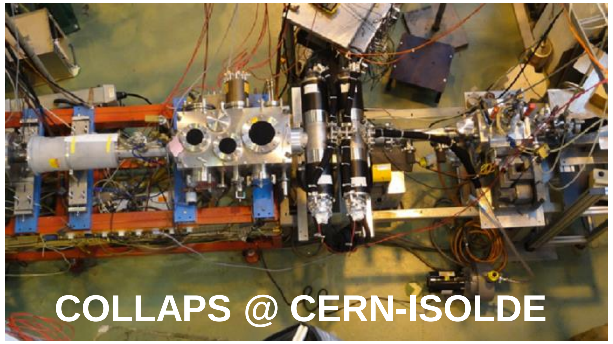
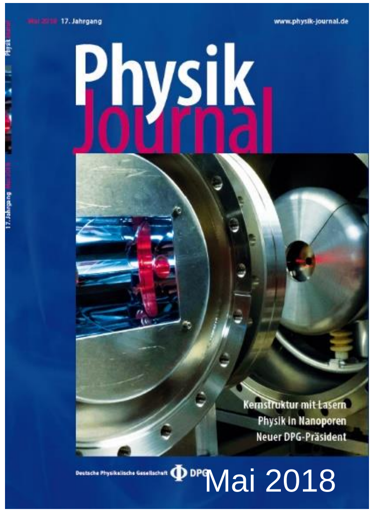
Meet ‘n Greet: About “absolute” and differential charge radii from laser spectroscopy


COLLAPS @ CERN-ISOLDE


LASPEC @ Argonne National Lab


Physik Journal

Kernstruktur mit Lasern

Physik in Nanoporen

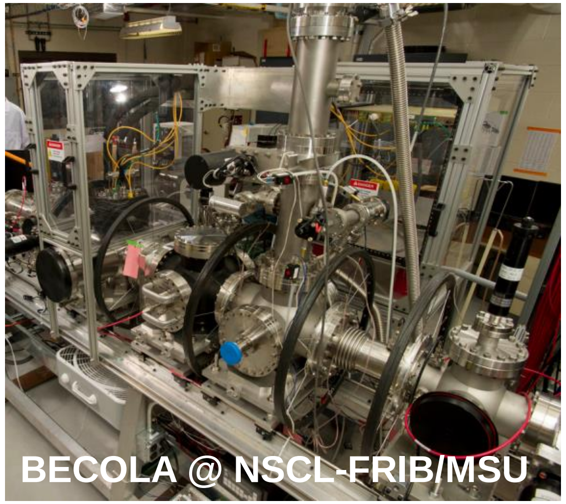
Neuer DPG-Präsident

Mai 2018

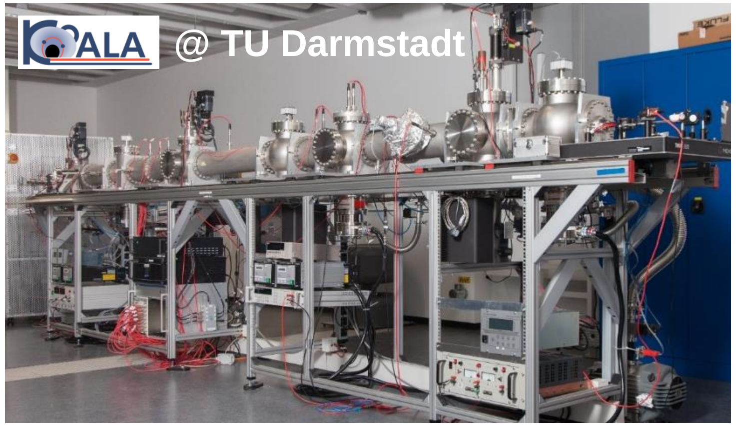
GEFÖRDERT VOM

Bundesministerium
für Bildung
und Forschung

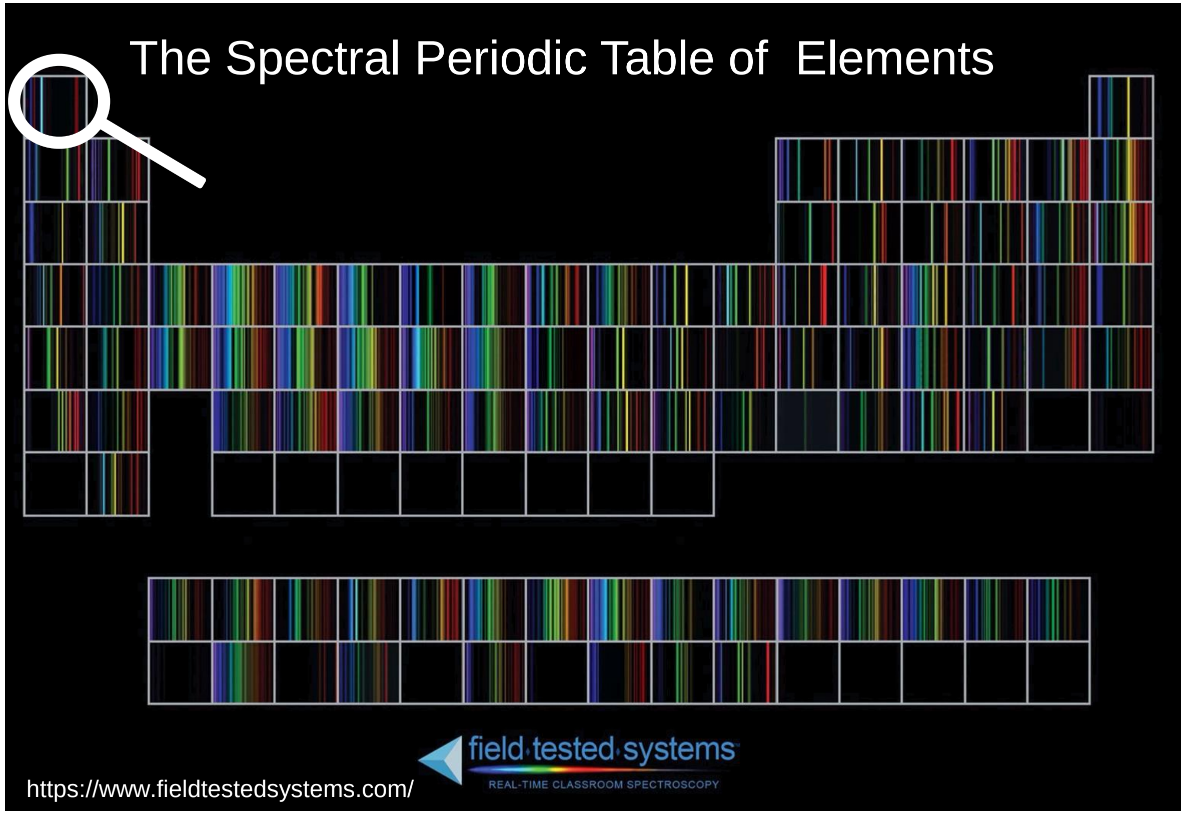
Gefördert durch
DFG Deutsche
Forschungsgemeinschaft


BECOLA @ NSCL-FRIB/MSU

SFB 1245 “Nuclei: From Fundamental Interactions to Structure and Stars

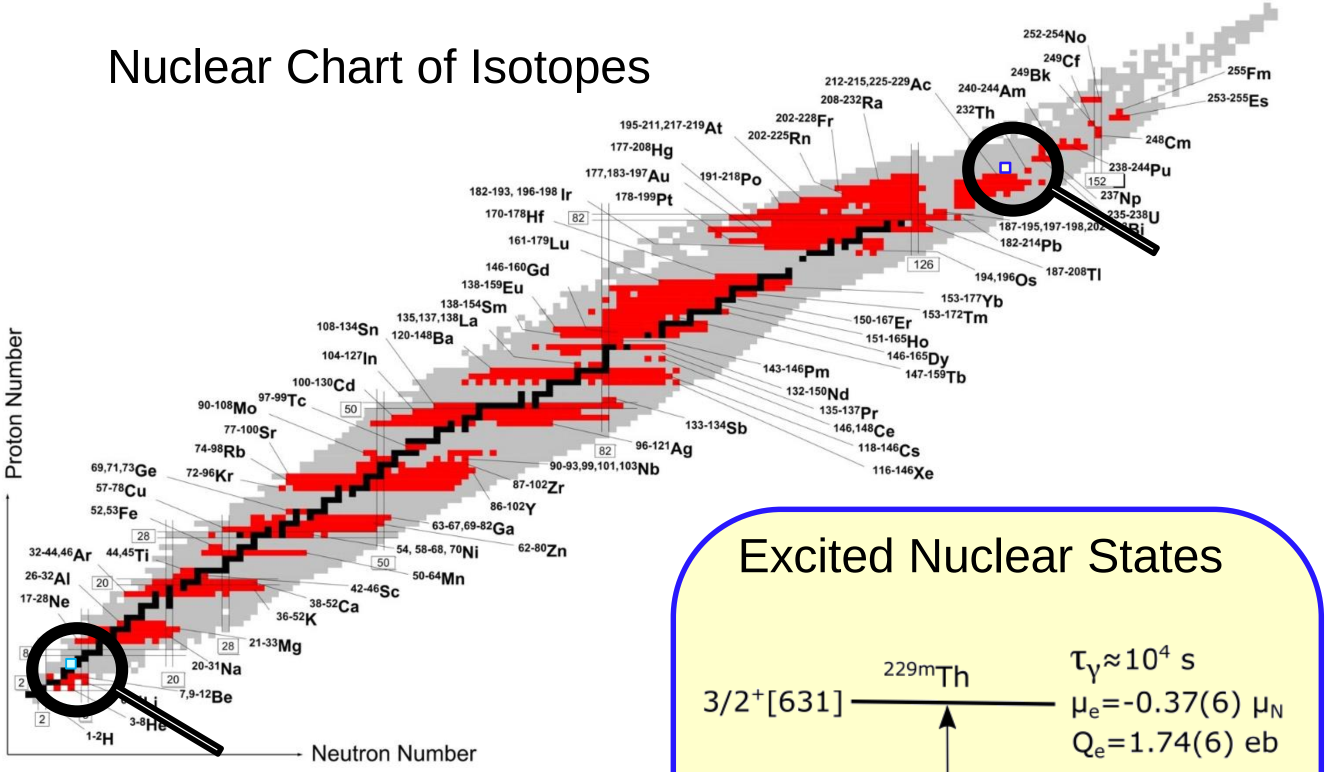

KOALA @ TU Darmstadt

Emission Spectra of the Elements

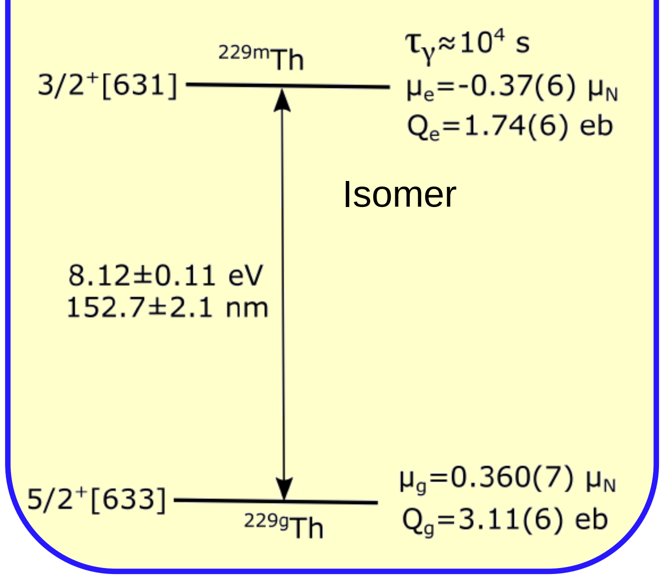


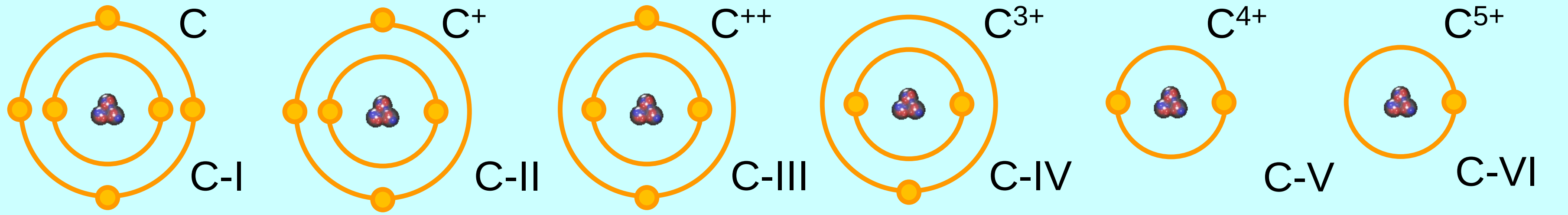
https://www.fieldtestedsystems.com/

Nuclear Chart of Isotopes



Excited Nuclear States



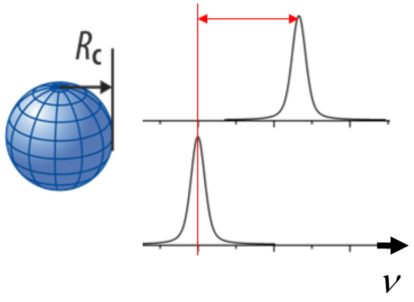


Isonuclear Series

History of Nuclear Effects in Atomic Spectra

N. Bohr, *The spectra of helium and hydrogen*
Nature, 92, 231 (1913)

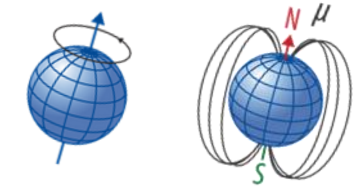
H. Urey et al., *A Hydrogen Isotope of Mass 2*
Physical Review 39, 164 (1932).



$$\delta\nu_{\mathrm{IS}} = \delta\nu_{\mathrm{MS}} + F\,\delta\left\langle r_{\mathrm{c}}^{2}\right\rangle^{AA'}$$

Nagaoka & Sugiura, *Spectroscopic evidence of isotopy*,
Jpn. J. Phys. 2, 167 (1923)

W. Pauli, *Zur Frage der theoretischen Deutung der Satelliten einiger Spektrallinien und ihrer Beeinflussung durch magnetische Felder*,
Naturwissenschaften **12**, 741 (1924).



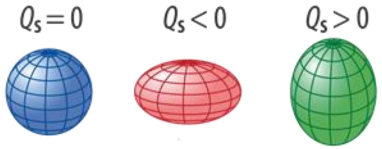
Spin I

Magnetic
Dipole
Moment μ

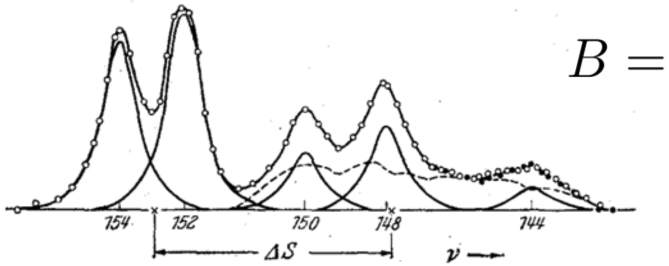
$$A = \frac{\mu_I\, B_e(0)}{I J}$$

H. Schüler and T. Schmidt, *Über Abweichungen des Atomkerns von der Kugelsymmetrie,*
Zeitschrift für Physik A **94**, 457 (1935).

Electric Quadrupole
Moment Q_s

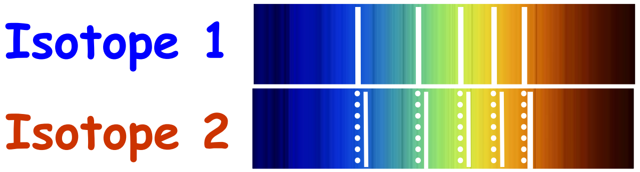


P. Brix and **H. Kopfermann**, *Zur Isotopieverschiebung im Spektrum des Samariums,* Zeitschrift für Physik **126**, 344 (1949).

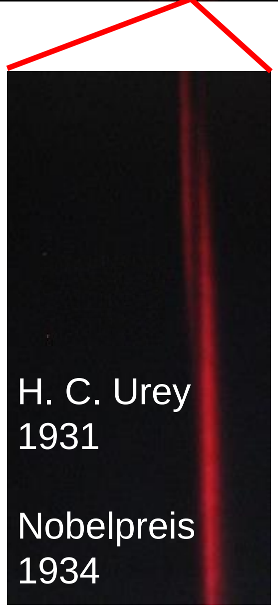


$$B = eQ_{\mathrm{s}}V_{zz}$$

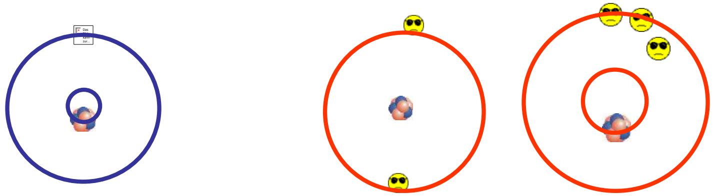
Mass Shift



$$\delta\nu_{\mathrm{IS}}^{AA'} = \nu^{A'} - \nu^{A}$$


H. C. Urey
1931

Nobelpreis
1934



Normal
mass effect
(NMS)

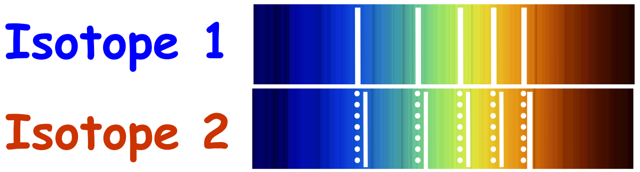
Spezific
mass effect
(SMS)

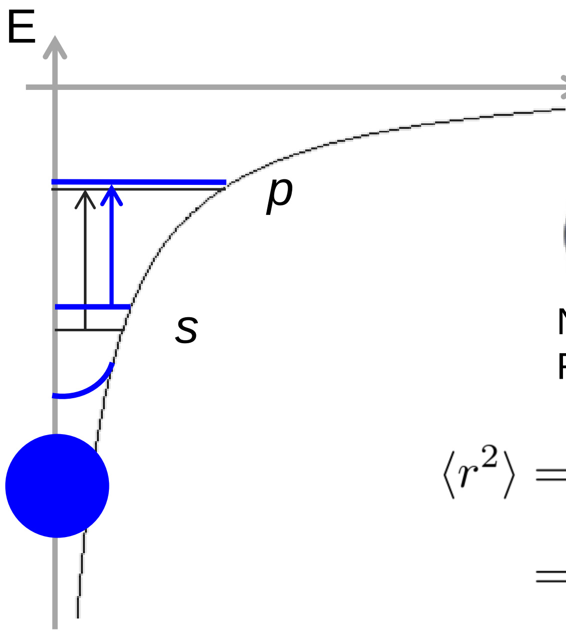
$$\delta\nu_{\mathrm{MS}}^{AA'} = \delta\nu_{\mathrm{NMS}}^{AA'} + \delta\nu_{\mathrm{SMS}}^{AA'}$$

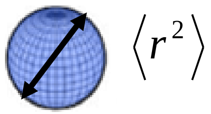
Mass shift (MS)

$$= (K_{\mathrm{NMS}} + K_{\mathrm{SMS}}) \cdot \frac{M_{A'} - M_{A}}{M_{A} M_{A'}}. \qquad K_{\mathrm{NMS}} = \nu_0 \cdot m_{\mathrm{e}}$$

Field Shift (Volume Shift, Finite Nuclear-Size Effect)

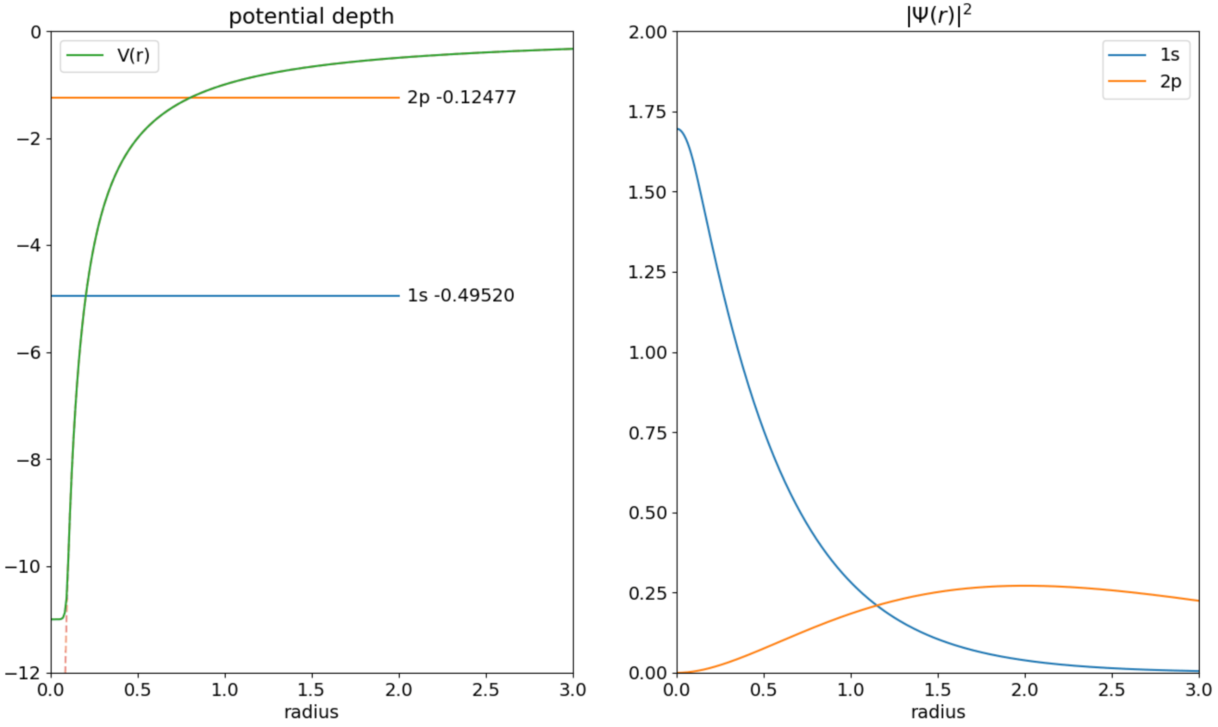






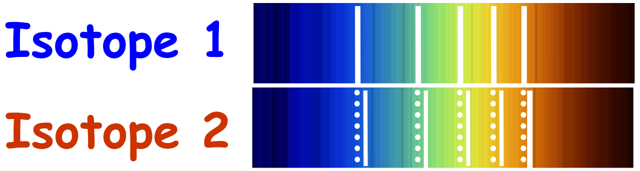
Nuclear **Charge** Radius

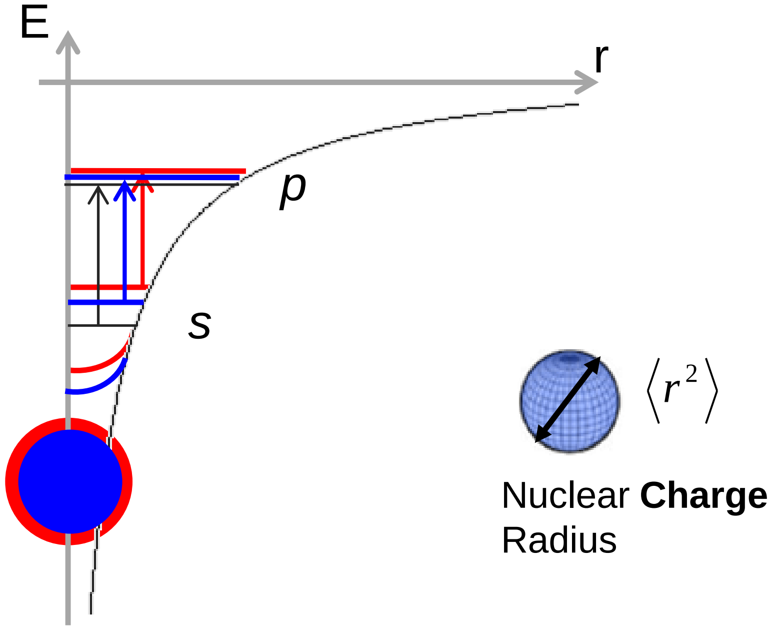
$$\begin{aligned}\langle r^2\rangle &= \int \rho(r) r^2 \mathrm{d}V \\ &= 4\pi \int_0^\infty \rho(r) r^4 \mathrm{d}r\end{aligned}$$



Animation by Tim Lellinger

Field Shift (Volume Shift, Finite Nuclear-Size Effect)

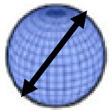




Nuclear **Charge** Radius

$$\delta\nu_{\mathrm{FS}}^{AA'} = \frac{2\pi Z e}{3} \Delta \left|\Psi(0)\right|^2 \left(\left\langle r^2 \right\rangle^{A'} - \left\langle r^2 \right\rangle^{A} \right)$$

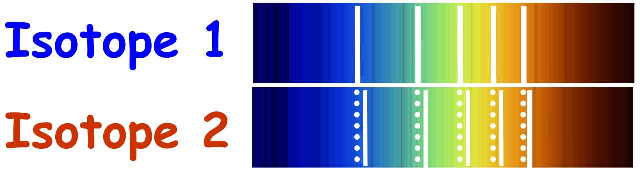
$$= \underbrace{\frac{2\pi Z e}{3} \Delta \left|\Psi(0)\right|^2}_{} \; \underbrace{\delta \left\langle r^2 \right\rangle^{AA'}}_{}$$



Electronic Factor
(→ Wavefunction)

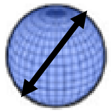
Nuclear Size:
Change of Nuclear
Charge Radius

Summary Isotope Shift



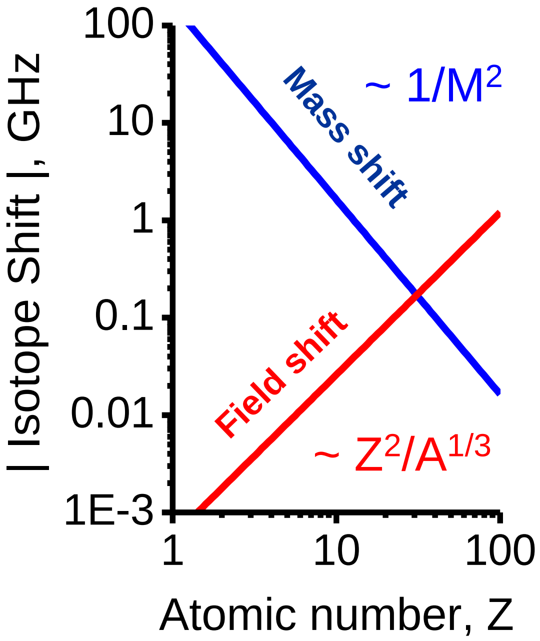
$$\delta\nu_{\mathrm{MS}}^{AA'} = \delta\nu_{\mathrm{NMS}}^{AA'} + \delta\nu_{\mathrm{SMS}}^{AA'}$$
$$= (K_{\mathrm{NMS}} + K_{\mathrm{SMS}}) \cdot \frac{M_{A'} - M_A}{M_A M_{A'}}$$

$$\delta\nu_{\mathrm{FS}}^{AA'} = \frac{2\pi Ze}{3}\Delta\left|\Psi(0)\right|^2 \left(\left\langle r^2\right\rangle^{A'} - \left\langle r^2\right\rangle^{A}\right)$$
$$= \underbrace{\frac{2\pi Ze}{3}\Delta\left|\Psi(0)\right|^2}\ \underbrace{\delta\left\langle r^2\right\rangle^{AA'}}$$

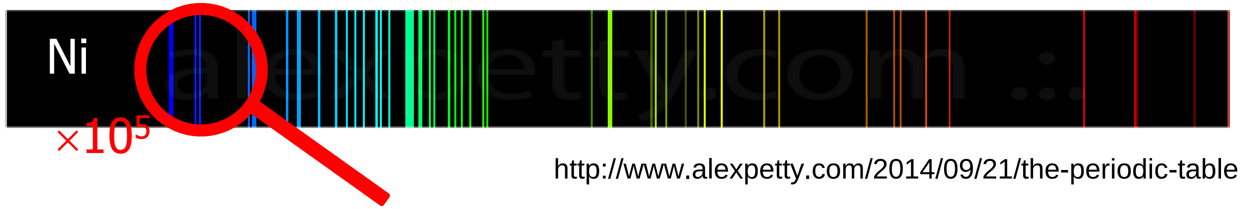


Electronic Factor
(→ Wavefunction)

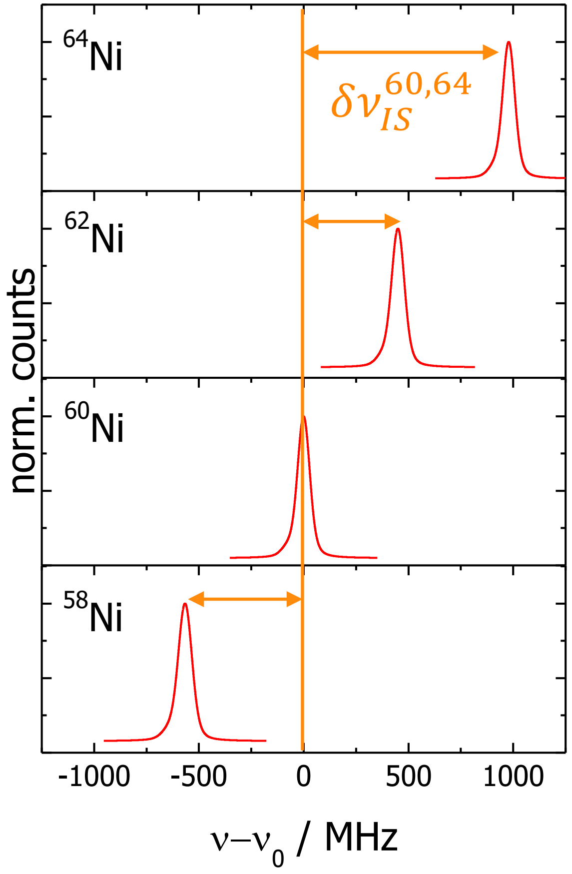
Nuclear Size:
Change of Nuclear
Charge Radius



Summary Isotope Shift (Example: Nickel)



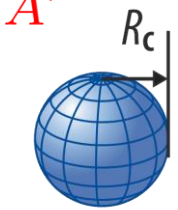
http://www.alexpetty.com/2014/09/21/the-periodic-table-of-light/



isotope shift

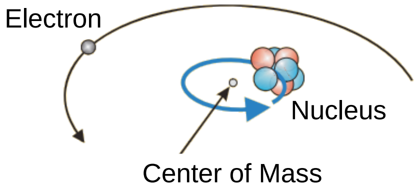
charge radius

$$\delta\nu_{\mathrm{IS}}^{AA'} = K \cdot M + F\,\delta\left\langle r_{\mathrm{c}}^{2}\right\rangle^{AA'}$$

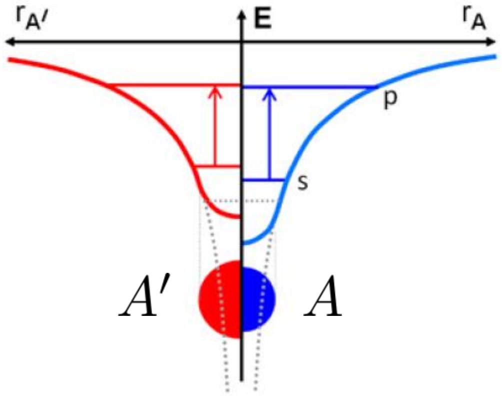


mass shift
(cm motion)

field shift



Mass Shift



Isotope Shift

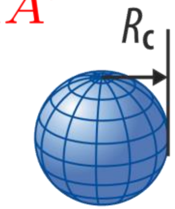
isotope shift

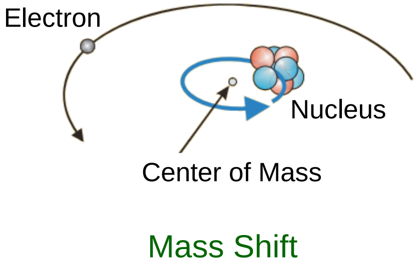
charge radius

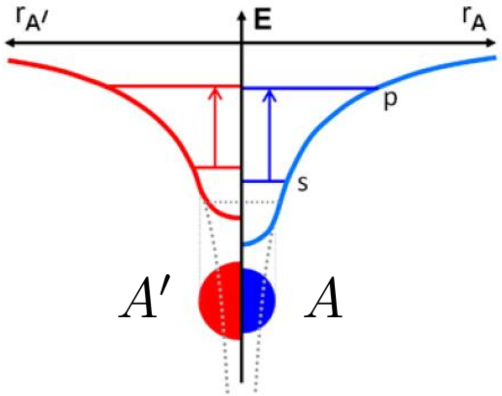
$$\delta\nu_{\mathrm{IS}}^{AA'} = K\cdot M + F\,\delta\left\langle r_{\mathrm{c}}^{2}\right\rangle^{AA'}$$

mass shift
(cm motion)

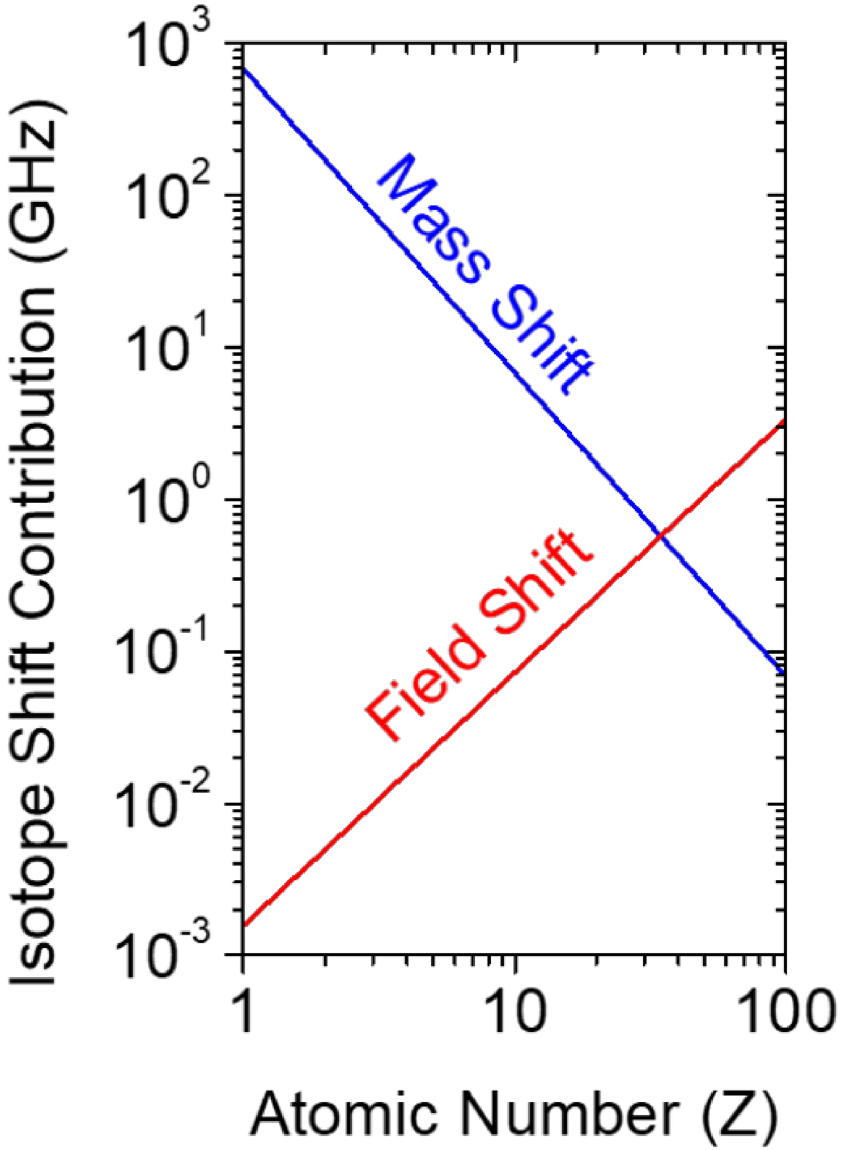
field shift







$$\delta\left\langle r_{\mathrm{c}}^{2}\right\rangle^{AA'} = \frac{\delta\nu_{\mathrm{IS}}^{AA'} - K\cdot M}{F}$$



Absolute Radii of light Isotopes

Required Accuracy $< 10^{-5}$

$$\delta\left\langle r_{\mathrm{c}}^{2}\right\rangle^{AA'} = \frac{\delta\nu_{\mathrm{IS}}^{AA'} - K\cdot M}{F}$$

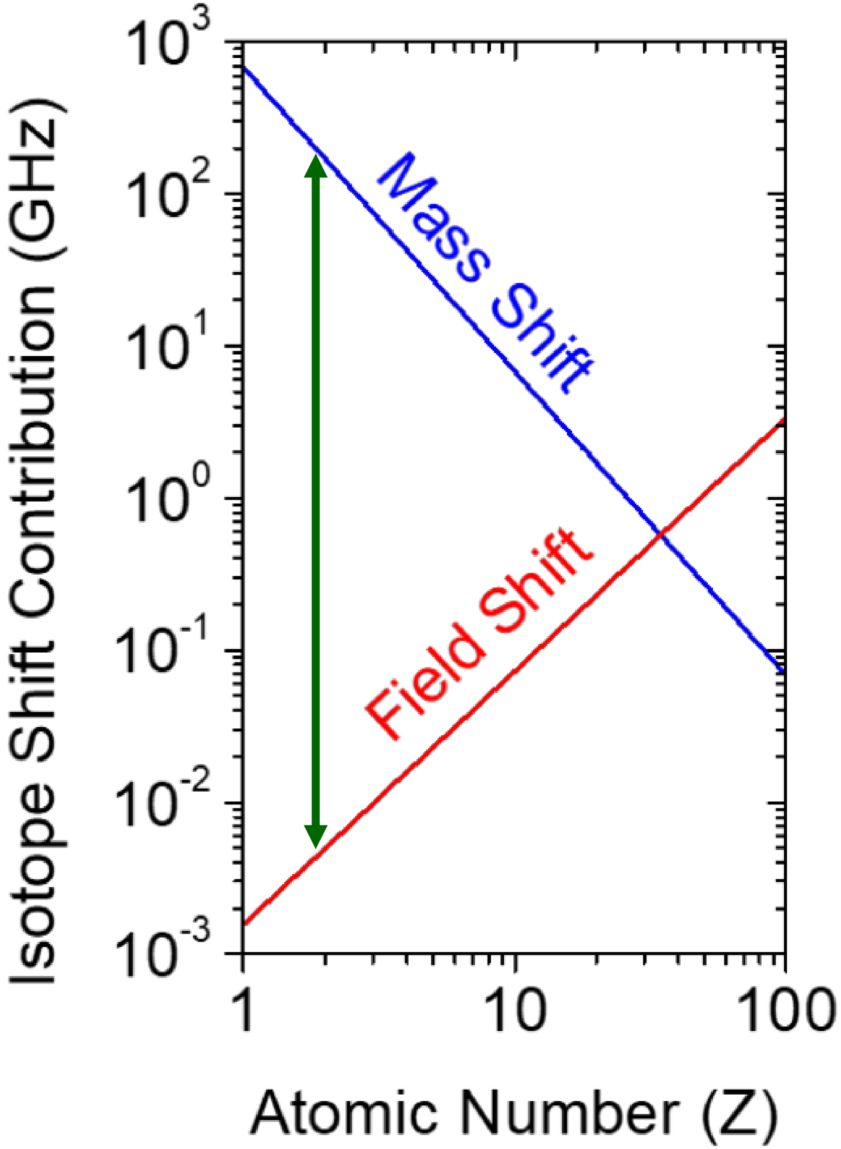
Theory

Mass Shift Calculations:

He (2 e⁻) : F. Marin et al., Z. Phys. D **32**, 285 (1995).
Li (3 e⁻) : Z.C. Yan and G.W.F. Drake, PRA **61**, 022504 (2000).
Be (4 e⁻) : M. Puchalski et al., PRA **89**, 012506 (2014).
B (5 e⁻) : B. Maaß et al, PRL **122**, 182501 (2019)

$$R_{\mathrm{c}}(A) = \underbrace{R_{\mathrm{c}}(A_{\mathrm{ref}})} + \delta\left\langle r_{\mathrm{c}}^{2}\right\rangle^{A_{\mathrm{ref}},A}$$

Reference radius required
from a different technique !



From where do we get a Reference Radius ?

(1) Elastic Electron Scattering

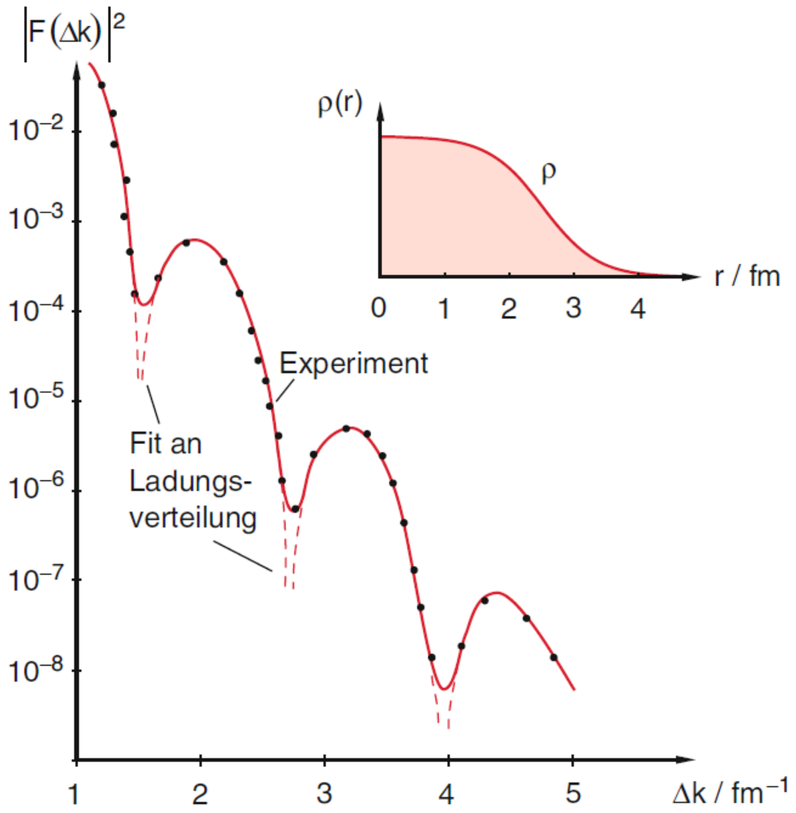
Form Factor: $$\left|F(q^2)\right|^2 = \frac{\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}}}{\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}}} \qquad q = \frac{2E}{\hbar c} \cdot \sin^2(\theta/2)$$

$$\rho_N(r) = \frac{1}{2\pi} \int_0^\infty \mathrm{d}q\; q^2\, F(q)\, \frac{\sin(qr)}{qr}$$

or in more general terms:

$$\rho_N(r) = \frac{1}{(2\pi)^3} \int \mathrm{d}^3 q\, e^{-i\vec{q}\cdot\vec{r}}\, F(q).$$

The form factor is the Fourier transform of the charge distribution.



Scattering of 750 MeV electrons on O atoms. The dashed curve is calculated based on the density distribution shown in the inlet.

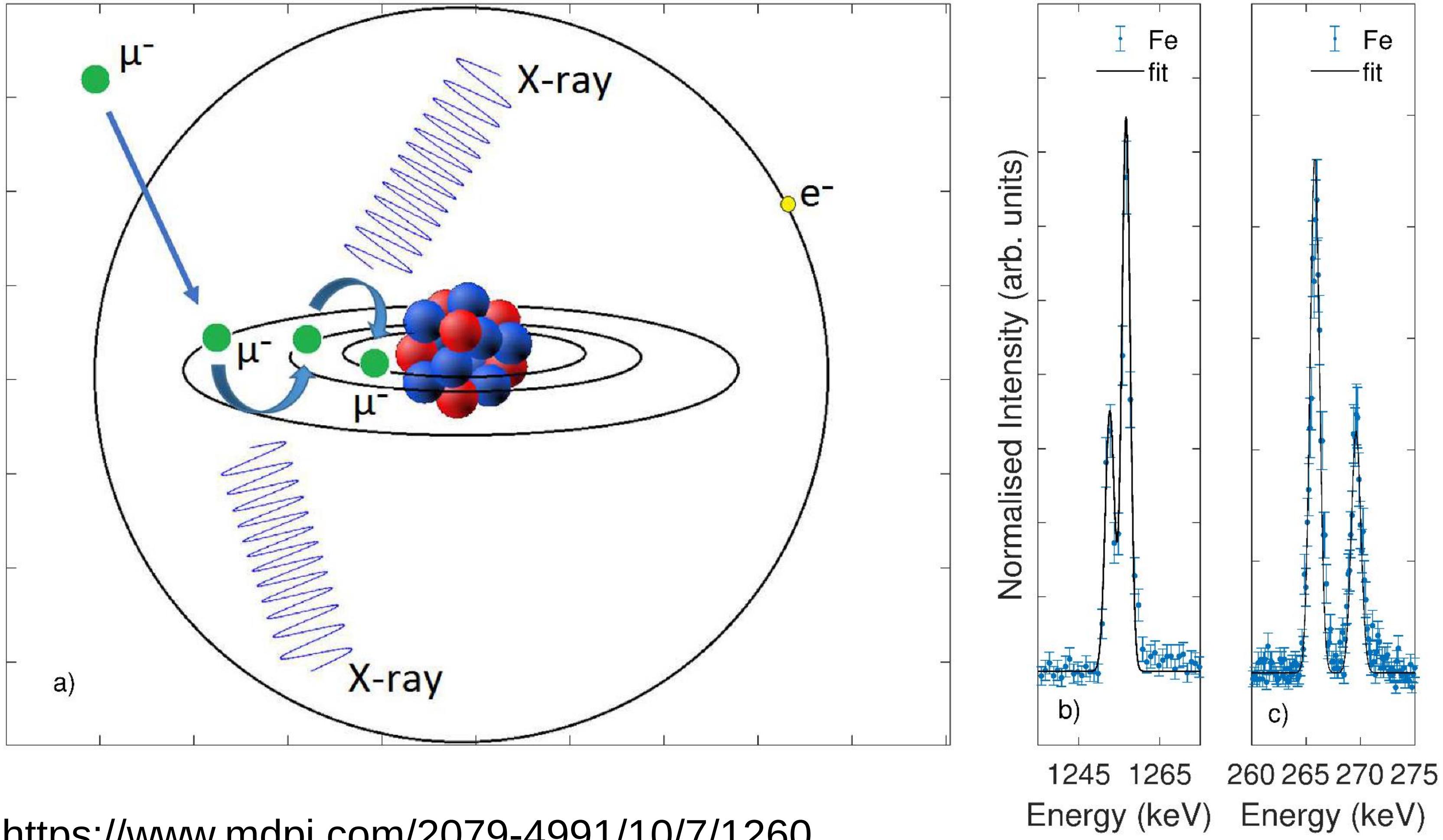
[Demtröder, Experimentalphysik IV]

- To some extent model-dependent
- Contributions of inelastic scattering and higher order contributions must be excluded

From where do we get a Reference Radius ?

(2) Muonic Atoms X-Ray Spectroscopy

$K_{\alpha 1,\, \alpha 2}$ $L_{\alpha},\, L_{\beta}$



https://www.mdpi.com/2079-4991/10/7/1260

Barret Radii:

$$\langle r^k e^{-\alpha r}\rangle = \frac{4\pi}{Ze}\int \rho_{\mathrm{c}}(r) r^k e^{-\alpha r} r^2 \, \mathrm{d}r$$

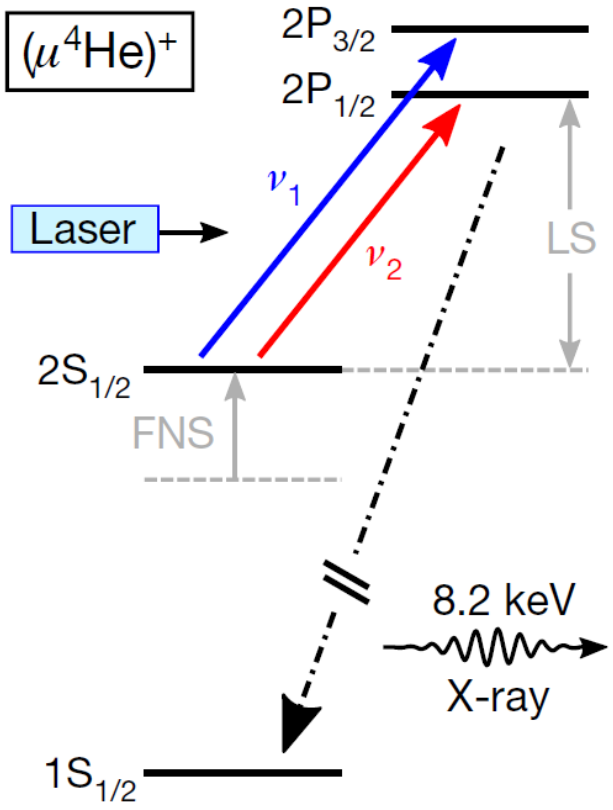
Largest uncertainty contribution:
Nuclear polarization effects

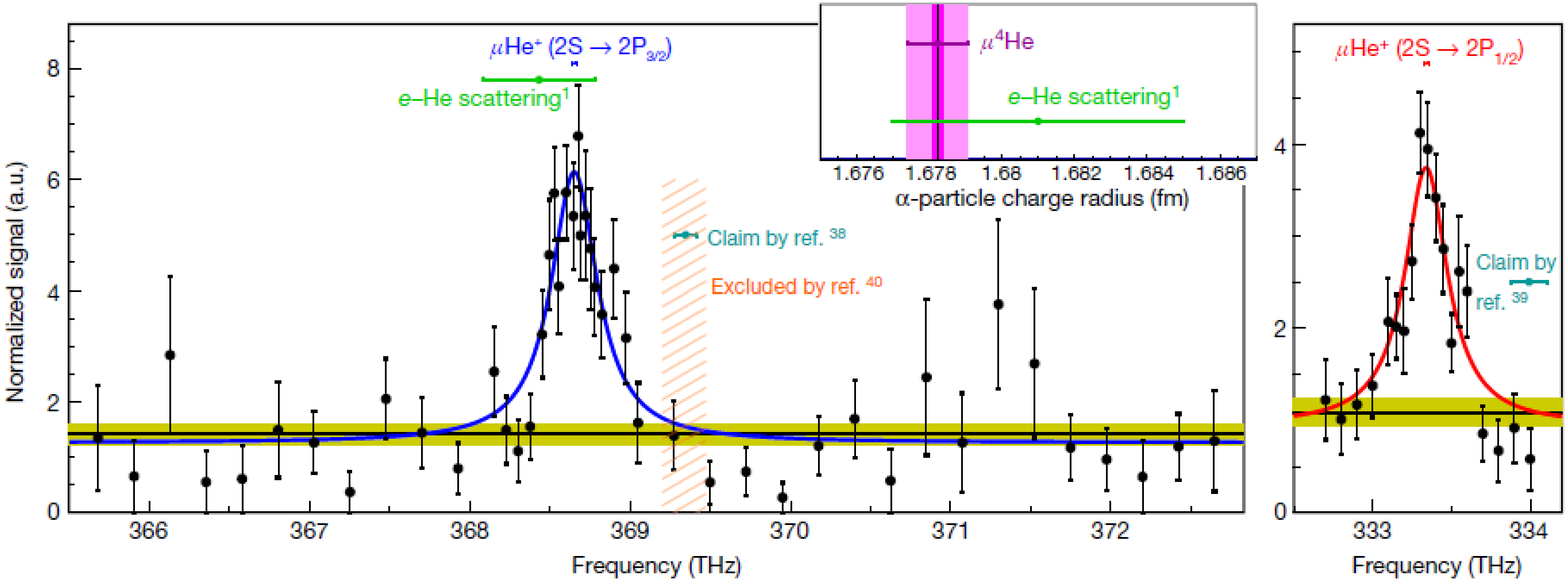
Combination with electron scattering data (ratio of moments) provides reliable reference radii.

From where do we get a Reference Radius ?

(3) Muonic Atoms Laser Spectroscopy

$$
\begin{aligned}
\Delta E^{\mathrm{theo}}_{2\mathrm{P}_{1/2}-2\mathrm{S}} = {} & 1{,}668.489(14)\ \mathrm{meV} \\
& - 106.220(8)\ \mathrm{meV\,fm}^{-2} \times r_\alpha^2 + 0.0112\ \mathrm{meV} \\
& + 9.340(250)\ \mathrm{meV} \\
& - 0.150(150)\ \mathrm{meV}.
\end{aligned}
$$





$$\Delta E^{\mathrm{exp}}_{2\mathrm{P}_{1/2}-2\mathrm{S}} = 1{,}378.521 \pm 0.048\ \mathrm{meV}, \qquad r_\alpha = 1.67824(13)_{\mathrm{exp}}\,(82)_{\mathrm{theo}}\ \mathrm{fm}.$$

Krauth *et al*., Nature 589, 527(2021)

Atomic Theory

Calculate the energy of an atomic state (Bohr):

$$E_n = -\frac{Z^2}{n^2}\frac{\alpha^2 m_e c^2}{2}$$

Employ pertubation theory (in α):

$$E_{tot} = E_{\mathrm{NR}} + \alpha^2 E_{\mathrm{rel}} + \alpha^3 E_{\mathrm{QED}} + \ldots + \Delta E_{\mathrm{nuc}}$$

$$\Delta E_{nuc} = \frac{2\pi Z e^2 r_c^2}{3}\sum_i \langle \delta^3(r_i)\rangle$$

$$= |\Psi(0)|^2$$

And again (in μ/m):

$$E_{\mathrm{NR}} = E_{\mathrm{NR}}^{(0)} + \left(\frac{\mu}{M}\right) E_{\mathrm{NR}}^{(1)} + \left(\frac{\mu}{M}\right)^2 E_{\mathrm{NR}}^{(2)} + \ldots$$

$$E_{\mathrm{rel}} = E_{\mathrm{rel}}^{(0)} + \left(\frac{\mu}{M}\right) E_{\mathrm{rel}}^{(1)} + \ldots$$

$$E_{\mathrm{QED}} = E_{\mathrm{QED}}^{(0)} + \left(\frac{\mu}{M}\right) E_{\mathrm{QED}}^{(1)} + \ldots$$

largest uncertainty

Exact energy calculation possible for hydrogen (1e⁻)

See: proton radius puzzle…

[Udem et a., Phys. Rev. Lett. 79, 2646 (1997)]
[R. Pohl et al., Nature 466, 213 EP (2010)]

Isotope Shift

Subtract Energies for two isotopes, a and b: Only mass-dependent terms stay!

$$
\begin{aligned}
\Delta E(a-b) = {} & \left[\left(\frac{\mu}{M}\right)_a - \left(\frac{\mu}{M}\right)_b\right]\left(E_{NR}^{(1)} + \alpha^2 E_{rel}^{(1)} + \alpha^3 E_{QED}^{(1)}\right) \\
& + \left[\left(\frac{\mu}{M}\right)_a^2 - \left(\frac{\mu}{M}\right)_b^2\right] E_{NR}^{(2)} + \ldots + \frac{2\pi Z e^2}{3}|\Psi(0)|^2\left[r_{c,a}^2 - r_{c,b}^2\right]
\end{aligned}
$$

Isotope Shift

Subtract Energies for two isotopes, a and b: Only mass-dependent terms stay!

$$\Delta E(a-b) = \left[\left(\frac{\mu}{M}\right)_a - \left(\frac{\mu}{M}\right)_b\right]\left(E^{(1)}_{NR} + \alpha^2 E^{(1)}_{rel} + \alpha^3 E^{(1)}_{QED}\right) + \left[\left(\frac{\mu}{M}\right)^2_a - \left(\frac{\mu}{M}\right)^2_b\right] E^{(2)}_{NR} + \ldots + \frac{2\pi Z e^2}{3}|\Psi(0)|^2\left[r^2_{c,a} - r^2_{c,b}\right]$$

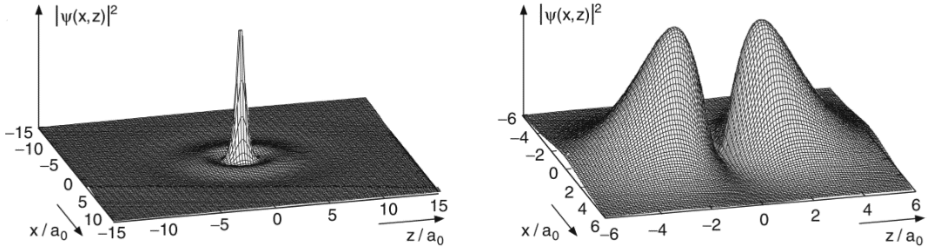
Test a *transition*, not an *energy*:

$$\Delta\nu_{\mathrm{IS}} = \delta\nu_{\mathrm{MS}} - F_{i\to f}\left[r^2_{c,a} - r^2_{c,b}\right]$$

$$F_{i\to f} \propto \left(|\Psi_f(0)|^2 - |\Psi_i(0)|^2\right)$$

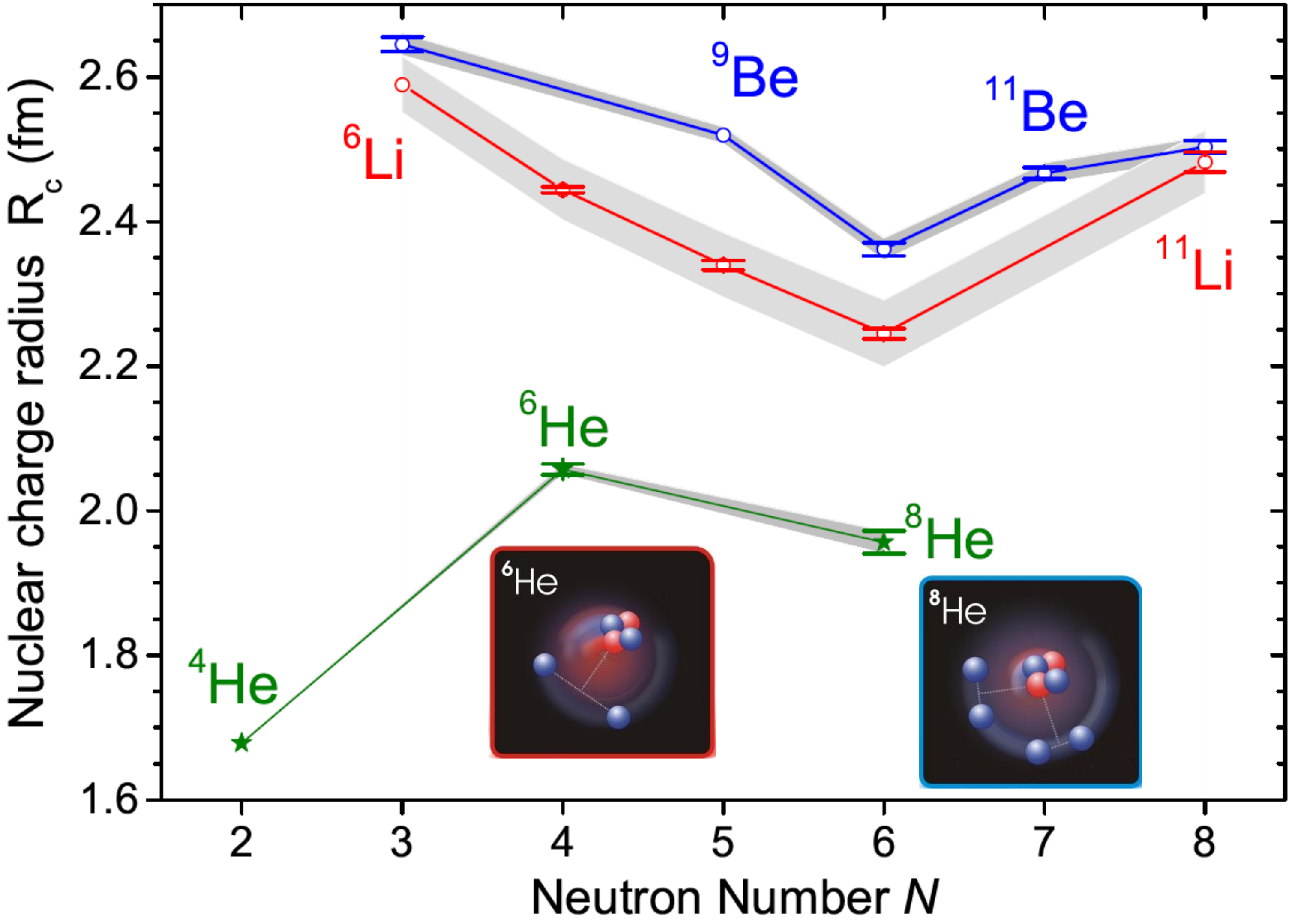
[Drake et al., Can. J. Phys. 83 311 (2005)]
[Lu et al., Rev. Mod. Phys. 85, 1383 (2013)]

Better probe the IS in a S-P transition!



[Demtröder Experimentalphysik 3, *Springer, 2005*]

Nuclear Radii of the Lightest Isotopes



Uncertainties in charge radii:

$$R_c(A) = \underbrace{R_c(A_{\text{ref}})}_{} + \underbrace{\delta\langle r_c^2\rangle^{A_{\text{ref}},A}}_{}$$

Grey Regions
$\sigma(R_c)$

Error Bars
$\sigma(\delta\nu_{IS})$

$R_c(^{9}\text{Be}) = 2.519\,(12)\ \text{fm}$

Jansen *et al*., Nucl. Phys. A 188, 337 (1972)

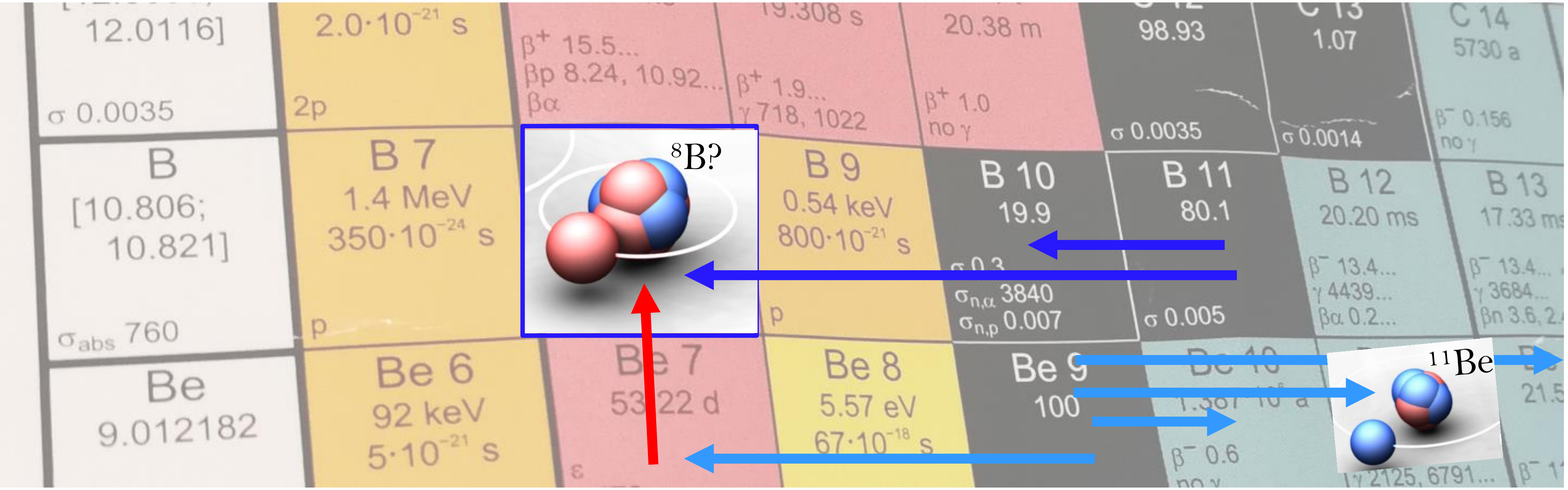
$R_c(^{6}\text{Li}) = 2.589\,(39)\ \text{fm}$

Nörtershäuser *et al*., Phys. Rev. C 84, 024307 (2011)

$\mathbf{r}_\alpha = \mathbf{1.678\,24(83)}\ \text{fm}$

Krauth *et al*., Nature 589, 527(2021)
from μHe^{+} (one-electron system)
"all- optical charge radius"

The Proton-Halo Nucleus ^{8}B



$$\delta\nu_{\mathrm{IS}} - \delta\nu_{\mathrm{MS}}^{\mathrm{Theory}} \propto \delta\langle r_{\mathrm{c}}^2\rangle$$

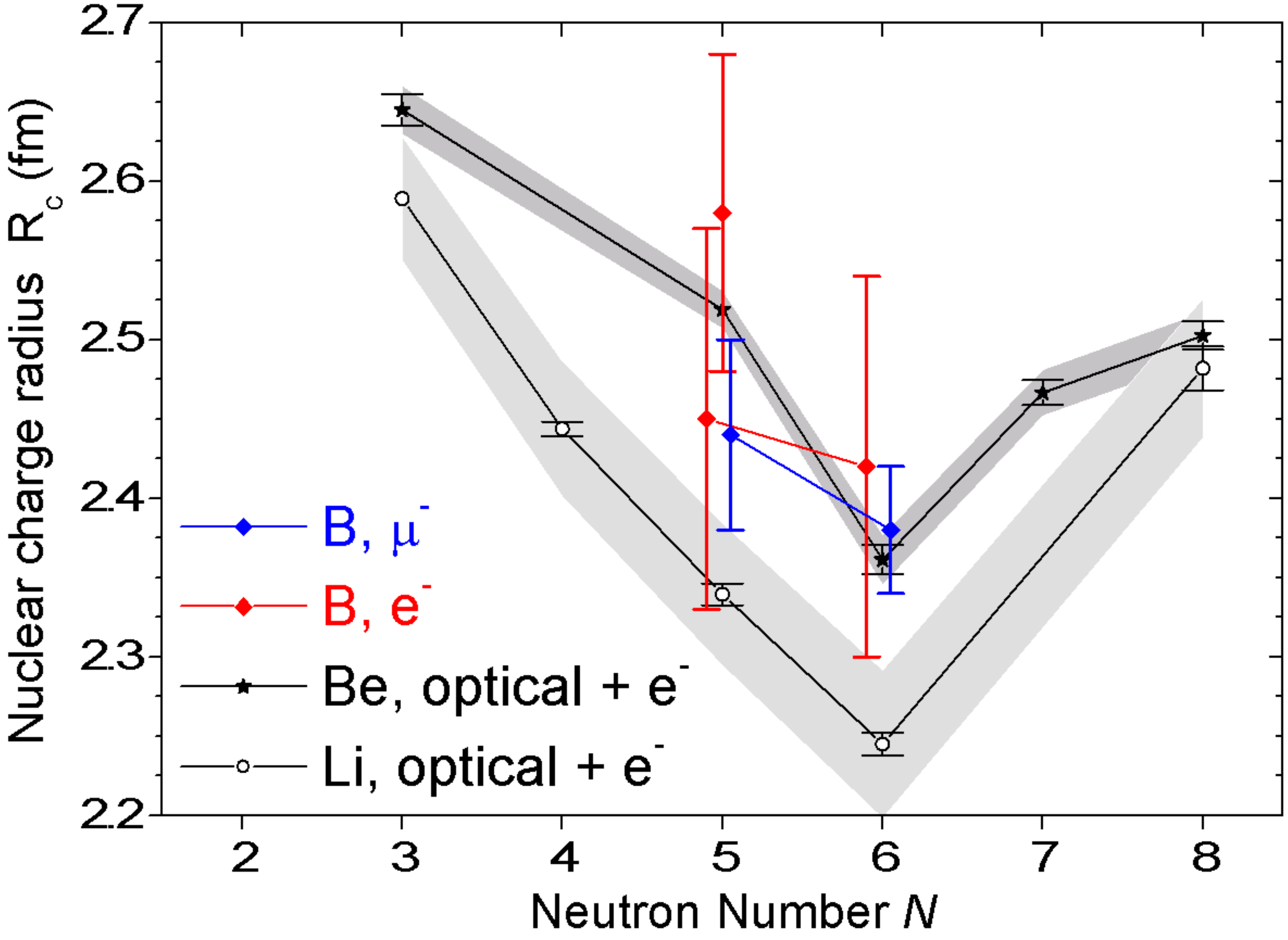
$$R_{\mathrm{c}}(A) = R_{\mathrm{c}}(A_{\mathrm{ref}}) + \delta\langle r_{\mathrm{c}}^2\rangle^{A_{\mathrm{ref}},A}$$

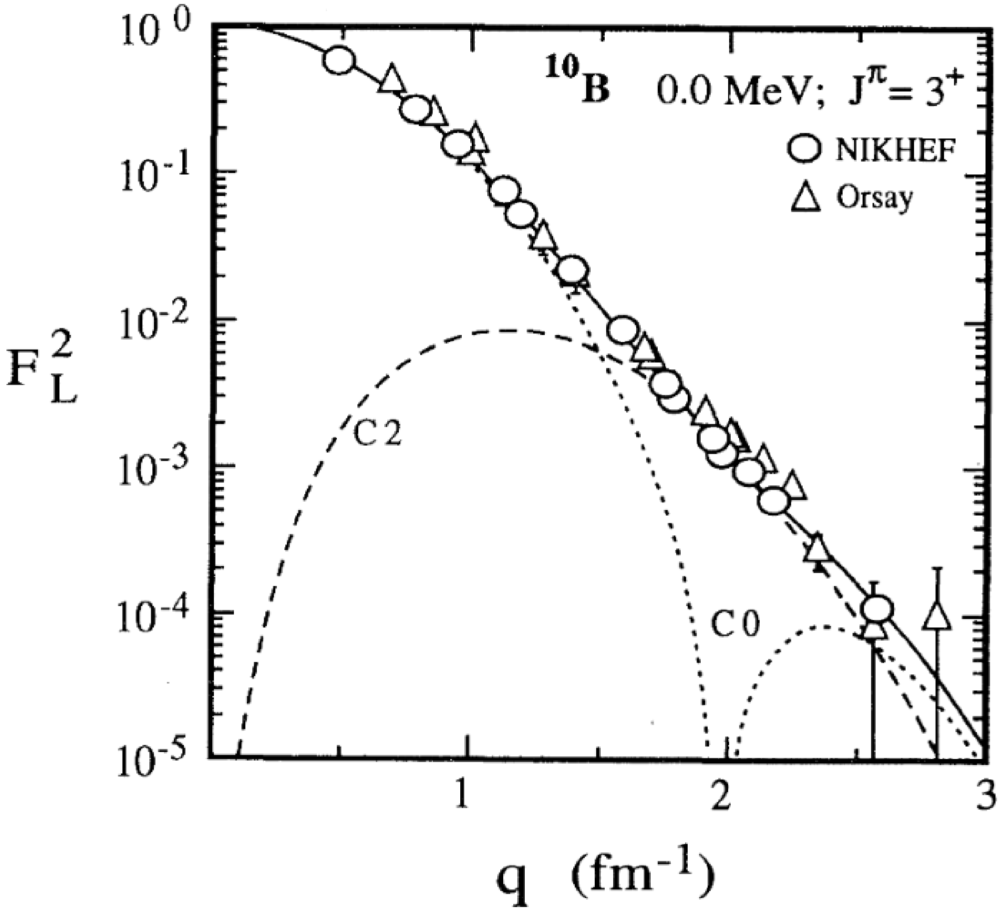
Reference Radius required

„Proton-halo size“: $R_{\mathrm{c}}(\mathrm{p_{halo}}) = R_{\mathrm{c}}(^8\mathrm{B}) - R_{\mathrm{c}}(^7\mathrm{Be})$

Conclusion: To gain information about the proton halo of ^{8}B, we need reliable reference radii for Be and B **on equal footing** !

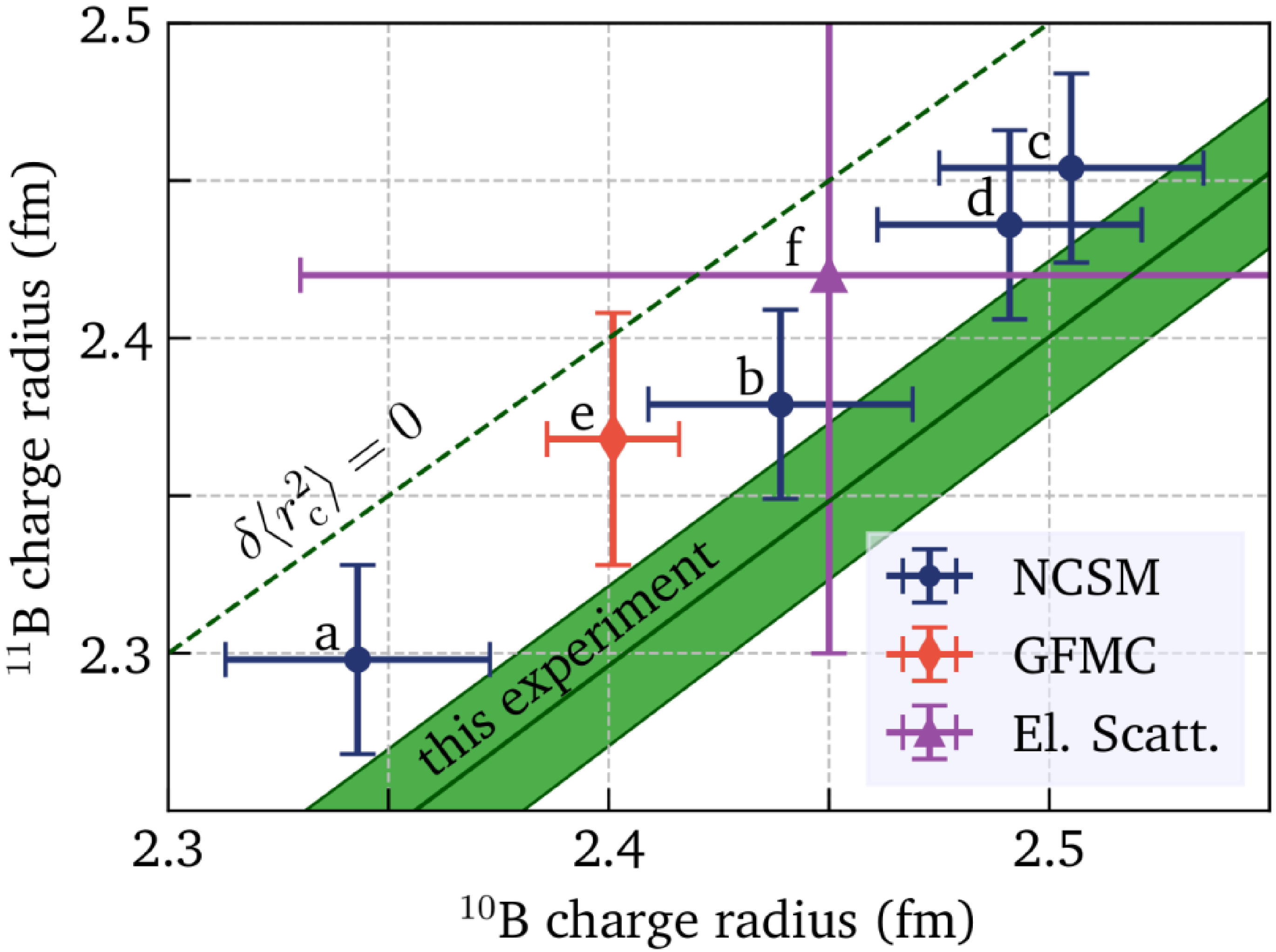
The „Tragedy“ of Boron Reference Radii





[1] Stovall, Nucl. Phys. 86, 225 (1966)
[2] Cichocki et al., PRC 51, 2406 (1995)
[3] Schaller *et al*., Nucl. Phys. A 343, 333 (1980)
[4] Olin *et al*., Nucl Phys A 360, 426 (1981)

Isotope Shift Measurement of ^{10,11}B



NCSM: No-Core Shell Model

(a) N2LO_{Sat}, (b,c) EM, N3LO (d) EMN, N4LO

Robert Roth, Thomas Hüther, TU Darmstadt

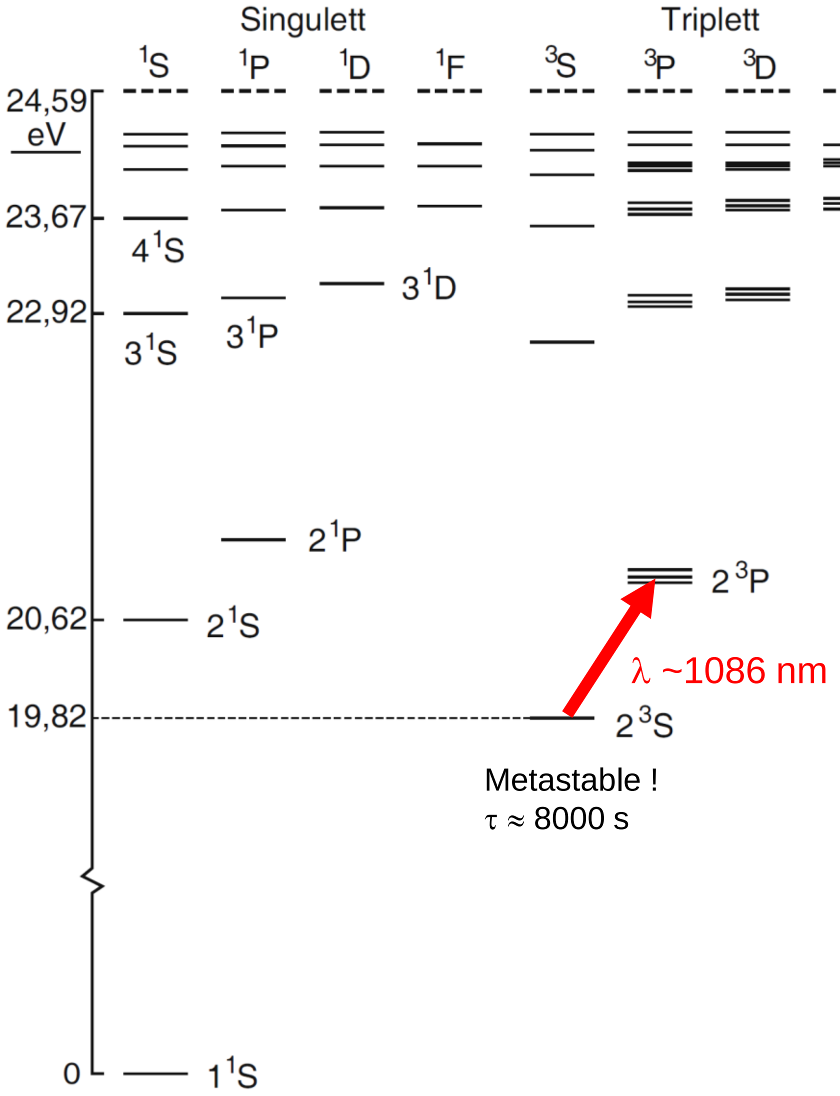
GFMC: Greens function Monte-Carlo (AV18 + IL7)

R.B. Wiringa, A. Lovato, ANL

Reference radii from eleastic electron scattering have very large uncertainties for both stable isotopes.

[Maaß et al., Phys. Rev. Lett. 122, 182501 (2019)]

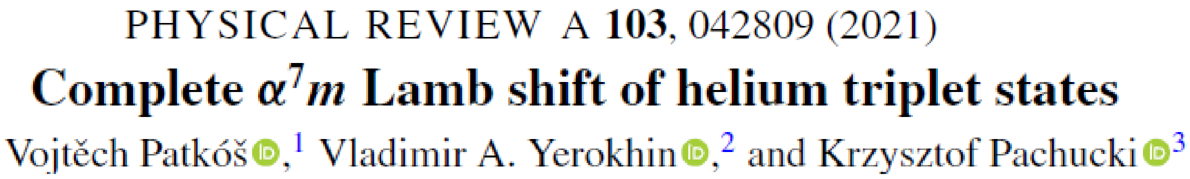
Helium



Metastable !
$\tau \approx 8000$ s

Status of Atomic Theory

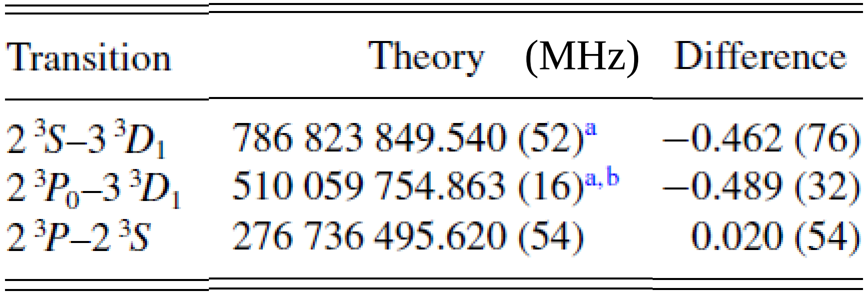
Helium


PHYSICAL REVIEW A **103**, 042809 (2021)

Complete $\alpha^7 m$ Lamb shift of helium triplet states

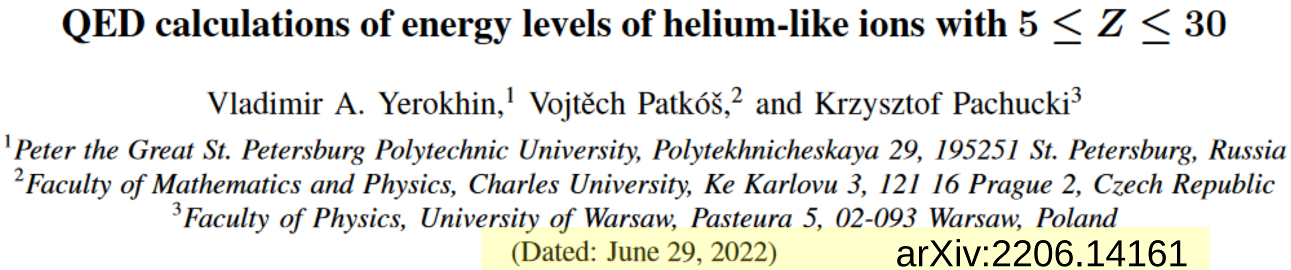
Vojtěch Patkóš,[1] Vladimir A. Yerokhin,[2] and Krzysztof Pachucki[3]

By comparing **the theoretical predictions with high-precision experimental results** (particularly, the 2 ^3S − 2 ^3P transition energy), **one can determine *R***. The present theoretical accuracy is, in principle, sufficient for a determination of the nuclear radius with an accuracy **of about 1%**.



Transition	Theory (MHz)	Difference
$2\,^3S$–$3\,^3D_1$	786 823 849.540 (52)[a]	−0.462 (76)
$2\,^3P_0$–$3\,^3D_1$	510 059 754.863 (16)[a, b]	−0.489 (32)
$2\,^3P$–$2\,^3S$	276 736 495.620 (54)	0.020 (54)

Helium-Like Systems


QED calculations of energy levels of helium-like ions with $5 \leq Z \leq 30$

Vladimir A. Yerokhin,[1] Vojtěch Patkóš,[2] and Krzysztof Pachucki[3]

[1]*Peter the Great St. Petersburg Polytechnic University, Polytekhnicheskaya 29, 195251 St. Petersburg, Russia*
[2]*Faculty of Mathematics and Physics, Charles University, Ke Karlovu 3, 121 16 Prague 2, Czech Republic*
[3]*Faculty of Physics, University of Warsaw, Pasteura 5, 02-093 Warsaw, Poland*

(Dated: June 29, 2022)

arXiv:2206.14161

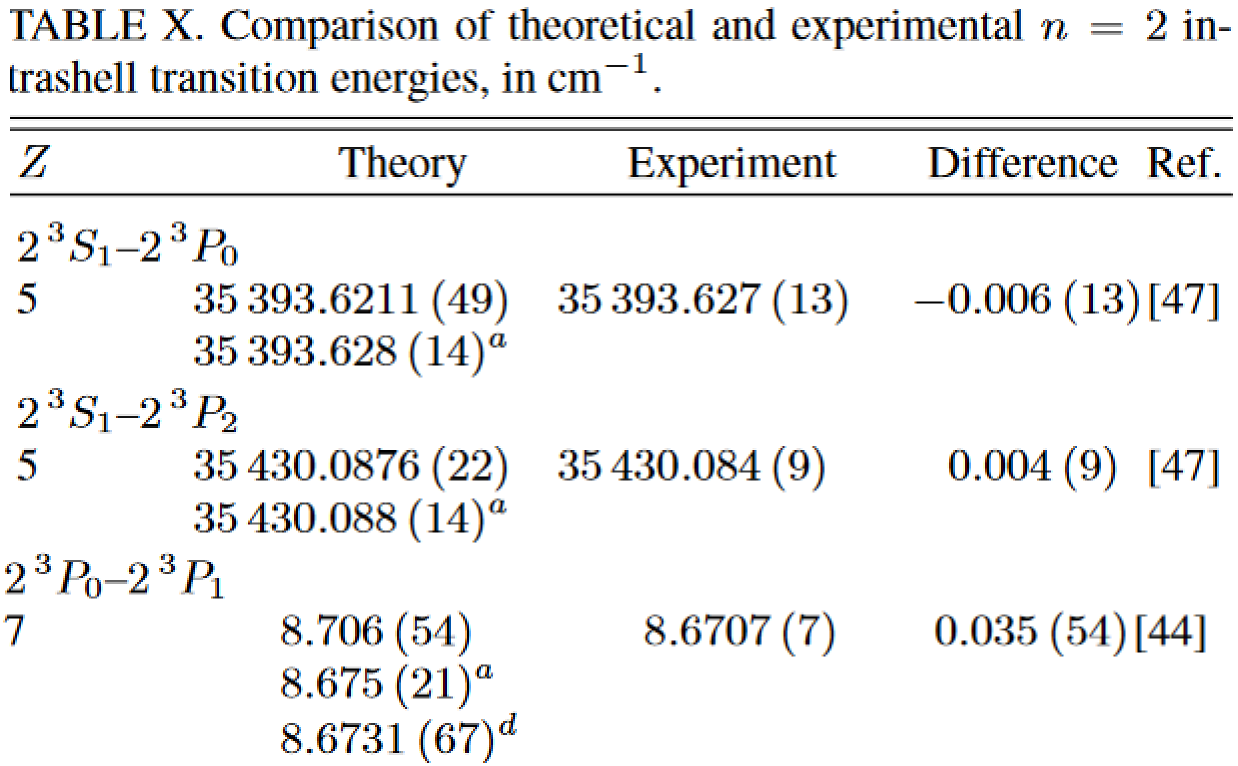
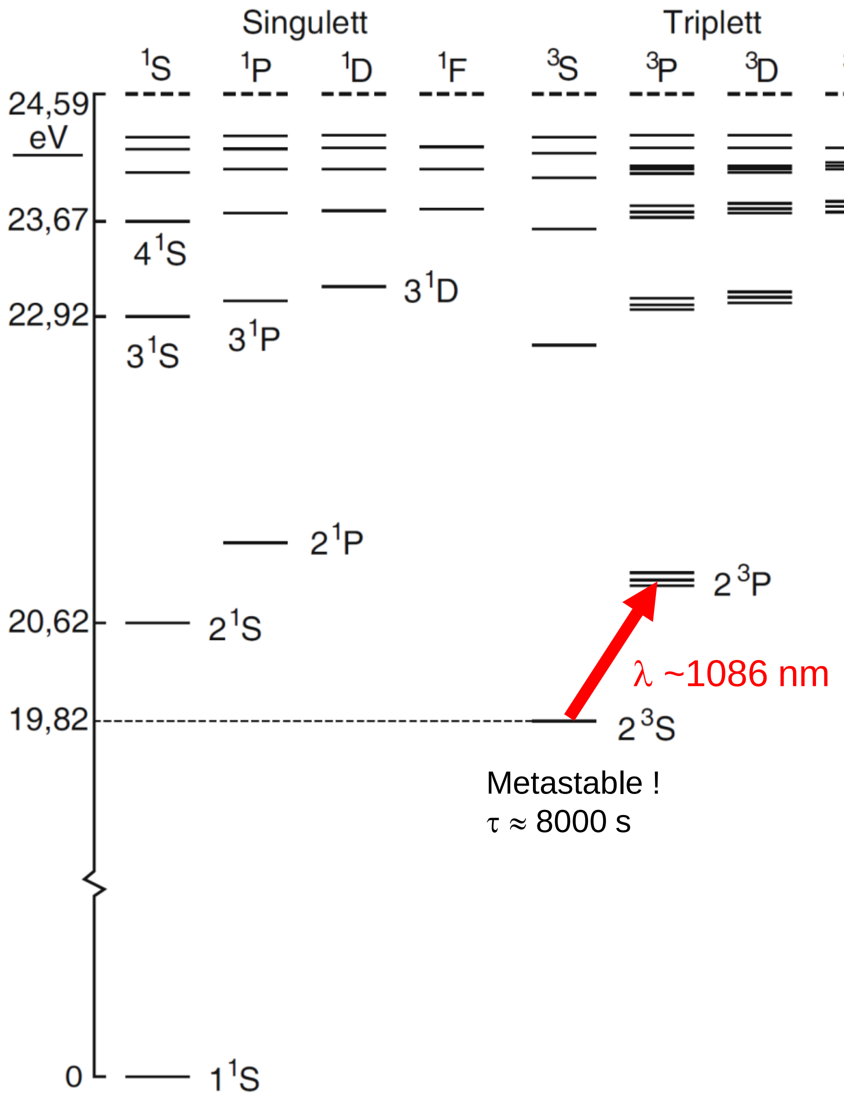
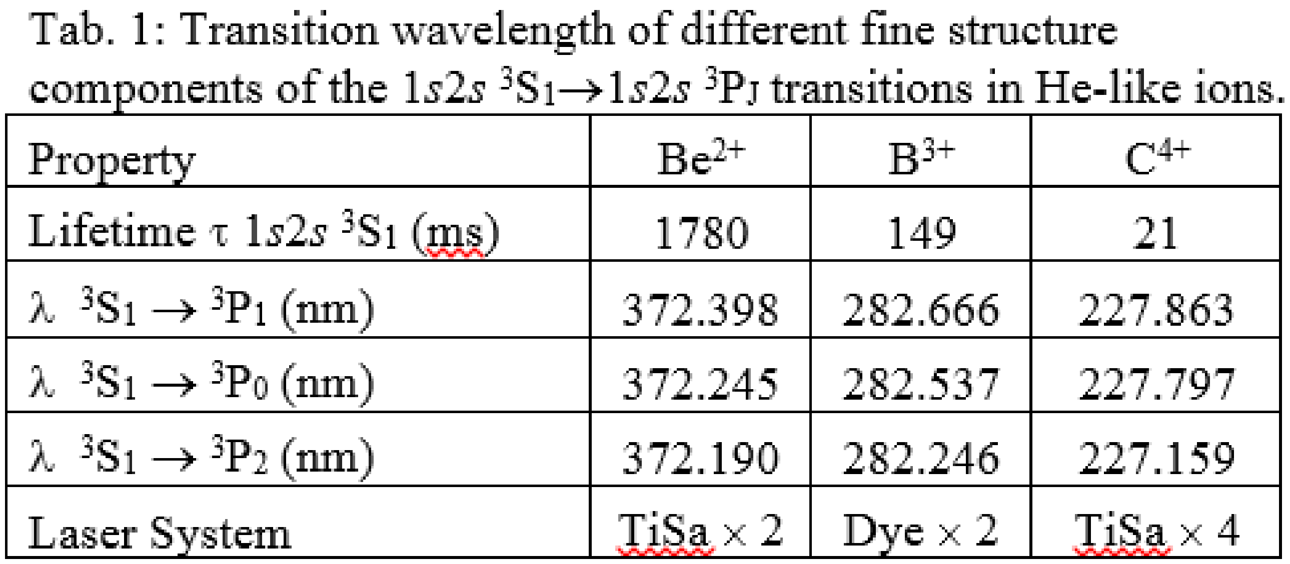


TABLE X. Comparison of theoretical and experimental $n = 2$ intrashell transition energies, in cm^{-1}.

Z	Theory	Experiment	Difference	Ref.
$2\,^3S_1$–$2\,^3P_0$				
5	35 393.6211 (49) 35 393.628 (14)[a]	35 393.627 (13)	−0.006 (13)	[47]
$2\,^3S_1$–$2\,^3P_2$				
5	35 430.0876 (22) 35 430.088 (14)[a]	35 430.084 (9)	0.004 (9)	[47]
$2\,^3P_0$–$2\,^3P_1$				
7	8.706 (54) 8.675 (21)[a] 8.6731 (67)[d]	8.6707 (7)	0.035 (54)	[44]

Helium-Like Systems

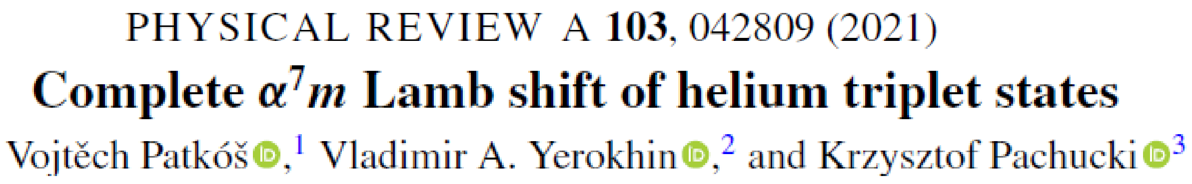



Tab. 1: Transition wavelength of different fine structure components of the $1s2s\ ^3S_1 \rightarrow 1s2s\ ^3P_J$ transitions in He-like ions.

Property	Be^{2+}	B^{3+}	C^{4+}
Lifetime τ $1s2s\ ^3S_1$ (ms)	1780	149	21
λ $^3S_1 \rightarrow {}^3P_1$ (nm)	372.398	282.666	227.863
λ $^3S_1 \rightarrow {}^3P_0$ (nm)	372.245	282.537	227.797
λ $^3S_1 \rightarrow {}^3P_2$ (nm)	372.190	282.246	227.159
Laser System	TiSa × 2	Dye × 2	TiSa × 4

Status of Atomic Theory

Helium



PHYSICAL REVIEW A **103**, 042809 (2021)

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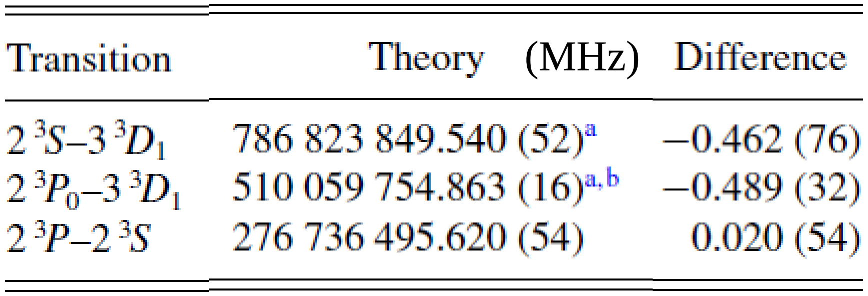

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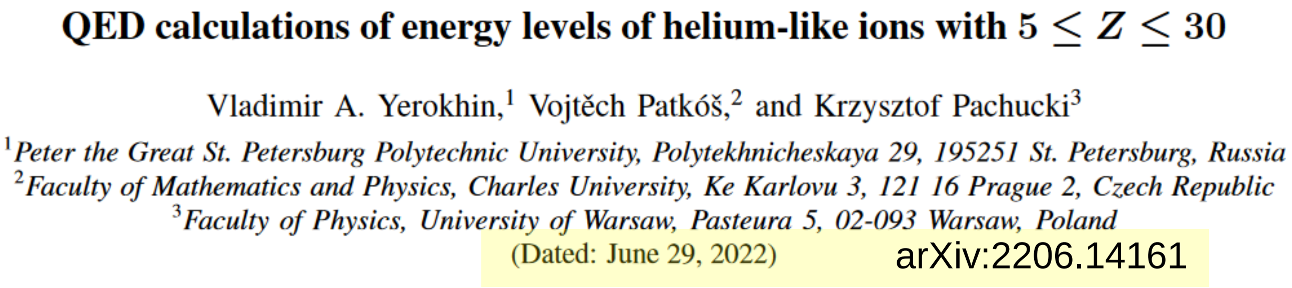

QED calculations of energy levels of helium-like ions with $5 \leq Z \leq 30$

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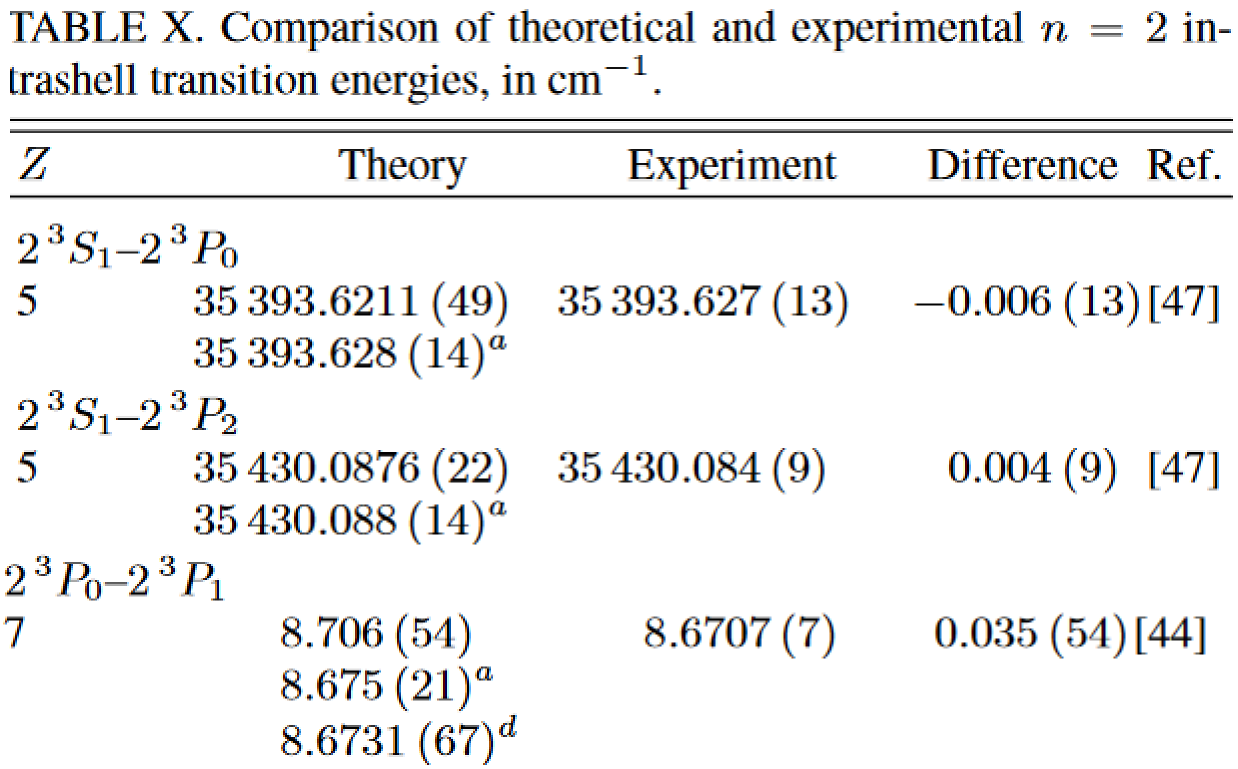
(Dated: June 29, 2022) arXiv:2206.14161



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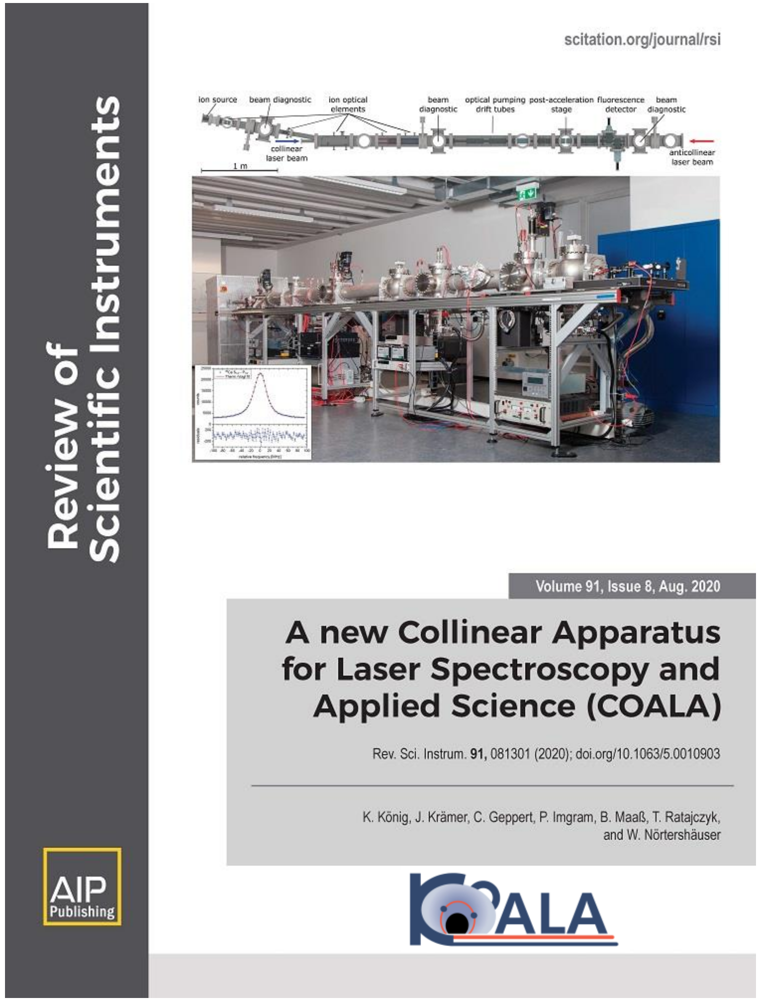
Measuring principle @ COALA, TU Darmstadt



scitation.org/journal/rsi

Review of Scientific Instruments

Volume 91, Issue 8, Aug. 2020

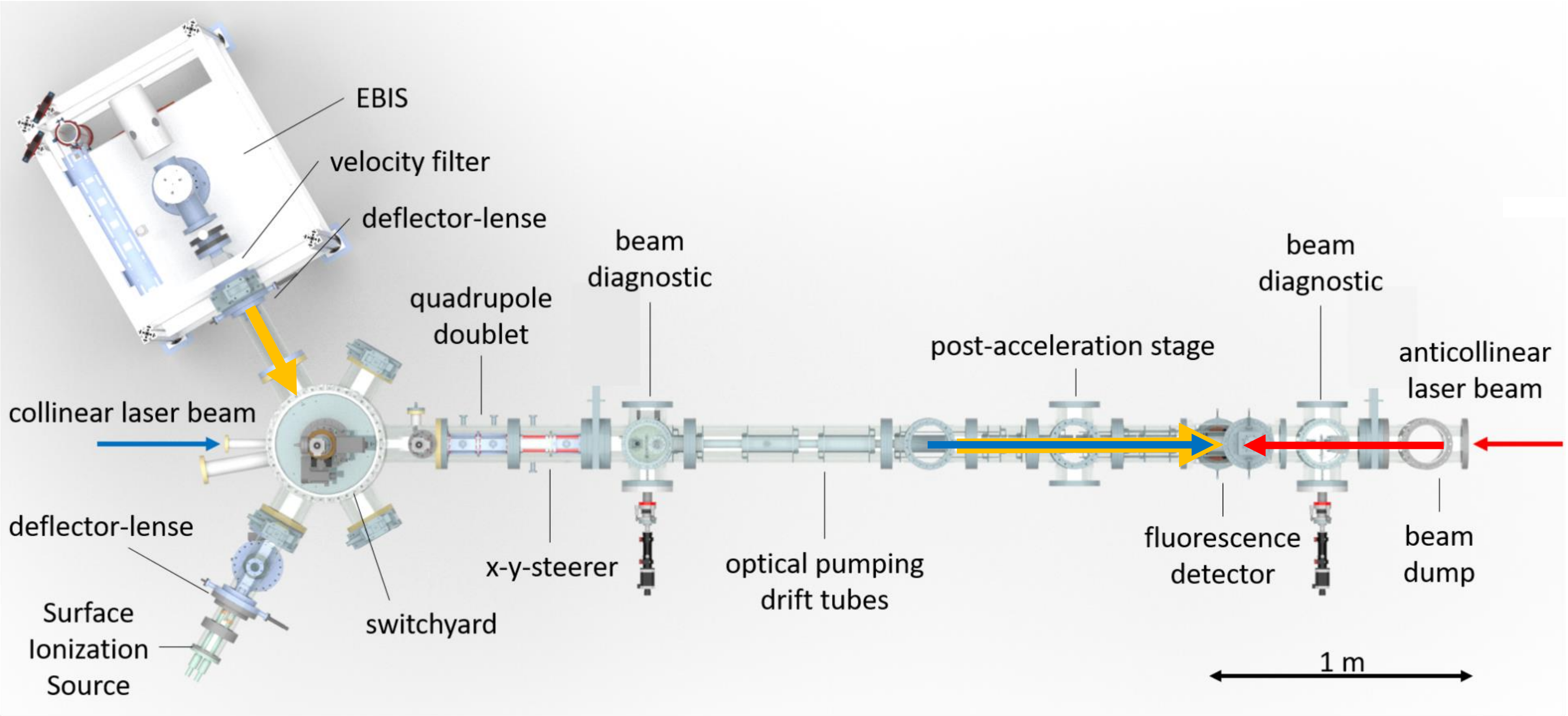
A new Collinear Apparatus for Laser Spectroscopy and Applied Science (COALA)

Rev. Sci. Instrum. **91,** 081301 (2020); doi.org/10.1063/5.0010903

K. König, J. Krämer, C. Geppert, P. Imgram, B. Maaß, T. Ratajczyk, and W. Nörtershäuser

AIP Publishing

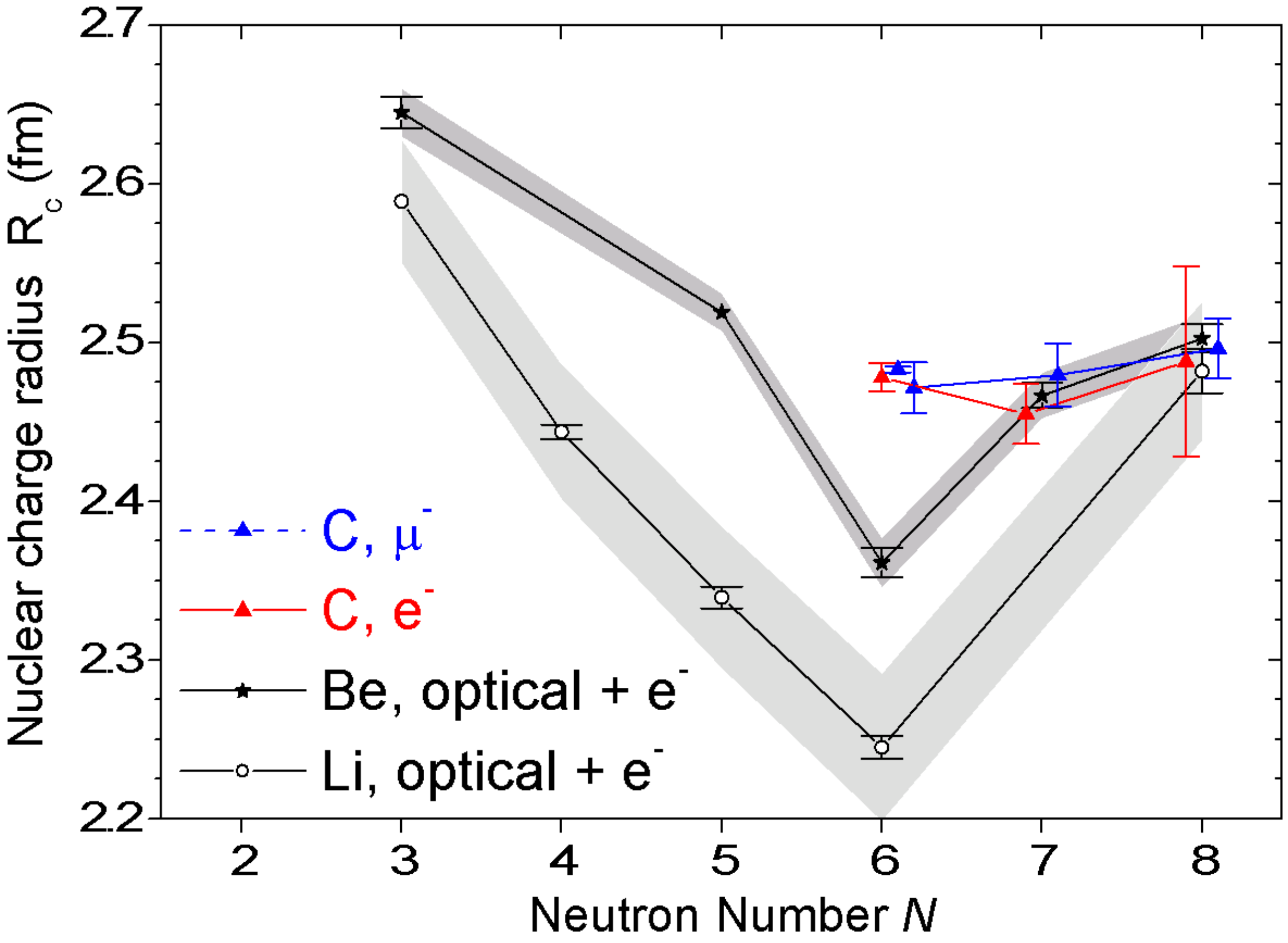
COALA



$$\left.\begin{aligned} \nu_c &= \nu_0 \gamma (1+\beta) \\ \nu_a &= \nu_0 \gamma (1-\beta) \end{aligned}\right\}$$

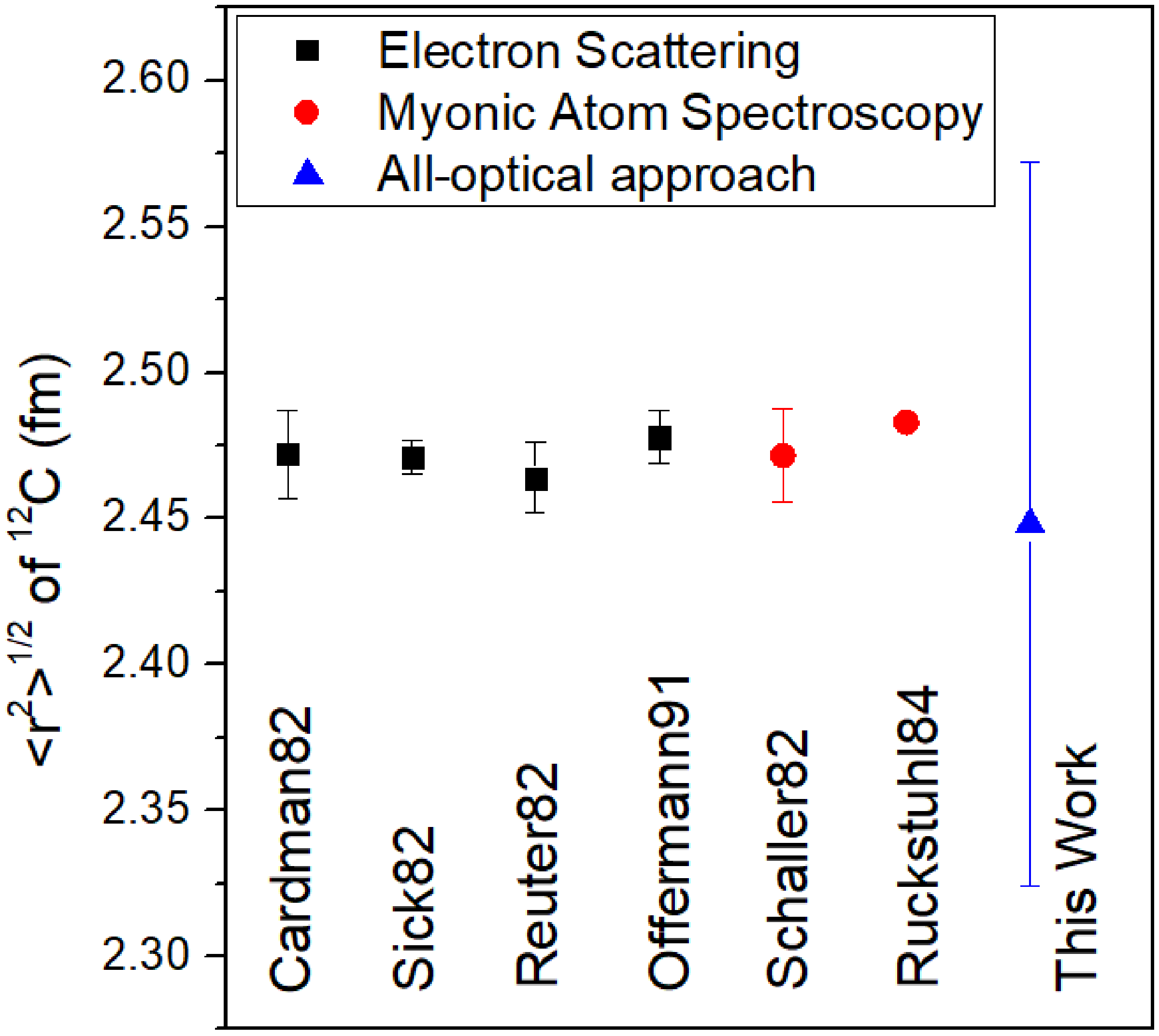
$$\boxed{\nu_c \cdot \nu_a = \nu_0^2 \gamma^2 \cdot (1+\beta)(1-\beta) = \nu_0^2}$$

^{12}C: Proof of Principle & Test of Theory



- Charge Radius of nucleus very well known
  → Test for theory
- Easy to produce in an EBIS
- Transition wavelength of 227 nm
  → Ti:Sa × 4 stabilized to frequency comb
- No hyperfine structure → no hyperfine-induced level mixing
- ^{13}C hyperfine structure requires a more elaborated experiment and theory
- Charge radius of ^{13}C can be improved based on the isotope shift method and compared to direct extraction from transition frequency

Extraction of the Nuclear Charge Radius



Current status:

based on Yerokhin, Patkos & Pachucki, arXiv:2206.14161
[Phys. Rev. A **106**, 022815 (2022)]

- uncertainty dominated by theory
- experimental accuracy sufficient to compete with muonic atom result

→ theory needs to be further improved

Fine structure splitting:

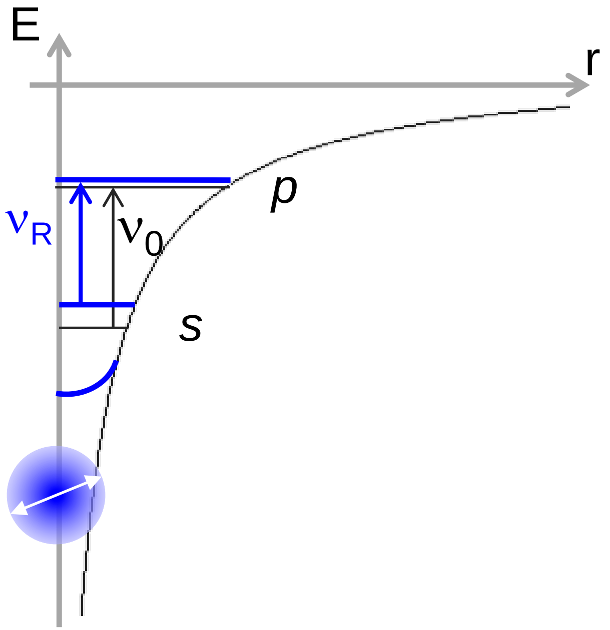
- measurement will provide mα^8-estimation for helium

→ contributes to α-determination

Outlook:

- ^{10,11}B^{3+} and ^{10,11}B^{2+} measurements at KOALA
- ^{8}B measurements at ANL
- Be^{2+} with external ion injection at KOALA
- Online application for carbon isotopes ??

The Idea of All-Optical Absolute Charge Radii



$$\delta\nu_{\mathrm{FS}} = \underbrace{-\frac{Ze^2}{6\varepsilon_0}\Delta\left|\Psi_{\mathrm{e}}(0)\right|^2_{\mathrm{i}\to\mathrm{f}}}_{\substack{\text{Electronic Factor}\\ (\to\text{ Wavefunction})}} \times \langle r_{\mathrm{c}}^2\rangle$$

$$= F_{\mathrm{i}\to\mathrm{f}}\,\langle r_{\mathrm{c}}^2\rangle$$

- Measure **transition frequency ν_R**
- Compare with high precision atomic calculation for a point-like nucleus ν_0
- Difference $\nu_R - \nu_0$ is finite-size effect and **proportional to the ms charge radius**
- So far applied **only for H-like systems**, i.e., H, μH and μHe
- Two-electron system requires elaborate QED calculations, which are now in reach
  Yerokhin, Patkóš & Pachucki, PRA 98, 032503 (2018)
  Patkóš, Yerokhin & Pachucki,, PRA 103, 042809 (2021)
- Laser spectroscopy on ^{12,13}C^{4+} performed.

Address He-like systems:
Li^{+}, Be^{2+}, **B^{3+}**, **C^{4+}**, N^{5+}