Wiederholung: Ableitung Bragg'sche Gleichung aus rez. Gitter

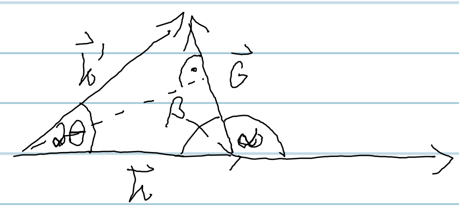
$$\vec{k}' = \vec{k} + \vec{G}_{hkl}$$

$$\cancel{k'^2} = (\vec{k}+\vec{G}_{hkl})^2 = \cancel{k^2} + 2\vec{k}\vec{G}_{hkl} + G_{hkl}^2$$

$\downarrow$ elastisch $k'^2 = k^2$

$$-G_{hkl}^2 = 2\vec{k}\vec{G}_{hkl} = 2kG_{hkl}\cos\alpha$$

$$= 2kG_{hkl}\underbrace{\cos(\theta+90°)}_{-\sin\theta}$$



$\beta = 180° - \alpha$

$180° - \beta = 90° = \theta$

$90° - 180° + \alpha = \theta$

$\alpha = \theta + 90°$

$$\cancel{-}G_{hkl}^{\cancel{2}} = \cancel{-}2k\cancel{G_{hkl}}\sin\theta$$

$$G_{hkl} = 2k\sin\theta = 2\cdot\frac{2\pi}{\lambda}\sin\theta \qquad G_{hkl} = \frac{2\pi}{d_{hkl}} \text{ (ÜB4)}$$

$$\frac{\cancel{2\pi}}{d_{hkl}} = 2\cdot\frac{\cancel{2\pi}}{\lambda}\sin\theta$$

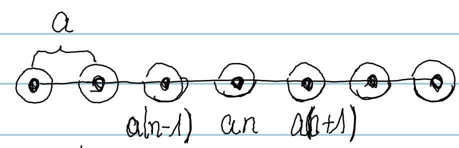
$$\boxed{\lambda = 2d_{hkl}\sin\theta} \quad \text{Bragg}$$

3. Schwingungen von Festkörpern

gesucht: $\omega = f(k)$ $\qquad k = \frac{2\pi}{\lambda}$

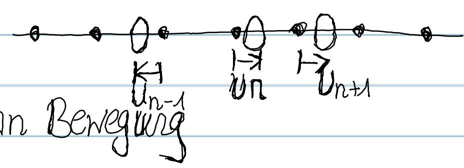
3.1 Dispersionsbeziehung

1 dim. Kette aus gleichen Atomen z.B. Na



$x_n = a \cdot n$

in Ruhe



in Bewegung

$$F = -K(u_n - u_{n+1} + u_n - u_{n-1})$$

$$F = -K(2u_n - u_{n+1} - u_{n-1})$$

Bew. Gleichung: $$m\frac{d^2 u_n}{dt^2} = -K(2u_n - u_{n+1} - u_{n-1})$$

Lösung: $$u_n = u_0 \cos(\omega t - k x_n)$$

↳ Amplitude

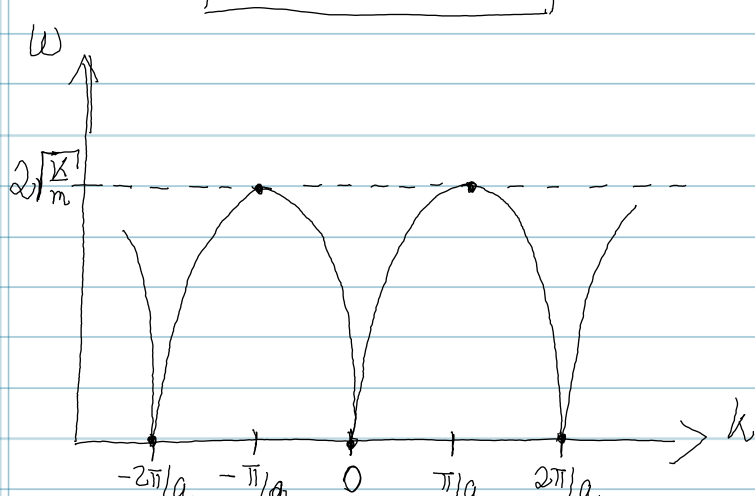
↳ Longitudinale Auslenkung

$$-m\omega^2 u_0 = -K(2 - 2\cos ka)u_0 = -4K\sin^2\left(\frac{ka}{2}\right)u_0$$

$$\boxed{\omega = 2\sqrt{\frac{K}{m}}\left|\sin\frac{ka}{2}\right|}$$

min: $k = 0$

max: $k = \pm\pi/a$



Konsequenzen:

① großes λ : $\omega \approx 2\sqrt{\frac{K}{m}} \cdot \frac{ka}{2} = a\sqrt{\frac{K}{m}}\, k$
(kleines k)

$\sin\frac{ka}{2} \approx \frac{ka}{2}$

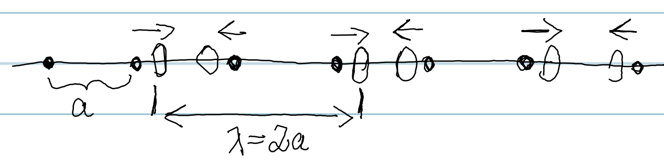
Ausbreitungsgeschw. ?

$$v = \frac{\lambda}{T} = \frac{2\pi}{k}\nu = \frac{\omega}{k} = a\sqrt{\frac{K}{m}} \cdot \frac{k}{k} = a\sqrt{\frac{K}{m}}$$

$\neq f(\lambda)$

② kleines λ : $v = f(\lambda)$ $\quad \lambda \downarrow \quad v \uparrow$
(großes k)

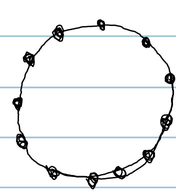
③ $k_{max} = \pm \pi/a \longrightarrow \frac{2\pi}{\lambda} = \frac{\pi}{a} \longrightarrow \lambda = 2a$



$u_n = u_0 \cos(\omega t - n\pi)$

$u_n = u_0 (-1)^n \cos \omega t$

④



$L = N \cdot a$ $\qquad$ $N =$ Anz. Atome

$\lambda = L, \frac{L}{2}, \frac{L}{3}, \ldots, 2a$

$$k = \underbrace{\pm\frac{2\pi}{L}, \pm\frac{4\pi}{L}, \ldots, \pm\pi/a}_{N\text{-Schwingungen}}$$

$$u_n = u_0 \cos(\omega t - k' n a)$$

$$k' = \frac{2\pi}{a} p + k \qquad p = 1, 2, 3, \dots$$

$$\cancel{u_n = u_0 \cos\left(\omega t - \frac{2\pi}{a} p n a\right)} \qquad u_n = u_0 \cos\left(\omega t - \left(k + \frac{2\pi}{a} p\right) n a\right)$$

$$u_n = u_0 \cos\left(\omega t - k n a - \frac{2\pi}{\cancel{a}} p n \cancel{a}\right)$$

$$u_n = u_0 \cos(\omega t - k n a - 2\pi p n)$$

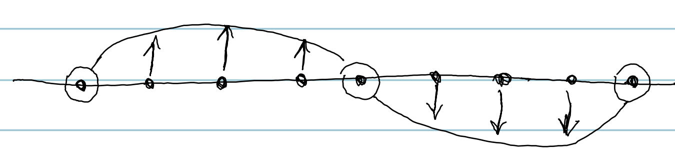
$$u_n = u_0 \cos(\omega t - k n a)$$

↳ im Intervall $[-\frac{\pi}{a}, \frac{\pi}{a}]$ alle physikalisch unterschiedlichen Schwingungen

3.2 Verschiedene Schwingungsarten

bisher: longitudinal → Auslenkung entlang der Kettenrichtung

transversal → " senkrecht "



N Atome → 3N Schwingungen
1N longitudinal
2N transversal