$. For the evaluation, we assume the following (the parameters are given in Table 5(a)):

1. Questionnaire (Q) and Provider Statements (PS): The resulting opinion about Cloud A's trustworthiness are extracted from the questionnaire CAIQ (Q) published by CSA in STAR and the provider statements (PS) published by Cloud A. The extracted opinions from both of the sources are supporting the trustworthiness of the cloud provider.
2. Feedback & Recommendation (FR): The resulting opinion is extracted from the users' feedback and recommendations. Users' opinion contradicts to the cloud provider's opinions (Q and PS).
3. Expert Assessment (EA): The extracted opinion from the experts' assessment about the trustworthiness of Cloud A also contradicts to that of the cloud provider A (Q and PS).

In this example, we assume an initial expectation value ($f = f_Q = f_{PS} = f_{FR} = f_{EA} = 0.1$), which reflects a rather pessimistic initial expectation [10].

To demonstrate the applicability and capability of different fusion operators, we consider three scenarios regarding the preferences of the users and considering conflicts when combining opinions. In scenario 1 and 3, we show that a user has different preferences on the impact of the different opinions, whereas in scenario 2 the user gives the same weight to all sources when combining the opinions. Furthermore, for scenario 1 and 2, conflicts between opinions are not considered

[10] Note that the user could either calculate the initial expectation value based on the provided opinion or replace this value with his own assumption.

Table 6. Resulting Opinions for the Different Scenarios

Scenarios	Cloud A: $o_{\widehat{\oplus}(FR,EA,Q,PS)}$	Cloud B: $o_{\widehat{\oplus}(FR,EA,Q,PS)}$
Scenario 1 (not considering conflict)	$(0.8062, 0.9723, 0.1)$ $E(o_{\widehat{\oplus}_w(FR,EA,Q,PS)}) = 0.7866$	$(0.8511, 0.8866, 0.1)$ $E(o_{\widehat{\oplus}_w(FR,EA,Q,PS)}) = 0.7659$
Scenario 2 (not considering conflict)	$(0.8165, 0.9707, 0.1)$ $E(o_{\widehat{\oplus}(FR,EA,Q,PS)}) = 0.7955$	$(0.8553, 0.8806, 0.1)$ $E(o_{\widehat{\oplus}(FR,EA,Q,PS)}) = 0.7651$
Scenario 3 (considering conflict)	$(0.8062, 0.5726, 0.1)$ $DoC = 0.4111$ $E(o_{\widehat{\oplus}_c(FR,EA,Q,PS)}) = 0.5043$	$(0.8511, 0.8534, 0.1)$ $DoC = 0.0374$ $E(o_{\widehat{\oplus}_c(FR,EA,Q,PS)}) = 0.7409$

whereas for scenario 3 (weighted fusion), conflicts between the opinions are considered (conflict-aware fusion). The discussion of the scenarios is given as follows (for brevity, we focus on discussion about *Cloud A* only):

Scenario 1: Based on the preferences (modelled by weights) given in Table 5(c), we use the *weighted fusion* to address different weights for scenario 1. The resulting opinion $o = (0.8062, 0.9723, 0.1)$ for the trustworthiness of Cloud A is given in Table 6, indicates that Cloud A is trustworthy with a probability of 0.7866. Though the *weighted fusion* operator can consider users' preferences when fusing dependent opinions, the operator is not able to deal with conflicts among opinions.

Scenario 2: This scenario demonstrates the application of the *average fusion* operator (which is equivalent to the *weighted fusion* using equal weights). The resulting opinion $((0.8165, 0.9707, 0.1))$ calculated in scenario 2 is different than the one in scenario 1 $((0.8062, 0.9723, 0.1))$. This is because of the influence of the variable weights in scenario 1. Scenario 1 and 2 show the comparison of the *weighted fusion* and *average fusion* operators in terms of their capabilities.

Scenario 3: In the previous scenarios, only the user's preferences are taken into account, but not the conflicts among the opinions. From the given opinions in Table 5(a), a user can be confused about the trustworthiness of Cloud A by observing the conflicting opinions (o_{FR} and o_{EA} in comparison to o_Q and o_{PS}). This is reflected in the result of the novel conflict-aware fusion operator. Using this operator, the opinion for the trustworthiness of Cloud A calculated as $(0.8062, 0.5726, 0.1)$ with a $DoC = 0.4111$ (cf. Table 6, Scenario 3). The impact of the *conflict-ware fusion* is clearly visible in the *certainty* value (0.5726) of this opinion compared to the certainty value (0.9723) in scenario 1 (weighted fusion). The expectation value (E) is also affected when conflict between opinions are taken into account. Considering the conflict, the final expectation value for Cloud A is (0.5043), which is clearly lower than in Scenario 1. We conclude that the *conflict-aware fusion* operator provides the most representative assessment of Cloud A's trustworthiness. Thus, this operator is best suited among the three operators that we have discussed. Note that the fusion operators in *subjective logic* do not consider preferential weights and conflicts when aggregating dependent opinions. Therefore, *conflict-aware fusion* operator is a better choice than the fusion operators in *subjective logic* when one requires the most representative trust assessment under conflict and personal preferences.

In a real world setting, we would assume that a user can choose between a couple of cloud providers. In this case, we propose to sort the available cloud providers based on their expectation value (using the *DoC* as a second criteria if necessary). In our example, having cloud A and cloud B (using the conflict-aware fusion – scenario 3 – see Tables 5(a), 5(b), 6) this means Cloud B is better ranked than cloud A. This comes from the fact that the proposition on cloud B is positive and the opinions (associated with the proposition) from the different sources are less conflicting. We argue that this again shows the strength of our conflict-aware fusion, as this order is more desirable than the order (Cloud A better than Cloud B) which we would get under the weighted fusion in scenario 1 and 2. We also have to note that in addition to the expectation value, especially, the certainty value is a good indicator to see whether the collected information is supposed to be representative or whether further analysis might be required.

8 Conclusion

In cloud marketplaces, users still require means for assessing the trustworthiness of the cloud providers up-front before signing any contract with them. Although we already see first steps in these directions, like the platforms envisioned by CloudCommons and multi-faceted Trust Management system for cloud marketplaces [25], elaborate metrics for aggregating information (in terms of multiple attributes) from different sources are still missing. We believe that our contribution presented in this paper is a useful tool to overcome this lack in current platforms and systems, and thus provides means for a more reliable and transparent assessment of the trustworthiness of cloud providers.

The novel fusion operator (i.e., conflict-aware) proposed in this paper is specifically designed to cope with dependent opinions under uncertainty and conflict that are associated with propositions. Hereby, the equivalence between the *CertainTrust* average fusion operator and the *subjective logic* averaging fusion operator as well as the consensus operator for dependent opinions provides the basis and justification for the validity of the *CertainTrust* average fusion operator. Finally, we provide the *conflict-aware fusion* operator – and the *weighted fusion* as an intermediate step. The *conflict-aware fusion* operator extends the state-of-the-art by considering the weights of different opinions and conflicts among the opinions. Moreover, the degree of conflict (DoC) is presented explicitly together with the resulting opinion and its corresponding expectation value (E) to support reliable and transparent decision-making in cloud marketplaces. We also argue that the graphical representation (CertainTrust HTI) of opinions can be especially useful when integrating the proposed approach for trust assessment in web pages and cloud platforms.

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Appendix

A Proof: Theorem 5.1

Proof. We prove the theorem component-wise for average fusion ($A.FUSION$) operator by verifying the following under different cases:

$$t_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = t_{A_1}; c_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = c_{A_1}; f_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = f_{A_1}. \quad (2)$$

At first, we prove $t_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = t_{A_1}$ under following three cases:

- Case 1: If $c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 1$.
- Case 2: If $c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 0$.
- Case 3: if $\{c_{A_i}, c_{A_j}\} \neq 1$.

Proof for Case 1:

$$t_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = \frac{t_{A_1} + t_{A_1} + \cdots + t_{A_1}}{n} \text{ [Using Table 1 and replace all } A_i\text{'s by } A_1] \\ = \frac{n * t_{A_1}}{n} \\ = t_{A_1} \quad (3)$$

Proof for Case 2:

$$t_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = 0.5 \text{ [Using Table 1 and replace } A_i\text{'s by } A_1] = t_{A_1} \quad (4)$$

Proof for Case 3:

$$t_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = \frac{(c_{A_1}t_{A_1}(1-c_{A_1})(1-c_{A_1})\cdots(1-c_{A_1})) + \cdots + (c_{A_1}t_{A_1}(1-c_{A_1})(1-c_{A_1})\cdots(1-c_{A_1}))}{(c_{A_1}(1-c_{A_1})(1-c_{A_1})\cdots(1-c_{A_1})) + \cdots + (c_{A_1}(1-c_{A_1})(1-c_{A_1})\cdots(1-c_{A_1}))} \\ \text{[Using Table 1 and replace } A_i\text{'s by } A_1] \\ = \frac{n * (t_{A_1}c_{A_1}(1-c_{A_1})}{n * (c_{A_1}(1-c_{A_1}))} \\ = t_{A_1} \quad (5)$$

Next, we prove $c_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = c_{A_1}$ under following two cases:

- Case 1: If $c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 1$.
- Case 2: if $\{c_{A_i}, c_{A_j}\} \neq 1$.

Proof for Case 1**:**

$$c_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = 1 \text{ [Using Table 1 and replace } A_i\text{'s by } A_1] = c_{A_1} \quad (6)$$

Proof for Case 2:

$$c_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = \frac{(c_{A_1}(1-c_{A_1})(1-c_{A_1})\cdots(1-c_{A_1})) + \cdots + (c_{A_1}(1-c_{A_1})(1-c_{A_1})\cdots(1-c_{A_1}))}{((1-c_{A_1})(1-c_{A_1})\cdots(1-c_{A_1})) + \cdots + ((1-c_{A_1})(1-c_{A_1})\cdots(1-c_{A_1}))}$$

[Using Table 1 and replace A_i's by A_1]

$$= \frac{n * (c_{A_1}(1-c_{A_1}))}{n * (1-c_{A_1})}$$

$$= c_{A_1} \tag{7}$$

Finally, we prove $f_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = f_{A_1}$

$$f_{\widehat{\oplus}(A_1,A_1,\cdots,A_1)} = \frac{f_{A_1} + f_{A_1} + \cdots + f_{A_1}}{n} \text{ [Using Table 1 and replace } A_i\text{'s by } A_1]$$

$$= \frac{n * f_{A_1}}{n} = f_{A_1} \tag{8}$$

The proof for the *W.FUSION* and *C.FUSION* operators of Theorem 5.1 can be carried out analogously.

B Sketch of the Proof: Theorem 5.2

Proof. The proof for $\widehat{\oplus}(o_{A_1}, o_{A_2}) = \widehat{\oplus}(o_{A_2}, o_{A_1})$ can be carried out component-wise by verifying $\widehat{\oplus}(t_{A_1}, t_{A_2}) = \widehat{\oplus}(t_{A_2}, t_{A_1})$, $\widehat{\oplus}(c_{A_1}, c_{A_2}) = \widehat{\oplus}(c_{A_2}, c_{A_1})$, and $\widehat{\oplus}(f_{A_1}, f_{A_2}) = \widehat{\oplus}(f_{A_2}, f_{A_1})$. Using Table 1, these can easily be verified.

The proof for $\widehat{\oplus}_w(o_{A_1}, o_{A_2}) = \widehat{\oplus}_w(o_{A_2}, o_{A_1})$ and $\widehat{\oplus}_c(o_{A_1}, o_{A_2}) = \widehat{\oplus}(o_{A_2}, o_{A_1})$ can be carried out analogously, using Table 2 and Table 3 respectively.

C Sketch of the Proof: Theorem 5.3

Looking at the definitions of the fusion operators (see Table 1, 2, 3), one sees that the theorem holds due to the commutativity of the summation.

D Proof: Theorem 5.4

Proof. We prove the theorem component-wise by verifying the following under different cases:

$$\widehat{\oplus}_w(t_{A_1}, t_{A_2}, \cdots, t_{A_n}) = \widehat{\oplus}(\underbrace{t_{A_1}, \cdots, t_{A_1}}_{a_1 \cdot \prod_{j \neq 1} b_j \textbf{ times}}, \underbrace{t_{A_2}, \cdots, t_{A_2}}_{a_2 \cdot \prod_{j \neq 2} b_j \textbf{ times}}, \cdots, \underbrace{t_{A_n}, \cdots, t_{A_n}}_{a_n \cdot \prod_{j \neq n} b_j \textbf{ times}}),$$

$$\widehat{\oplus}_w(c_{A_1}, c_{A_2}, \cdots, c_{A_n}) = \widehat{\oplus}(\underbrace{c_{A_1}, \cdots, c_{A_1}}_{a_1 \cdot \prod_{j \neq 1} b_j \textbf{ times}}, \underbrace{c_{A_2}, \cdots, c_{A_2}}_{a_2 \cdot \prod_{j \neq 2} b_j \textbf{ times}}, \cdots, \underbrace{c_{A_n}, \cdots, c_{A_n}}_{a_n \cdot \prod_{j \neq n} b_j \textbf{ times}}),$$

$$\widehat{\oplus}_w(f_{A_1}, f_{A_2}, \cdots, f_{A_n}) = \widehat{\oplus}(\underbrace{f_{A_1}, \cdots, f_{A_1}}_{a_1 \cdot \prod_{j \neq 1} b_j \textbf{ times}}, \underbrace{f_{A_2}, \cdots, f_{A_2}}_{a_2 \cdot \prod_{j \neq 2} b_j \textbf{ times}}, \cdots, \underbrace{f_{A_n}, \cdots, f_{A_n}}_{a_n \cdot \prod_{j \neq n} b_j \textbf{ times}}).$$

We prove $\widehat{\oplus}_w(t_{A_1}, t_{A_2}, \cdots, t_{A_n}) = \widehat{\oplus}(\underbrace{t_{A_1}, \cdots, t_{A_1}}_{a_1 \cdot \prod_{j \neq 1} b_j \textbf{ times}}, \underbrace{t_{A_2}, \cdots, t_{A_2}}_{a_2 \cdot \prod_{j \neq 2} b_j \textbf{ times}}, \cdots, \underbrace{t_{A_n}, \cdots, t_{A_n}}_{a_n \cdot \prod_{j \neq n} b_j \textbf{ times}})$ under following three cases:

- Case 1: If $c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 1$.
- Case 2: If $c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 0$.
- Case 3: if $\{c_{A_i}, c_{A_j}\} \neq 1$.

Proof for Case 1:

$$\widehat{\oplus}_w(t_{A_1}, t_{A_2}, \cdots, t_{A_n}) = t_{\widehat{\oplus}_w A_1, A_2, \cdots, A_n} = \frac{\sum_{i=1}^{n} w_i t_{A_i}}{\sum_{i=1}^{n} w_i} \text{ [Using Table 2]} = \frac{\sum_{i=1}^{n} \frac{a_i}{b_i} t_{A_i}}{\sum_{i=1}^{n} \frac{a_i}{b_i}} = \frac{\sum_{i=1}^{n} \frac{a_i \cdot \prod_{j \neq i}^{n} b_j}{\prod_{k=1}^{n} b_k} t_{A_i}}{\sum_{i=1}^{n} \frac{a_i \cdot \prod_{j \neq i}^{n} b_j}{\prod_{k=1}^{n} b_k}}$$

$$= \frac{\sum_{i=1}^{n} t_{A_i} a_i \cdot \prod_{j \neq i}^{n} b_j}{\sum_{i=1}^{n} a_i \cdot \prod_{j \neq i}^{n} b_j} = \frac{\sum_{i=1}^{n} t_{A_i} m_i}{\sum_{i=1}^{n} m_i} \text{ [Replace } a_i \cdot \prod_{j \neq i}^{n} b_j \text{ with } m_i\text{;product of integers is an integer]}$$

$$= \frac{\sum_{i=1}^{n} (\sum_{j=1}^{m_i} t_{A_i})}{\sum_{i=1}^{n} m_i} = \widehat{\oplus}(\underbrace{t_{A_1}, \cdots, t_{A_1}}_{a_1 \cdot \prod_{j \neq 1} b_j \textbf{ times}}, \underbrace{t_{A_2}, \cdots, t_{A_2}}_{a_2 \cdot \prod_{j \neq 2} b_j \textbf{ times}}, \cdots, \underbrace{t_{A_n}, \cdots, t_{A_n}}_{a_n \cdot \prod_{j \neq n} b_j \textbf{ times}}) \text{ [where } m_i = a_i \cdot \prod_{j \neq i}^{n} b_j] \tag{9}$$

Proof for Case 2:

$$\widehat{\oplus}_w(t_{A_1}, t_{A_2}, \cdots, t_{A_n}) = t_{\widehat{\oplus}_w A_1, A_2, \cdots, A_n} = 0.5 = t_{\widehat{\oplus} A_1, A_2, \cdots, A_n} \tag{10}$$

Proof for Case 3:

$$\widehat{\oplus}_w(t_{A_1}, t_{A_2}, \cdots, t_{A_n}) = t_{\widehat{\oplus}_w A_1, A_2, \cdots, A_n} = \frac{\sum_{i=1}^{n}(c_{A_i} t_{A_i} w_i \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n}(c_{A_i} w_i \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))} \text{ [Using Table 2]}$$

$$= \frac{\sum_{i=1}^{n} \frac{a_i}{b_i}(c_{A_i} t_{A_i} \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n} \frac{a_i}{b_i}(c_{A_i} \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))} = \frac{\sum_{i=1}^{n} \frac{a_i \cdot \prod_{j\neq i}^{n} b_j}{\prod_{k=1}^{n} b_k}(c_{A_i} t_{A_i} \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n} \frac{a_i \cdot \prod_{j\neq i}^{n} b_j}{\prod_{k=1}^{n} b_k}(c_{A_i} \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))}$$

$$= \frac{\sum_{i=1}^{n}(a_i \cdot \prod_{j\neq i}^{n} b_j \cdot c_{A_i} t_{A_i} \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n}(a_i \cdot \prod_{j\neq i}^{n} b_j \cdot c_{A_i} \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))}$$

$$= \frac{\sum_{i=1}^{n}(m_i \cdot c_{A_i} t_{A_i} \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n}(m_i \cdot c_{A_i} \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))} \text{ [Replace } a_i \cdot \prod_{j\neq i}^{n} b_j \text{ with } m_i\text{;product of integers is an integer]}$$

$$= \frac{\sum_{i=1}^{n}(\sum_{i=1}^{m_i} c_{A_i} t_{A_i} \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))}{\sum_{i=1}^{n}(m_i \cdot c_{A_i} \prod_{i=1,\ j\neq i}^{n} (1 - c_{A_j}))}$$

$$= \frac{\sum_{i=1}^{n}(\sum_{j=1}^{m_i} t_{A_i})}{\sum_{i=1}^{n} m_i} \text{ [Expanding the equation and reducing the common terms]}$$

$$= \widehat{\oplus}(\underbrace{t_{A_1}, \cdots, t_{A_1}}_{a_1 \cdot \prod_{j\neq 1} b_j \textbf{ times}}, \underbrace{t_{A_2}, \cdots, t_{A_2}}_{a_2 \cdot \prod_{j\neq 2} b_j \textbf{ times}}, \cdots, \underbrace{t_{A_n}, \cdots, t_{A_n}}_{a_n \cdot \prod_{j\neq n} b_j \textbf{ times}}) \text{ [where } m_i = a_i \cdot \prod_{j\neq i}^{n} b_j] \tag{11}$$

The proof for the following two equations can be carried out analogously using Table 1 and Table 2:

$$\widehat{\oplus}_w(c_{A_1}, c_{A_2}, \cdots, c_{A_n}) = \widehat{\oplus}(\underbrace{c_{A_1}, \cdots, c_{A_1}}_{a_1 \cdot \prod_{j\neq 1} b_j \textbf{ times}}, \underbrace{c_{A_2}, \cdots, c_{A_2}}_{a_2 \cdot \prod_{j\neq 2} b_j \textbf{ times}}, \cdots, \underbrace{c_{A_n}, \cdots, c_{A_n}}_{a_n \cdot \prod_{j\neq n} b_j \textbf{ times}}),$$

$$\widehat{\oplus}_w(f_{A_1}, f_{A_2}, \cdots, f_{A_n}) = \widehat{\oplus}(\underbrace{f_{A_1}, \cdots, f_{A_1}}_{a_1 \cdot \prod_{j\neq 1} b_j \textbf{ times}}, \underbrace{f_{A_2}, \cdots, f_{A_2}}_{a_2 \cdot \prod_{j\neq 2} b_j \textbf{ times}}, \cdots, \underbrace{f_{A_n}, \cdots, f_{A_n}}_{a_n \cdot \prod_{j\neq n} b_j \textbf{ times}})$$

Finally, the component-wise algebraic verifications implies the verification of the theorem itself.

The proof of the same theorem for $C.FUSION$ operator can be carried out analogously using Table 1 and Table 3

E Proof: Theorem 5.5

Proof. We prove the theorem component-wise by verifying the following equations under different cases:

$\widehat{\oplus}_w((t_{A_1}, w_1 * k), (t_{A_2}, w_2 * k), \cdots, (t_{A_n}, w_n * k) = \widehat{\oplus}_w((t_{A_1}, w_1), (t_{A_2}, w_2), \cdots, (t_{A_n}, w_n)$
$\widehat{\oplus}_w((c_{A_1}, w_1 * k), (c_{A_2}, c_2 * k), \cdots, (c_{A_n}, w_n * k) = \widehat{\oplus}_w((c_{A_1}, w_1), (c_{A_2}, w_2), \cdots, (c_{A_n}, w_n)$
$\widehat{\oplus}_w((f_{A_1}, w_1 * k), (f_{A_2}, w_2 * k), \cdots, (f_{A_n}, w_n * k) = \widehat{\oplus}_w((f_{A_1}, w_1), (f_{A_2}, w_2), \cdots, (f_{A_n}, w_n)$

We prove $\widehat{\oplus}_w((t_{A_1}, w_1 * k), (t_{A_2}, w_2 * k), \cdots, (t_{A_n}, w_n * k) = \widehat{\oplus}_w((t_{A_1}, w_1), (t_{A_2}, w_2), \cdots, (t_{A_n}, w_n))$ under following three cases:

- Case 1: If $c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 1$.
- Case 2: If $c_{A_1} = c_{A_2} = \cdots = c_{A_n} = 0$.
- Case 3: if $\{c_{A_i}, c_{A_j}\} \neq 1$.

Proof for Case 1:

$$\begin{aligned}
&\widehat{\oplus}_w((t_{A_1}, w_1 * k), (t_{A_2}, w_2 * k), \cdots, (t_{A_n}, w_n * k) \\
&= \frac{\sum_{i=1}^{n} k * w_i t_{A_i}}{\sum_{i=1}^{n} w_i} \text{ [Using Table 2 and replace } w_i \text{ with scaling factor } k] \\
&= \frac{\sum_{i=1}^{n} k \cdot w_i t_{A_i}}{\sum_{i=1}^{n} k \cdot w_i} \text{ [Multiply with a scaling factor } k] = \frac{k \cdot w_1 t_{A_1} + k \cdot w_2 t_{A_2} + \cdots + k \cdot w_n t_{A_n}}{k \cdot w_1 + k \cdot w_2 + \cdots + k \cdot w_n} \\
&= \frac{\sum_{i=1}^{n} w_i t_{A_i}}{\sum_{i=1}^{n} w_i} \text{ [Reducing common constant } k] = \widehat{\oplus}_w((t_{A_1}, w_1), (t_{A_2}, w_2), \cdots, (t_{A_n}, w_n))
\end{aligned} \tag{12}$$

The proof for Case 2 and 3 can be carried out analogously using Table 2. Moreover, the proof for the following two equations can be carried out analogously using Table 2:
$\widehat{\oplus}_w((c_{A_1}, w_1 * k), (c_{A_2}, c_2 * k), \cdots, (c_{A_n}, w_n * k) = \widehat{\oplus}_w((c_{A_1}, w_1), (c_{A_2}, w_2), \cdots, (c_{A_n}, w_n)$
$\widehat{\oplus}_w((f_{A_1}, w_1 * k), (f_{A_2}, w_2 * k), \cdots, (f_{A_n}, w_n * k) = \widehat{\oplus}_w((f_{A_1}, w_1), (f_{A_2}, w_2), \cdots, (f_{A_n}, w_n)$

Finally, the component-wise algebraic verifications implies the verification of the theorem itself.

The proof of the same theorem for $C.FUSION$ operator can be carried out analogously using Table 1 and Table 3.

F Proof: Theorem 5.6

Proof. We prove the theorem and omit detail algebraic simplifications for brevity.

First, we prove $E(\widehat{\oplus}_w(o_{A_1}, w), (o_{A_2}, w), \cdots, (o_{A_n}, w)) = E(\widehat{\oplus}(o_{A_1}, o_{A_2}, ..., o_{A_n}))$

$$\begin{aligned}
&E(\widehat{\oplus}_w(o_{A_1}, w), (o_{A_2}, w), \cdots, (o_{A_n}, w)) \\
&= E(\widehat{\oplus}_w(t_{(A_1,w),\cdots,(A_n,w)}), \widehat{\oplus}_w(c_{(A_1,w),\cdots,(A_n,w)}), \widehat{\oplus}_w(f_{(A_1,w),\cdots,(A_n,w)})) \\
&= t_{\widehat{\oplus}_w((A_1,w),\cdots,(A_n,w))} * c_{\widehat{\oplus}_w((A_1,w),\cdots,(A_n,w))} + (1 - c_{\widehat{\oplus}_w((A_1,w),\cdots,(A_n,w))}) * f_{\widehat{\oplus}_w((A_1,w),\cdots,(A_n,w))} \\
&\ldots\text{[Substitution of } t_{\widehat{\oplus}_w((A_1,w),\cdots,(A_n,w))}, c_{\widehat{\oplus}_w((A_1,w),\cdots,(A_n,w))}, f_{\widehat{\oplus}_w((A_1,w),\cdots,(A_n,w))}\text{[using Table 2]]} \\
&\ldots\text{[replace } w_i \text{ by } w \text{ where } i = 1 \cdots n \text{ and algebraic simplifications]} \\
&= t_{\widehat{\oplus}(A_1,\cdots,A_n)} * c_{\widehat{\oplus}(A_1,\cdots,A_n)} + (1 - c_{\widehat{\oplus}(A_1,\cdots,A_n)}) * f_{\widehat{\oplus}(A_1,\cdots,A_n)} \\
&= E(\widehat{\oplus}(o_{A_1}, o_{A_2}, ..., o_{A_n})) \text{ [Using the Equation 1]}
\end{aligned} \tag{13}$$

Next, we prove $E(\widehat{\oplus}(o_{A_1}, o_{A_2}, ..., o_{A_n})) = \frac{\sum_{i=1}^{n} E(o_{A_i})}{n}$

$$\begin{aligned}
&E(\widehat{\oplus}(o_{A_1}, o_{A_2}, ..., o_{A_n})) = t_{\widehat{\oplus}(A_1,\cdots,A_n)} * c_{\widehat{\oplus}(A_1,\cdots,A_n)} + (1 - c_{\widehat{\oplus}(A_1,\cdots,A_n)}) * f_{\widehat{\oplus}(A_1,\cdots,A_n)} \\
&\ldots\text{[Substitution of } t_{\widehat{\oplus}(A_1,\cdots,A_n)}, c_{\widehat{\oplus}_w(A_1,\cdots,A_n)}, f_{\widehat{\oplus}_w(A_1,\cdots,A_n)} \text{ under different cases [using Table 1]]} \\
&\ldots\text{[after several steps of algebraic simplifications and using the Equation 1 for each } A_i\text{]} \\
&= \frac{E(o_{A_1}) + E(o_{A_2}) + \cdots + E(o_{A_n})}{n} = \frac{\sum_{i=1}^{n} E(o_{A_i})}{n}
\end{aligned} \tag{14}$$

G Proof: Theorem 5.7

Proof. We prove the theorem and omit detail algebraic simplifications for brevity.

$$
\begin{aligned}
&E(\widehat{\oplus}_w(o_{A_1}, w_1), (o_{A_2}, w_2), \cdots, (o_{A_n}, w_n)) \\
&= E(\widehat{\oplus}_w(t_{(A_1,w_1),\cdots,(A_n,w_n)}), \widehat{\oplus}_w(c_{(A_1,w_1),\cdots,(A_n,w_n)}), \widehat{\oplus}_w(f_{(A_1,w_1),\cdots,(A_n,w_n)})) \\
&= t_{\widehat{\oplus}_w((A_1,w_1),\cdots,(A_n,w_n))} * c_{\widehat{\oplus}_w((A_1,w_1),\cdots,(A_n,w_n))} + (1 - c_{\widehat{\oplus}_w((A_1,w_1),\cdots,(A_n,w_n))}) * f_{\widehat{\oplus}_w((A_1,w_1),\cdots,(A_n,w_n))} \\
&\ldots \text{[Substitution of } t_{\widehat{\oplus}_w((A_1,w_1),\cdots,(A_n,w_n))}, c_{\widehat{\oplus}_w((A_1,w_1),\cdots,(A_n,w_n))}, f_{\widehat{\oplus}_w((A_1,w_1),\cdots,(A_n,w_n))} \text{[using Table 2]]} \\
&\ldots \text{[replace } c_i \text{ by } c \text{ where } i = 1 \cdots n \text{ and algebraic simplifications]} \\
&= \frac{w_1 * (t_{A_1} * c + (1 - c) * f_{A_1}) + \cdots + w_i * (t_{A_n} * c + (1 - c) * f_{A_n})}{w_1 + w_2 + \cdots + w_i} \\
&= \frac{\sum_{i=1}^{n} w_i E(o_{A_i})}{\sum_{i=1}^{n} w_i} \text{ [Using Equation 1 for each } A_i \text{ and } c = c_{A_1} = c_{A_2} = \cdots = c_{A_n}]
\end{aligned} \tag{15}
$$

H Proof: Theorem 5.8

Proof. We prove the theorem and omit detail algebraic simplifications for brevity.

$$
\begin{aligned}
&\frac{\sum_{i=1}^{n} E(o_{A_i})}{n} = \frac{E(t_{A_1}, c, f_{A_1}) + E(t_{A_2}, c, f_{A_2}) + \cdots + E(t_{A_n}, c, f_{A_n})}{n} \\
&\text{[If } c = c_{A_1} = c_{A_2} = \cdots = c_{A_n}] \\
&= \frac{(t_{A_1} * c + (1 - c) * f_{A_1}) + (t_{A_2} * c + (1 - c) * f_{A_2}) + \cdots + (t_{A_n} * c + (1 - c) * f_{A_n})}{n} \\
&\text{[Using Equation 1]} \\
&= \frac{(t_{A_1} + t_{A_2} + \cdots + t_{A_n}) * c + (f_{A_1} + f_{A_2} + \cdots + f_{A_n}) * (1 - c)}{n} \\
&= t_{\widehat{\oplus}_w(A_1,A_2,\cdots,A_n)} * c + f_{(A_1,A_2,\cdots,A_n)} * (1 - c) \\
&\text{[Substitute } c_{A_i} \text{ by } c \text{ and } w_i \text{ by } w \text{ where } i = 1 \cdots n \text{ in Table 2]} \\
&= E(\widehat{\oplus}_w((o_{A_1}, w), (o_{A_2}, w), \cdots, (o_{A_n}, w))) \\
&\text{[Using Equation 1 and Definition 4.2]}
\end{aligned} \tag{16}
$$

The proof of the same theorem for $C.FUSION$ operator can be carried out analogously using Table 3.

I Sketch of the Proof: Equivalence with Averaging Fusion in Subjective Logic

For brevity we just provide a sketch of the proof.

A bijective mapping between an opinion in CertainLogic/CertainTrust given by its parameters, $o = (t, c, f)$ and subjective logic, where the opinion is given as $o = (b, d, u, a)$ has been provided in [14]. To prove the equivalence between the *average fusion* proposed in this paper and the *averaging fusion and consensus operator for dependent opinions* proposed for subjective logic in [18, 19] one can start with the definition of the average fusion/consensus operator for dependent opinions in subjective logic and replace the parameters b, d, u, and a by the corresponding parameters of CertainLogic following the bijective mapping. Finally, one applies the bijective mapping another time to convert the resulting equations which are still calculating the parameters b, d, u and a (a is assumed to be static for *average fusion* in [18]) to calculate the parameters of CertainLogic t, c, and f. The result will be equivalent to the *average fusion* defined in this paper.